



STUDY GUIDE

LESSONS, STRATEGIES, AND DIAGNOSTIC TESTS

ACT[®]

2015 EDITION

- INCLUDES**
- ▶ 800+ Professionally Written Questions
 - ▶ Fully Interactive Lesson Plans
 - ▶ Thousands of Additional Practice Problems Online

LESSONS, STRATEGIES, AND DIAGNOSTIC TESTS

ACT[®]

—

STUDY GUIDE

VARSLTY TUTORS

St. Louis



Eric Forster

Editor-in-Chief

Laura Torp

Managing Editor

Kara Levis

Science Editor

Staff Editors

Andrew Battershill

Andrew Ockert

Joseph Ritchie

Armen Stromquist

Kalea Sebesta

Aubtin Saedi

Database Manager

Kara Levis

Layout

Laura Torp

VARSITY TUTORS

St. Louis

Copyright © 2015 by Varsity Tutors LLC

Published by Varsity Tutors LLC
8000 Maryland, #150
Clayton, MO 63105
www.varsitytutors.com
Email: books@varsitytutors.com

All rights reserved. The text of this publication, or any part thereof, may not be reproduced in any manner whatsoever without written permission from the publisher. This prohibition includes storage in a retrieval system or transmission in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act.

ISBN-13: 978-1-944204-00-6

2015 Edition

LIMIT OF LIABILITY/DISCLAIMER OF WARRANTY:

The publisher and the author make no representations or warranties with respect to the accuracy or completeness of the contents of this work and specifically disclaim all warranties, including without limitation warranties of fitness for a particular purpose. no warranty may be created or extended by sales or promotional materials. the advice and strategies contained herein may not be suitable for every situation. this work is provided with the understanding that the publisher is not engaged in rendering legal, accounting, or other professional services. if professional assistance is required, the services of a competent, professional person should be sought. neither the publisher nor the author shall be liable for damages arising herefrom.

Table of Contents

How to Use This Book	1
The ACT: An Introduction	2
The ACT English Section	4
Chapter Outline	6
Review	7
Review: Parts of Speech	8
Review: Subject and Predicate	11
Review: Independent and Dependent Clauses	12
Identifying Grammatical Errors	13
Usage Errors	14
Comparative and Superlative Errors	15
Preposition Errors	17
Pronoun Usage Errors	20
Verb Usage Errors	24
Punctuation Errors	30
Possessive Apostrophes	31
Apostrophes for Contractions	33
Quotation Mark Errors	36
Colon Errors	40
Semicolon Errors	42
End-of-Sentence Punctuation Errors	44
Hyphen and Dash Errors	45
Comma Errors	47
Commas in Lists	48
Comma Splices	50
Appositives	52
Commas for Phrases and Clauses	54
Agreement Errors	56
Modifier-Word Modified Agreement Errors	56
Pronoun-Antecedent Agreement Errors	57
Noun-Noun Agreement Errors	57
Subject-Verb Agreement Errors	57
Phrase and Clause Errors	61
Conjunction Errors	62
Modifier Placement Errors	66
Parallel Structure Errors	71
Sentence Completeness Errors	74
Sentence Fragment Errors	75
Sentence Combination Errors	77
Making and Analyzing Editing Decisions	78
Editing in Context vs. Error Identification	79
Direct Editing Decisions: Which Option is Best?	81
Adding Content	82
Choosing the Best Sentence for a Spot	82
Choosing the Best Spot for a Sentence	88
Rearranging Content	93
Moving Sentences	93
Paragraph Breaks	95
Rephrasing for a Particular Effect	97
To Edit or Not—and Why	104



To Add Content or Not—and Why	104
To Remove Content or Not—and Why	107
To Rephrase Content or Not—and Why	109
Identifying the Function of Passage Elements	116
Identifying the Effects of Editing	123
Identifying the Effects of Adding Content	124
Identifying the Effects of Removing Content	127
Identifying the Effects of Rephrasing Content	130
Analyzing Passage Effectiveness	135
 The ACT Reading Section	 137
Chapter Outline	139
Review: Close Reading	140
What is Said	142
Main Idea	143
Finding the Main Idea of a Selection from a Passage	144
Finding the Main Idea of an Entire Passage	145
Analyzing Argument	151
Questions About the Author’s Argument	152
Questions About Other People’s Arguments	153
Questions About the Structure of an Argument	154
Identifying Details	158
Identifying Details in a Specific Paragraph	159
Identifying Details in a Full Passage	160
How It’s Said	165
Determining Context-Based Meaning	166
Context-Based Meaning of Unfamiliar Words	167
Words with Multiple Meanings	168
Context-Dependent Meaning of Phrases and Clauses	171
Analyzing Tone	176
Tone of a Sentence or Paragraph	176
Tone of an Entire Passage	179
Words That Describe Tone	181
Point of View	184
Identifying Author and Audience	185
Identifying the Author’s Directly Stated Views	186
Inferring the Author’s Views on a Topic	188
Sequence	195
Sequence of Passage Text	195
Sequence of Events	196
When Passage Text Sequence $\neq$ Passage Event Sequence	197
Identifying the Reasoning Behind Sequence	199
Inferring the Previous Topic	201
Inferring the Next Topic	202
Why It’s Said	207
Purpose of Lines and Passage	208
Questions About the Purpose of a Single Word	209
Questions About the Purpose of a Sentence	210
Questions About a General Part of the Passage	210
Questions About the Purpose of the Passage as a Whole	211
Purpose of Dialogue	213
Engaging with What’s Said	218



Comparing and Contrasting	219
Identifying Comparisons and Contrasts	220
Comparing and Contrasting Two Textual Features	221
Comparing More than Two Textual Features	223
Comparing and Contrasting Passage Structures	224
Identifying Cause and Effect	229
Identifying Cause	230
Identifying Effect	231
Identifying Cause and Effect Simultaneously	232
Summarizing and Paraphrasing	236
Paraphrasing an Excerpt	237
Summarizing an Excerpt	238
Summarizing a Passage	239
Generalizations, Inferences, and Predictions	243
Making Inferences	244
Making Predictions	245
Making Generalizations	246
Comparing What's Said: Paired Passages	250
Comparing and Contrasting Ideas in Paired Passages	251
Identifying the Main Idea of Each Passage	251
Comparing the Main Ideas of Paired Passages	252
Comparing and Contrasting Arguments in Paired Passages	255
Comparing Arguments: Where Do the Authors Agree?	257
Contrasting Arguments: Where Do the Authors Disagree?	258
Hypothetical Changes to Arguments	259
Making Inferences Based on Newly Introduced Information	260
Comparing and Contrasting Language and Rhetoric in Paired Passages	261
Choosing Which Passage Best Fits a Description of Style	263
Direct Comparisons of Style Descriptions	264
Direct Comparisons of Textual Features	265
Making Inferences About Passage Genre	266
Making Inferences About the Authors	266
Multi-Passage Critical Thinking: Conclusions, Inferences, and Predictions	267
Drawing Inferences and Conclusions from Paired Passages	267
Making Predictions Based on Paired Passages	270
 The ACT Science Section	 279
Chapter Outline	281
Review	282
Review: Experiments & Procedures	283
Control Groups	283
Independent Variable	283
Dependent Variable	284
Sample Passage: Experiments and Procedures	285
Review: Graphs	288
Axes and Labels	288
Dual Axes	289
Linear Relationships	290
Exponential Relationships	291
Graphing an Equation	292
Determining an Equation for a Graph	293
Practice Questions: Graphs	294
Review: Math, Science, and Equations	298



Reading an Equation	298
Applying an Equation	298
Units	300
Terminology and Jargon	302
Data Interpretation	303
Identifying a Point from a Table	304
Identifying a Trend from a Table	305
Identifying a Point from a Graph	306
Identifying a Trend from a Graph	307
Anticipating Placement of New Data Points	308
Practice Passage: Data Interpretation	309
Data Interpretation: Further Applications	313
Experimental Setups and Results	314
Distinguishing Experimental Purpose	315
Interpreting Textual Setups	318
Interpreting Diagrammed Setups	319
Describing Results	323
Visualizing Results	325
Applying Results to New Scenarios	328
Experimental Integrity and Flaws	330
Experimental Setups and Results: Further Applications	332
Evaluating Conflicting Theories	333
Given Background	334
Identifying Theses and Arguments	336
Identifying Agreement	338
Comparing Arguments	340
Applying Arguments	342
Integrating New Information	344
Evaluating Conflicting Theories: Further Applications	346
The ACT Mathematics Section	347
Chapter Outline	349
Arithmetic	352
Number Theory and Basic Operations	353
Classifying Numbers	353
Order of Operations	354
Absolute Value	356
Sets and Sequences	357
Simplifying Fractions	362
Mixed Fractions	362
Fractions and Decimals	362
Operations with Fractions	363
Compound and Complex Fractions	365
Ratios and Proportions	366
Decimals	367
Combinations and Permutations	368
Combinations	368
Permutations	369
Squaring, Exponents, and Square Roots	370
Perfect Squares	370
Simplifying Square Roots	371
Equations with Square Roots	372
Squaring and Square Roots with Fractions	375



Squaring and Square Roots with Decimals	377
Data Analysis	379
Probability	380
Probability of Single Events	380
Probability of Multiple Events: “And” and “Or”	383
Dependent vs. Independent Events	383
Mean	388
Basic Statistics	388
Median	391
Mode	393
Range	395
Basic Statistics Review	397
Algebra	399
Rules of Exponents	400
Monomials	403
Adding and Subtracting Monomials	403
Variables and Algebraic Notation	403
Multiplying and Dividing Monomials	404
Factoring Monomials	406
Practice Problems: Monomials	408
Binomials	412
Adding and Subtracting Binomials	412
Multiplying and Dividing Binomials	415
Factoring Binomials	416
FOIL	417
Practice Problems: Binomials	419
Adding Polynomials	425
Polynomials	425
Subtracting Polynomials	426
Multiplying Polynomials	426
Dividing Polynomials	428
Factoring Polynomials	429
Practice Problems: Polynomials	431
Equations and Inequalities	436
Linear Equations	436
Linear Inequalities	437
Absolute Value Equations	437
Direct and Inverse Variation	438
Rational Equations	440
Radical Equations	443
Quadratic Equations	445
Function Notation	448
Inequalities	451
Rules of Inequalities	452
Linear Inequalities	453
Non-Linear Inequalities	455
Solving Systems of Equations	457
Solving Using Substitution	457
Solving Using Elimination	458
Geometry	460
Plane Geometry	461
Squares	464



Area of a Square	464
Perimeter of a Square	465
Diagonal of a Square	465
Rectangles	467
Rectangular Area	467
Rectangular Diagonals	467
Triangles	469
Area of a Triangle	469
Equilateral Triangles	470
Isosceles Triangles	472
Circles	473
Bringing It All Together	474
Complex Shapes	476
Solid Geometry	479
Cubes	479
Cylinders	481
Spheres	482
Angles	484
Triangular Angles	484
Circular Angles	486
Clock Math	487
Coordinate Geometry	488
x- and y-Intercept	489
Lines	497
Distance Formula	497
Midpoint Formula	500
Parallel, Perpendicular, and Tangent Lines	504
Graphing Points	506
Graphing	506
Graphing Lines	509
Graphing Quadratic Functions	518
Graphing Exponential Functions	527
Graphing Inequalities	535
Trigonometry	544
SOHCAHTOA	545
Trigonometric Identities	547
Reciprocal Identities (Trigonometric Ratios)	547
Quotient Identities	549
Pythagorean Identities	549
Conversions Between Radians and Degrees	550
Unit Circle	552
Trigonometric Function Graphs	554
Sine Graphs	554
Cosine Graphs	555
Tangent Graphs	555
Amplitude, Period, Frequency, and Phase Shifts	556
Solutions for Triangles	557
Practice Problems: Trigonometry	561
Full-Length ACT Diagnostic Test	569
English Test	570
Mathematics Test	582
Reading Test	590



Science Test	599
English Test: Answer Key	613
Mathematics Test: Answer Key	636
Reading Test: Answer Key	663
Science Test: Answer Key	678
Score Conversion Tables	691
Calculating Your Composite Score	693



How to Use This Book

Welcome to Varsity Tutors' *2015 ACT Study Guide: Lessons, Strategies, and Diagnostic Tests*. We hope that you'll find the lessons contained herein useful for improving your scores on the ACT, which can help you get into the college or university of your choice.

While you will find some advice for test-taking in this book, its primary focus is skills-based. In other words, if you can learn how to do the math or how to read passages effectively, just to name two specific examples, you'll do better on the test. Being able to apply your skills in a timely fashion is the main consideration in approaching the ACT as a test, but the skills you learn and use are broadly applicable, and will help you succeed in college. What that means for you is that we're not trying to sell you on a proprietary "method" or some made-up approach. This book is designed to teach you the skills you don't know and remind you of the skills you have, but may have forgotten.

This is an e-book, and the version of this book you are reading is a PDF, which is a feature-rich format. It may be available in other e-formats, like as an iBook or on Kindle, so be sure to check for availability on your preferred device.

Software Note: We highly recommend that you use Adobe Reader to view this PDF so that all its interactive features work properly. A free version of this software is available for Mac and PC here:

<https://get.adobe.com/reader/>

In this book, you will find things that are familiar to you from other books, both of the physical and the electronic variety, but you may come across some features you haven't seen before. We're striving to push the electronic format in ways that are useful to you, our reader, while maintaining a recognizable and clean layout.

The first thing you might notice is that this book is heavily linked to itself. If you browse the Table of Contents, you'll see that there are many active links that can get you to other parts of this book quickly and accurately. These links are not always perfectly obvious, so you should always be hovering your mouse over anything that might interest you, in order to see if you can click on it.

Try clicking this link: [This is an example of a link. This link takes you back to the Table of Contents.](#)

As you go through this book, another thing you may notice are various blue boxes that prompt you to see the solutions to problems or give you a walkthrough of a reading passage. When you hover over these boxes, you'll see more text or graphics appear on the page in front of you. If you click on these buttons, you will make the text or graphics persistent, so you can scroll around the page as you will. If you want the text or graphics to disappear again, just click the button.

Sample Button

Incorrect answers are displayed with an "x" [X] next to them, so that you can immediately comprehend that they are answers to be avoided. Correct answers will appear with an arrow [➡] next to them.

It's a big book, and there's a lot to cover: explore and have fun. We certainly had fun writing this for you!



The ACT: An Introduction

What is the ACT? What does ACT stand for?

The ACT, or American College Test, is a multiple-choice standardized test used by colleges and universities in the United States for the admission and placement of incoming students. Students typically take the ACT in high school as part of their college application process; however, students not yet in high school—as well as adults—can also take the ACT. Along with the SAT, it is one of the two main collegiate admissions tests, and each year it is taken by more than 1.6 million prospective college students.

What's the difference between the ACT and the SAT?

While both the ACT and the SAT are college entrance exams, the ACT is an achievement test, which tests what students have already learned. It consists of either four or five sections, depending on whether one takes the ACT or the ACT Plus Writing. In contrast, the SAT is an aptitude test, which aims to predict how well a student will do in college or university by measuring his or her reasoning and verbal abilities in three sections: Critical Reasoning; Mathematics; and a non-optional Writing section, (which actually tests grammar and expression).

The ACT and SAT also differ significantly in the way that they are scored. Your score on the ACT is determined by the number of questions that you get correct, so there is no penalty for guessing—it can only ever increase your score. The SAT, in contrast, determines your score by the number of questions which you answer incorrectly. So, guessing on the SAT might not be the best idea, because you could decrease your score if you guess incorrectly.

What does the ACT test? What sections are on the ACT?

The ACT is designed to test how well a student has learned material that has been covered in the standard high school curriculum. Students may elect to take either the ACT or the ACT Plus Writing. The ACT consists of an English section, a Math section, a Reading section, and a Science section. The ACT Plus Writing consists of the four aforementioned sections as well as an essay-based Writing test.

Are the ACT's sections always given in the same order?

Yes, the ACT always gives its sections in the same order. The English section is always given first, followed by the Math section, the Reading section, and the Science section. On the ACT Plus Writing, the Writing section is always given last.

Tip:

Some colleges require the SAT or the ACT, while others accept both. Research the schools to which you want to apply, and make sure you're studying for the correct test!



Tip

You can consume a small snack or a bottle of water during your break.

Tip

Certain states may incorporate the ACT or ACT Plus Writing into their mandatory statewide educational assessments, which students do not pay to take.

Are students given breaks during the ACT?

In the ACT (without the writing section), students are given one break after completing the English and Math sections, before completing the Reading and Science sections. The ACT Plus Writing includes a second break after the Science section, before the Writing section.

How long is the ACT?

Altogether, the ACT (without the Writing section) takes about four hours and fifteen minutes to complete. The ACT Plus Writing takes about five hours to complete. These times are calculated to include the break students receive after completing the English and Math sections, prior to the Reading and Science sections, along with the second break, between the Science and Writing sections for the ACT Plus Writing.

How much does it cost to take the ACT?

It currently costs \$36.50 to register for the ACT, and \$52.50 to register for the ACT Plus Writing. These prices include score transmission to up to four colleges or universities. Fee waivers are available for students in their junior or senior year of high school in the United States who demonstrate financial need.



The ACT English Section

At a Glance

Tests: Knowledge of English grammar as well as writing features like tone, style, and word choice

Timing: $\frac{45 \text{ Minutes}}{75 \text{ Questions}} = \frac{36 \text{ Seconds}}{\text{Question}}$

Question Types:

- 1) Error identification in underlined portions of a presented passage;
- 2) Questions testing passage-level editing decisions like inserting, rearranging, or omitting content

If the idea of spotting rogue commas and dangling participles makes your blood run cold, the ACT English section likely presents a major source of apprehension as you approach the conclusion of your high school career. Made up of seventy-five multiple-choice questions to be answered in forty-five minutes, even students who have succeeded in their English and writing courses can find themselves stumped. In preparing for the ACT English section, it is important to recognize that the test employs two general types of questions and to ready yourself for each one.

In most questions, the ACT English section tests students' abilities to identify and fix a wide variety of common grammatical errors introduced by a specified word or as part of an indicated phrase or sentence. Where do commas go in lists? How do you format indirect quotations? Which conjunction makes logical sense? Reviewing the rules of standard English grammar can help you prepare to answer these questions, and reading appropriately difficult material of any genre can reinforce what you learn by presenting you with examples of

grammatical concepts in action. Don't forget: you don't need to be able to name a particular type of error to answer an ACT English question correctly—you just have to be able to pick out which answer choice fixes it!

Just as it is important to bolster your grammatical knowledge when preparing for the ACT English section, it is also important to recognize that not all ACT English questions focus on nitty-gritty details. The section also asks students to act as editors in a larger sense, proposing changes to the passage at hand and asking them to select the most effective option. Which transition works best? Is this phrase adding anything to the argument? Where should this sentence be placed in the first paragraph? All of these are questions you may find yourself asking on the ACT English section. These questions often require you to consider the context, tone, and main idea of the passage as a whole. Practicing the description of the features of articles or passages can help you improve in your ability to understand exactly what goes into determining a passage's character."

The ACT English section can seem particularly difficult to review for because of the many different kinds of grammatical errors that can appear on the test, as well as the variety of forms that those grammatical errors can take. Varsity Tutors' free ACT English resources can help you create a detailed study plan by helping you to recognize and work on the concepts that you find to be most tricky.

Concepts Tested

- Complete vs. Incomplete Sentences
- Correlative and Subordinating Conjunctions
- Dangling Modifiers
- Parallel Structure Errors
- Comparatives and Superlatives
- Comma Usage
- Errors in the Use of Colons, Semicolons, and Other Types of Punctuation
- Agreement
- Verb Tense
- Verb Mood
- Pronoun Case (e.g. "Who" vs. "Whom")
- Active vs. Passive Voice
- Conjunction Logic
- Ambiguity
- Redundancy
- Word Choice
- Where Best to Insert New Content
- How to Rearrange Content Most Clearly
- Effects of Removing Content
- ... and More!



Reviewing for the different types of questions featured on the ACT English section has benefits other than preparing you for your test date. Virtually every college class requires some form of written assignments, whether in the form of essays or lab reports, and tightening your grasp on tricky grammatical concepts can help you avoid being ambiguous, or worse, misunderstood by your readers. Furthermore, looking beyond the collegiate environment, written communication forms a part of most jobs, meaning that the ability to write clearly and precisely can help you present yourself as an attractive candidate when applying to the job of your dreams. After all, in many circumstances, your first impression is conveyed through a resumé and cover letter, written documents that can suffer from all of the grammatical errors the ACT English section challenges you to recognize and correct. So, even if the ACT English section seems challenging, know that with careful preparation, you can be ready to feel confident on test day, and that the skills you develop while reviewing will serve you well in your future endeavors.

ACT English Sample Question

Select the answer choice that makes the passage correct in standard modern English.

During the final months of 2007, the prices of basic grains nearly doubled in Northern Africa, Latin America, and much of Asia, the high prices¹ caused a global food crisis.

1. **A.** NO CHANGE
B. of Asia. The
C. of Asia, the quite
D. for Asia. The

See Answer and Explanation



How to Use This Book

Each section in this chapter investigates one ACT English topic in-depth and walks you step-by-step through relevant practice questions. Each section concludes with a practice quiz so that you can test your skills before trying some of the full ACT diagnostic tests at the end of the book.

Chapter Outline

[Review](#)

[Review: Parts of Speech](#)

[Review: Subject and Predicate](#)

[Review: Independent and Dependent Clauses](#)

[Identifying Grammatical Errors](#)

[Usage Errors](#)

[Comparative and Superlative Errors](#)

[Preposition Errors](#)

[Pronoun Usage Errors](#)

[Verb Usage Errors](#)

[Punctuation Errors](#)

[Possessive Apostrophes](#)

[Apostrophes for Contractions](#)

[Colon Errors](#)

[Semicolon Errors](#)

[Quotation Mark Errors](#)

[End-of-Sentence Punctuation Errors](#)

[Hyphen and Dash Errors](#)

[Comma Errors](#)

[Commas in Lists](#)

[Comma Splices](#)

[Appositives](#)

[Commas for Dependent Clauses](#)

[Agreement Errors](#)

[Phrase- and Clause-level Errors](#)

[Conjunction Errors](#)

[Modifier Placement Errors](#)

[Parallel Structure Errors](#)

[Sentence Completeness Errors](#)

[Sentence Fragment Errors](#)

[Sentence Combination Errors](#)

[Making and Analyzing Effective Editing Decisions](#)

[Introduction: Editing in Context vs. Error Identification](#)

[Direct Editing Decisions: Which Option is Best?](#)

[Adding Content](#)

[Rearranging Content](#)

[Rephrasing for a Particular Effect](#)

[To Edit Content or Not—and Why](#)

[To Add Content or Not—and Why](#)

[To Remove Content or Not—and Why](#)

[To Rephrase or Not—and Why](#)

[Identifying the Function of Passage Elements](#)

[Identifying the Effects of Editing](#)

[Analyzing Passage Purpose](#)



Review

How do you tell an adjective from an adverb? What's a participle? What makes a sentence "complex" or "compound?" Skills like these, while not directly tested on the ACT English section, are still necessary for success on the test. They provide the groundwork necessary for you to understand the specific grammatical principles that *are* tested. A quick but thorough review of basic but crucial concepts can help you get off to a good start as you begin reviewing.

Section Summary

[Parts of Speech](#)

- There are eight parts of speech: nouns, pronouns, verbs, adjectives, adverbs, prepositions, conjunctions, articles, and interjections. Each is used in a particular way.
- Pronouns in English have number, case, and gender.
- Verbs in English have mood, tense, and voice.

[Subject and Predicate](#)

- The subject of a sentence is the noun phrase that performs the verb in the sentence.
- The predicate of a sentence is the verb phrase that represents the action the subject performs.

[Independent and Dependent Clauses](#)

- Dependent clauses contain either a subject or a predicate, but not both. They cannot stand on their own as complete sentences. They attach to independent clauses.
- Independent clauses contain both a subject and a predicate and can stand on their own as complete sentences.
- A "complex" sentence contains an independent clause and at least one dependent clause.
- A "compound" sentence contains two or more independent clauses.



Review: Parts of Speech

The ACT English section tests your ability to recognize errors and make editing decisions; it never asks you to identify what part of speech a particular word is. Despite this, you're going to have to know words' parts of speech in order to understand how they are interacting grammatically, so a brief review can be helpful.

There are eight parts of speech—

Nouns, pronouns, verbs, adjectives, adverbs, prepositions, conjunctions, articles, and interjections.

Nouns

Nouns are the concrete or abstract “things” in language. Nouns directly represent people, places, things, and abstract concepts like ideas and emotions.

Examples: “Bill,” “San Diego,” “trees,” “improvement,” “embarrassment”

Proper nouns, including names and titles, are always capitalized.

Examples: “President Washington,” “*Alice in Wonderland*,” “the Statue of Liberty”

Pronouns

If you repeat someone's name every time you referred to him or her in a typical paragraph, it can get pretty tiring:

✗ *Sarah went to the grocery store, where Sarah found a good deal on sliced bread. Sarah decided to go home and make toast in Sarah's new toaster. Sarah's new toaster improved greatly upon Sarah's old toaster, which always set off Sarah's fire alarm when Sarah tried to make Sarah's toast.*

Exhausting to read, right? Pronouns solve this problem. Pronouns are placeholders for nouns that represent them so that we don't have to repeat a noun (e.g. “Sarah”) every time we want to refer to that noun.

The meaning of a particular pronoun is determined by the context in which it is used. (After all, pronouns would be a lot less useful if “it” only meant one thing!) A pronoun only refers to one noun at a time—the last noun used in a sentence that agrees with the pronoun. The noun to which a pronoun refers is called its **antecedent**. In the above paragraph, “Sarah” is the antecedent of both the pronouns “she” and “her.”

In modern English, pronouns can be subjective (“I”), objective (“me”), or possessive (“my”) in case, as well as singular (“him”) or plural (“them”) and gendered (“she” and “him”) or neuter (“it”).

Case Studies

Having a hard time telling subjective and objective pronouns apart, as in the common head-scratcher of deciding whether “who” or “whom” is correct in a sentence? [Click here](#) to check out our section on pronoun case errors to get a jump on this error and be ready for it on the exam.

Verbs

Verbs convey action or being. Action verbs convey actions, and linking verbs (also called “helping verbs”) work along action verbs to form various tenses and convey being.

Examples of Action Verbs: “run,” “wonder,” “flap,” “focus”

Examples of Linking Verbs: “am,” “is,” “are,” “was,” “were,” “be,” “being,” “been”



Verbs have three main characteristics: tense, mood, and voice.

Tense

The tense of a verb tells you when that verb's action is occurring, has occurred, or will occur.

On the ACT English section, you will never be required to name a tense, merely to identify errors. Tense errors primarily focus on situations in which a verb's tense does not agree with or make sense in the context of the rest of the sentence.

Tenses in Action

Present:	"I write a letter."
Present Progressive:	"I am writing a letter."
Simple Past:	"I wrote a letter."
Past Progressive:	"I was writing a letter."
Pluperfect:	"I had written a letter."
Future:	"I will write a letter."
Future Progressive:	"I will be writing a letter."
Future Perfect:	"I will have written a letter."

Mood

The mood of a verb tells you what the verb is doing:

- **Indicative** verbs tell you about the state of something
"I ate the cookies."
- **Imperative** verbs form commands
"Eat the cookies!"
- **Subjunctive** verbs are perhaps the trickiest for most people to grasp; the subjunctive mood describes a state that is recognized as not being reality—a hypothetical situation.
"If I were to eat those cookies, I bet they'd taste really good!"
- **Infinitive** verbs convey an abstract notion of an action and are always formed with "to"
"I want to eat the cookies."

Voice

Verbs (and the sentences that contain them) have two options for voice: active voice or passive voice. Which of these a given verb demonstrates is wrapped up in the concept of **transitivity**. You can think of transitivity as involving an actor and something or someone being acted upon. The order in which the actor and the acted-upon appear in a sentence determine its voice and the voice

Active Voice: Subject-Verb-Object (SVO)

In active voice sentences, the actor is the subject of the verb and the acted-upon is the object.

"The cat chases the mouse."

Subject Verb Object

"The man ate the sandwich."

Subject Verb Object

"The coach will encourage her team."

Subject Verb Object

Passive Voice: Object-Verb-Subject (OVS)

In passive voice sentences, the acted-upon is the subject of the verb and the actor is the object.

"The mouse is being chased by the cat."

Object Verb Subject

"The sandwich was eaten by the man."

Object Verb Subject

"The coach's team will be encouraged by her."

Object Verb Subject

of its verb(s).

While the passive voice certainly has its uses and is not grammatically incorrect, ACT English section can include problems where the correct answer is to rearrange the sentence to avoid the use of the passive voice. If a problem comes down to the decision of picking between two options that only differ in voice, pick the active voice option! The test-writers prefer it.



Special Case: Verbals

Certain words can act as nouns or verbs depending on the context, and identifying these can present a challenge in that they might fall into a special case of word somewhere between nouns and verbs: verbals. Verbals can be described as certain types of words “acting like nouns.”

Gerunds

Gerunds are verbs, usually ending in “-ing,” that act as nouns in a given sentence.

Painting is fun, but my favorite hobby is knitting.

Participles

Participles are verbs that act as adjectives in a given sentence.

The laughing campers overcooked the burnt marshmallow over the roaring fire.

Infinitives

Infinitives may seem familiar, but they are technically verbals.

To get to the point, “To be or not to be” is a line used to open a very famous monologue in *Hamlet*.

Adjectives

Adjectives are descriptors—they provide descriptive details about nouns or other adjectives.

The bright blue songbird flew into its stable nest in the tall strong oak tree.

Multiple words can combine to form compound adjectives, which are hyphenated:

The twenty-year-old architect’s self-designed studio is in a six-story building.

Adverbs

Adverbs, like adjectives, are descriptors, but adverbs describe verbs or adjectives.

The very speedy car zoomed by at a blindingly fast pace, and the onlookers cheered loudly.

Prepositions

Prepositions convey the temporal or spatial relationship between two objects, providing information about what is happening where.

The squirrel ran over the porch, under the chair, around the fence, through the tire swing, up the tree, across the branch, and into its nest, from where it could keep an eye on the dog.

Quick Tip

Can’t tell if a word is a preposition? Prepositions are anything you can be in relation to a cloud. In a cloud, above a cloud, beside a cloud, near a cloud, beneath a cloud . . .

Conjunctions

Conjunctions link phrases, clauses, and sentences together.

Coordinating conjunctions are single words that do this:

Cakes and cookies usually need some sugar or sweetener, but I forgot that when cooking.

Subordinate conjunctions are pairs of words that always work together to do this. You can’t use one without the other or your sentence will be grammatically incorrect! Three common subordinate conjunctions are “either . . . or,” “neither . . . nor,” “if . . . then,” and “not only . . . but also.”

Diners can enjoy either tea or coffee after their meals, but we offer neither milk nor sugar for those drinks; however, we offer not only regular tea and coffee, but also decaf.

Articles

Articles are words like “a,” “an,” and “the” that convey varying levels of specificity. Technically, articles can be categorized as adjectives, but the ACT English section will never ask you about that!

Interjections

Interjections convey sudden bursts of emotion and usually mimic things people say.

Examples: “wow,” “whoa,” “um”

“Wow, that was way easier than I thought it would be!”



Review: Subject and Predicate

All sentences that are not commands need to contain both a subject and a verb. Taken with their surrounding words and phrases, these two parts of the sentence are called the subject and the predicate. Subjects contain a noun or word acting as a noun, and predicates contain a verb. While the nouns in the subjects of sentences are often the actors that perform the verb, this does not hold true for all sentences. ([See Active vs. Passive Voice](#))

The following are examples of complete sentences:

The **ocean cycles** between high tide and low tide twice a day.

Subject

Predicate

Some sea **creatures**, such as mollusks and starfish, **are visible** at the water's edge during low tide.

Subject

Predicate

I **think** that sea creatures are very interesting.

Subject

Predicate

The bolded words in the above sentences are the **simple subjects** and **simple predicates**, or the single words or phrases that form the subject's main noun(s) and the predicate's main verb(s).

The following are examples of incomplete sentences missing necessary elements:

✗ The interesting story of how sharks evolved to use cartilage instead of bone.

Subject

✗ Snorkeling in the Great Barrier Reef.

Subject

✗ Enjoyed my class's field trip to the science museum very much, especially the exhibit on fish.

Predicate

Questions may look like they do not follow this rule, but they do: rearranging them can demonstrate that they contain all the required elements of a complete sentence, but are just inverted.

Have you been to the ocean? --> You have been to the ocean.

What is your favorite sea creature? --> Your favorite sea creature is what.

Who is in charge at the local aquarium? --> Who is in charge at the local aquarium.

Note that the rearranged sentences are grammatically complete but don't always make sense, especially when the sentence includes words like "who," "what," "when," "where," or "why."

Commands may seem like they also form an important exception to this rule, but you can think of a command as having an implied subject—"you."

(You) Take out the trash.

(You) Explain to me what's going on.

(You) Please make dinner reservations for next Friday.

Some sentences' subjects contain more than one noun, and some sentences' predicates contain more than one verb. We call these "compound subjects" and "compound predicates," respectively. Some sentences contain both a compound subject and a compound predicate!

Blue whales and bottlenose dolphins are classified as mammals, not fish, and live in the ocean.

Subject 1

Subject 2

Verb 1

Verb 2



Review: Independent and Dependent Clauses

To be grammatically complete, sentences that are not commands must be what are called **independent clauses**. You can think of “independent clause” as shorthand for “can stand on its own as a complete sentence.” **Dependent clauses**, on the other hand, add on to independent clauses, but cannot stand as complete sentences on their own. Every independent clause needs a subject and a predicate, and every dependent clause needs one or the other, but not both.

A dependent clause sitting by itself is not grammatically correct; it structurally “depends” on being linked to an independent clause.

X Which are poisonous and found in tropical waters.

Dependent Clause

X Because they are part of the phylum Cnidaria.

Dependent Clause

X If you see a jellyfish in the wild.

Dependent Clause

Complex sentences include at least one dependent clause.

My favorite sea creature is the jellyfish, which is a type of invertebrate organism.

Independent Clause

Dependent Clause

Invertebrate organisms are so named because they don't have spines.

Independent Clause

Dependent Clause

If you want to learn about jellyfish, I recommend going to the aquarium, as you can see some there.

Dependent Clause

Independent Clause

Dependent Clause

Compound sentences include multiple independent clauses properly linked together.

Invertebrate organisms don't have spines; however, that doesn't mean that they scare easily.

Independent Clause

Independent Clause

Some invertebrates protect themselves by living in shells, and they move to larger shells as they grow.

Independent Clause

Independent Clause

Some people think invertebrates are boring, but I think they're fascinating.

Independent Clause

Independent Clause



Identifying Grammatical Errors

Perhaps the signature question type on the ACT English section, many problems you'll encounter on this part of the test will ask you to identify grammatical errors. As an added wrinkle, you'll always be presented with the option "NO ERROR" to select in the event that the indicated portion of the passage is error-free. Thus, these questions test your ability to discern a wide variety of errors and edit selections so that the indicated prose is correct or flows most smoothly. Reviewing common error types and often-missed rules of English grammar can help you prepare to correct your way to correct answers on test day!

Section Summary

[Usage Errors](#)

- This section discusses errors in the usage of a single term, like using the wrong preposition or a subjective pronoun where an objective pronoun should be used

[Punctuation Errors](#)

- This section details a wide variety of punctuation errors, paying special attention to the myriad ways in which commas can create errors when used incorrectly.

[Agreement Errors](#)

- This section goes over errors created when one word doesn't agree with another word or the rest of the sentence.

[Phrase- and Clause-Level Errors](#)

- This section walks you through errors in conjunction usage, modifier placement, and parallel structure.

[Sentence Completeness Errors](#)

- This section teaches you to identify and fix sentence fragments and run-on sentences.



Usage Errors

Sometimes, a single word can contain a sentence's error due to the way it clashes with the rest of the sentence's context. Familiarizing yourself with common usage errors and training yourself to spot these rogue terms quickly can help save you time on test day as you encounter errors with which you are familiar.

Section Summary

[Comparative and Superlative Errors](#)

- Comparative adjectives are used to compare two things or people. Comparative adjectives often end in the suffix “-er.”
- Superlative adjectives are used to compare three or more things or people. Superlative adjectives often end in the suffix “-est.”
- Certain adjectives like “good” and “bad” have irregular comparative and superlative forms, e.g. “good—better—best” and “bad—worse—worst.”

[Pronoun Usage Errors](#)

[Subjective and Objective Pronoun Errors](#)

- Subjective pronouns are used as the subjects of clauses and sentences.
- Objective pronouns are used as the objects of verbs and prepositions.

[Possessive Pronoun Errors](#)

- A possessive pronoun is used to show the ownership of a pronoun's antecedent over the noun that follows it.

[Gendered Pronoun Errors](#)

- Pronouns in standard modern English can have one of three genders: masculine (e.g. “he,” “him”), feminine (e.g. “she,” “her”), or neuter (“it”).
- Plural pronouns like “they” and “us” don't have a gender grammatically associated with them, but can represent groups of objects, males, females, or groups of mixed gender depending on the context in which they're used.



Comparative and Superlative Errors

Comparatives: When comparing two things or people, the comparative form of an adjective is used. Many comparative adjectives end in the suffix “-er,” but not all of them do.

Norbert prefers the larger sundae.

Comparison: The sundae Norbert prefers vs. another, smaller, implicit option

I can run faster than she can.

Comparison: The speed at which I can run vs. the speed at which she can run

Is it better to overprepare or underprepare when packing for a camping trip?

Comparison: The relative value of overpreparing vs. the relative value of underpreparing

Gemma is more excited about going on vacation than doing her last homework of the year.

Comparison: Gemma’s excitement about going on vacation vs. her excitement about doing her last homework of the year.

In this last sentence, notice the use of the phrasing “more _____ . . . than.” When making a comparison using the word “more,” you need to follow it with the word “than” between the two items being compared.

Superlatives: When comparing three or more things or people, the superlative form of an adjective is used. Many superlative adjectives end in the suffix “-est,” but not all of them do.

I ordered the biggest pizza they had, but it was still gone minutes after I served it at the party.

Comparison: The pizza I ordered vs. all of the other pizza sizes offered where I ordered it

Those were the hungriest guests I’d ever invited!

Comparison: The hunger of my guests vs. the hunger of all the other guests I ever invited

We want the most trustworthy candidate to be elected as president of the club.

Comparison: the trustworthiness of the candidate we want to be elected vs. the trustworthiness of all other candidates

I think that Penelope’s experiment will yield the best results because she did the most research.

Comparison 1: The results of Penelope’s experiment vs. the results of all the other experiments in the group

Comparison 2: The amount of research Penelope did vs. the amount of research all other people in the group did individually

Common Tricky Situations

“Less” vs. “Fewer”: “Fewer” is used for countable nouns like “pennies” and “less” is used for non-countable nouns, like “water.”

I dumped less charcoal into the grill and cooked fewer hamburgers to avoid starting another fire.

“Farther” vs. “Further”: “Farther” is used to describe comparisons involving physical distance, and “further” is used to describe figurative distance.

She has traveled farther than I have? I should get to know her further.

“Good” and “Bad”: The comparative of “good” is “better” and the superlative is “best.” The comparative of “bad” is “worse” and the superlative is “worst.”

I thought that I was going to do worse in the competition than Ralph, but I did the best out of all the participants!



Practice Problems: Comparatives and Superlatives

Identify which of the following sentences contain comparative or superlative errors.

See Correct Answers and Explanations

1. People who experience less health problems are more likely to be happier and less depressed than their unhealthy counterparts.
2. Even though Joe, Pat and Lisa are all neighbors, Joe's house is further from Lisa's than it is from Pat's.
3. After examining the two cookies on the plate, Rita choose the sweeter for her afternoon treat.
4. Shaking himself out of his reverie, the author reflected that he had rarely been more wearier.
5. Most sweaters are made of acrylic, wool, or cashmere, and though they are the most expensive of the three, I prefer cashmere sweaters because they are so soft.
6. Carbohydrates and proteins both have less calories per gram than fats do.
7. Amy drinks lesser water than Johnny, so Johnny refills his water bottle more often than Amy does.
8. The teacher was reluctant to say which of the twin brothers was the worst musician.



Preposition Errors

Prepositions are words that convey temporal and/or spatial relationships between two objects or concepts. Like [using the wrong conjunction](#), using the wrong preposition can completely convolute the logic of a sentence. Consider the following sentences:

I sent the package from my local U.S. post office to Zimbabwe.

I sent the package to my local U.S. post office from Zimbabwe.

“To” and “as” can be used as prepositions, but they can also function as other parts of speech, so pay attention to the context when working with these!

In the first sentence, the speaker sent a package that began its trip at his or her local post office in the United States and ended up in Zimbabwe. The second sentence suggests that the speaker is in Zimbabwe, as the sentence’s context suggests that the package was sent from there “to” the speaker’s local U.S. post office. A lot of prepositions convey a sort of directionality to actions being described, and reversing them, as you can see, can have drastic effects on the sentence’s meaning. Even on test day, make sure that prepositions make sense. They may often be small words, but their impact on writing can’t be underestimated.

There aren’t any convenient tricks to making sure a sentence is using the correct preposition. Just make sure you’re familiar with the specific meaning and usage of each one, and keep an eye out for any logical inconsistencies within sentences. Then, you’ll be all set to handle prepositions on the ACT English section.

Common Prepositions Defined and Used Correctly

In, Inside, Within: These specify a location with a sense of internalness or describe the source of something internal, such as emotions.

In Japan, green tea is a very popular beverage.

It was so dark inside the cave that we needed to use flashlights to see anything.

I found the drive within myself to stick with swimming lessons, even though they were difficult.

Into: Conveys external to internal movement of something.

The spy ducked into the ice cream shop to shake the people who were trailing him.

From: Conveys the source of something, its original location after it has moved to a new location, or the giver of something that has changed possession.

Those cookies were special-ordered from the most famous bakery in the city, and brought here from their storefront this morning.

To: Conveys the movement of something with the destination specified or the receiver of something that has changed possession

On: being located atop of

I asked you to go to the library and pick up the books on my reading list.

For: (to be used) with the specific purpose of or in recognition of

I got you a present for Arbor Day, even though that’s a bit unusual.

What are you planning to use that fishing pole for if not to go fishing?

For the birds to make a warm nest, they needed sticks and bits of fluff.



Before: preceding spatially or chronologically

I can't believe you got to see the sequel to my favorite movie before I did!

After: proceeding spatially or chronologically

Z comes after Q in the alphabet.

Above, Over: physically or metaphorically located directly higher on a vertical axis than

You did far above average on your last exam.

If you look over the fence, you can see the neighbors' new puppy.

Below, Beneath, Under, Underneath: physically or metaphorically located directly lower on a vertical axis than

I think something is living underneath our porch, since I see tracks beneath the loose boards.

With: alongside, functioning as a part of the same group or unit as

I always like eating chocolate with marshmallows, so smores are a favorite food of mine.

Beside: located next to

I almost ran into a branch sticking out from the tree beside the grocery store entrance as I walked in.

Among, Amongst: located as a point in a larger group

He hid the bill amongst junk mail in the hopes his family would pass it over and not notice how much money he'd spent.

Near: located at a relatively close distance from

Luckily, the hospital is near the skate park, so when I broke my leg, I didn't have far to hobble.

As, Like: having the definition or status of the thing specified. Can be used to create similes in which different things are compared in order to emphasize a similarity between them.

We got into an argument as strangers, but we left the store as friends.



Practice Problems: Preposition Errors

Identify which of the following sentences contain errors in the usage of prepositions.

See Correct Answers and Explanations

1. Jamie prefers reptiles for cats or dogs since she is allergic to fur.
2. Complaining of the problem will do nothing to solve it.
3. Spanning from Washington to Florida and from California to Maine, the continental United States stretches across a great deal of territory.
4. The bride thought it would be difficult for her to choose between the three dresses, but it wasn't.
5. While I tried to avoid from getting caught in the rain, it was no use, and I was soaked to the bone by the time I got home.
6. I'd eaten half from the batch of cookies before I remembered that I made them for the party.
7. Glancing down at the street from the window on the fifty-fifth floor, I discovered a fear of heights and felt afraid of my life.
8. I tried to prevent the cats fighting with the dog, but there was still a lot of barking and hissing.



Pronoun Usage Errors

The case and gender of pronouns can create grammatical errors that have nothing to do with [a pronoun's agreement with its antecedent](#).

Pronoun Case Errors

Considering pronoun case in standard modern English brings up two concerns: deciding whether a subjective or objective pronoun is called for in a given sentence, and using possessive pronouns when necessary.

Subjective Pronouns vs. Objective Pronouns

Pronouns have two cases—subjective and objective—that are used in non-possessive scenarios. Which one is correct depends on the role the pronoun is playing in the sentence.

The subjective case is used when a pronoun is a subject of a sentence, phrase, or clause.

I think that he would enjoy that movie.

We talked to them yesterday.

Who is going to fix this leak?

The objective case is used when the pronoun is an object of a verb or a preposition.

That gift is to him from us.

Did you ask them about what's going on?

To whom did you speak at the ticket office?

Predicate Nominatives

A predicate nominative is a noun or pronoun used in the predicate of a sentence that directly renames the subject of the sentence directly after a form of “be.” Predicate nominatives call for the subjective form of a pronoun.

Who is she?

We are they.

This is he.

Clearing Up Clauses

What do you when a sentence has a subordinate clause calling for a subjective pronoun, but that pronoun is also the object of a verb in the main clause? The pronoun's role in the clause always takes precedence:

I don't know who I should call.

In these sentence, “who” is the subject of the phrase “who I should call,” but it is the object of the verb “don't know.” It takes the subjective form “who” because the clause calls for a subjective pronoun.

Guess who I sat next to at dinner!

The same thing is occurring here: “who” is the subject of “who I sat next to at dinner” but the object of “Guess.” It takes the subjective form because that is what the clause calls for.

Subjective	Objective	Possessive
I	Me	Mine
You	You	Yours
He	Him	His
She	Her	Hers
It	It	Its
We	Us	Ours
They	Them	Theirs
Who	Whom	Whose
Whoever	Whomever	

Quick Tip

A quick way to determine which pronoun you need involves substituting either “she” and “her” or “he” and “him” for the pronoun in question. If “she” or “he” makes a correct sentence, you need a subjective pronoun. If “her” or “him” makes sense, you need an objective pronoun.



Possessive Adjectives vs. Possessive Nouns

If a possessive word is called for in a sentence, make sure that you are clear as to whether a possessive adjective (e.g. “my,” “his,” “our”) or a possessive pronoun (e.g. “mine,” “hers,” “ours”) is called for. Certain pronouns use possessive adjectives and possessive pronouns that are identical, but for others, these are different words.

That’s my blanket—it’s mine, not yours! Your blanket is over there.

Pronouns and Gerunds

Verbs ending in “-ing” acting as nouns are called gerunds. Because gerunds act as nouns, pronouns immediately preceding them referring to their action should be possessive adjectives, not subjective pronouns.

✗ Do you think they will mind me eating a snack before dinner?

→ Do you think they will mind my eating a snack before dinner?

✗ I think they might object to him jumping in puddles in his nice new shoes.

→ I think they might object to his jumping in puddles in his nice new shoes.

Reflexive Pronouns

Reflexive pronouns end in “-self” or “-selves” and are used to refer back to a subject that has already been named while emphasizing a sense of reflexive (self-directed) activity.

He shined his shoes himself.

They did the work themselves.

She introduced herself as the team’s head strategist.

Are you saying that I myself should deliver the message?

Note that the bold and underlined reflexive pronouns refer back to the underlined subjective pronouns.

Choices, Choices

Certain pronouns end in “-ever” and emphasize the wide number of choices present in a given situation.

I will order food from whichever restaurant is closest.

Whoever is the thirtieth caller will win these concert tickets!



Gender

In standard modern English, pronouns can be masculine (e.g. “he”), feminine (e.g. “she”), or neuter, (e.g. “it”). Only third-person singular pronouns have assigned gender in English. Note that third-person plural pronouns “we” and “they” can represent groups entirely composed of individuals of one gender or the other or groups of objects, but the pronouns themselves do not have a gender assigned and vary in meaning based on the context in which they are used.

Make sure to use masculine pronouns for male people and feminine pronouns for female people. “It” should not be used to refer to a person, but to animals and objects.

Paul said that he was going to have to work late on his project.

Darlene is quite the expert at chess; I’ve never seen her lose a game.

The vase’s vibrant colors and organic shape helped it fetch a high price when sold.

This rule may be blurred somewhat when referring to pets by name; in these cases, it is appropriate to use “he” or “she,” even if the name itself is not gendered, as in the following example:

My dog Spot is the greatest dog ever—he even knows how to fetch the paper!

My cat Mittens loves to sunbathe—you can often find her on the back porch.

Ships are conventionally referred to as “she,” especially when referred to by name.

Example: My uncle’s boat is named the *Queen Mab*; **she** is a sturdy craft, and we’re going to take **her** out on the lake this weekend.



Practice Problems: Pronoun Usage

Identify which of the following sentences contain errors related to the use of pronouns.

See Correct Answers and Explanations

1. My pet turtle Fred is great; he's always active and usually seems interested in what I'm doing.
2. Many people find that they can understand them and their own emotions better after taking some time to relax and decompress.
3. From whom should I say the package is?
4. "To me, from I," she scribbled on the gift card she was buying for herself because it was on sale.
5. I recommended that he revise his work, but it doesn't look like he took that advice.
6. I don't think they'll mind me playing the kazoo during karaoke night so much as you yodeling.
7. A student asked she and I if we wanted to buy any cookies to support the local marching band.
8. I will recommend for a promotion whoever solves this problem first, and order a celebratory lunch for that person from which restaurant they want.



Verb Usage Errors

The importance of verbs is clear and can be overstated only with great difficulty. Verbs are an essential part of every grammatically correct and complete sentence, and are the words concerned with the actions taken by a subject. Everything that is done by a noun is a verb. As a fundamental aspect of any basic sentence structure, it is important to understand the rules governing the use of verbs, and to know about, in order to avoid, some of the most common species of verb errors. The three species of verb error we will be reviewing are: Verb Tense Errors, Verb Mood Errors and Verb Voice Errors.

What are Verb Tenses?

Verb tenses are used to indicate the time and order in which events occur. Tenses both tell us the time, and are regulated by that temporal information. If an event occurred at one discrete time in the past, you must use the past tense to describe it. There are three simple verb tenses, and three “perfect” verb tenses.

Simple Tenses

The Simple Present is used to describe events that are currently happening. It is formed with a “-s” verb ending.

Kevin cooks.

The Simple Past is used to describe events that have already occurred. It is formed with an “-ed” ending.

Kevin cooked.

The Simple Future is used to describe events that will occur, but have not yet occurred. It is formed with “will” and a verb or “is-verb,” “going to,” and a verb.

Kevin will cook.

Kevin is going to cook.

Perfect Tenses

The Present Perfect is used to describe events that began in the past and have continued into the present. It is also used to describe events that occurred in the past whose consequences continue to be relevant in the present. The present perfect is formed with a past participle and “has” or “have.”

Kevin has cooked for years.

The Future Perfect is used to describe events that will have been completed by some future date. It is formed with “will have” and a past tense verb.

Kevin will have cooked the bird by Thanksgiving.

The Past Perfect is used to describe events or actions that have occurred before another action or event that has also occurred in the past. It is formed with “had” and a past tense verb.

Kevin ate the bird that he had cooked.



Verb Tense Errors

Many verb tense errors occur simply in the formation of the verb. If you know the correct way to form each verb tense you can avoid this species of error.

Errors of verb tense consistency are much more complex, and are worthy of some explanation. As a general rule, you must avoid shifting verb tenses if the time frame of the events or actions discussed has not changed.

✗ The wind blew, clouds appeared on the horizon, and a huge wave crashes into the seawall.

This sentence incorrectly shifts between the past tense to the present when no shift in time has been indicated. To make the correction more clear, let's add a specific time.

Yesterday evening, the wind blew, clouds appeared on the horizon, and a huge wave crashed into the seawall.

Over the course of a paragraph, especially a paragraph describing a complex series of events, tenses may shift between simple and perfect tenses. For instance, a simple past action might precede the discussion of other events that occurred in the past and whose actions or consequences continued into the present. But, as long as the tenses are correctly used, tense shifts between different sentences are allowed, and often needed. An error occurs when the verb tense shifts within a sentence and without cause.

What are Verb Moods?

Verb moods show the mode or manner in which a thought, idea, or action is expressed. Whereas verb tenses are obviously used to show time and order, verb moods are used to show states of being. There are four verb moods in English, and knowing the form and conditions in which these moods are used will help you both to recognize errors and to compose your own writing.

The Indicative Mood is by far the most commonly used verb mood. It expresses a state of reality. As the name suggests, the indicative mood is used to simply indicate that an event or action has or will occur in actuality. You can think of the indicative verb mood as the default mood in which verbs are usually phrased, unless one of the other verb mood conditions exist.

The sky is blue.

Roses are red.

That movie was upsetting.

All of these examples correctly express facts. Note that verb tense is immaterial to verb mood. The indicative mood can be used with any [verb tense](#).

The Imperative Mood is used to indicate a command or request, or to give permission or issue a restriction.

Correct examples of the imperative mood:

Please, give me some more.

Move over so I can sit down.



Stop doing that!

I am going to be waiting in the surveillance truck across the street. Signal with your arms when you want me to pick you up.

All four of these examples correctly use the imperative mood. The first three examples are isolated and fairly clear. The verb comes at the beginning of the sentence and issues a command/request or places a restriction on someone's behavior. The final example includes multiple sentences, the first is in the indicative mood and the second is in the imperative, as the speaker is asking or telling his/her interlocutor which action to take.

The Subjunctive Mood is the most commonly misused verb mood in English. This verb mood should be used to express a statement contrary to fact, an untrue or hypothetical condition, a wish, or a doubt. An excellent indicator that the subjunctive mood should be used is the word "if."

I would if I could.

The judge demanded that he be banished from the county.

If I were a better fiscal planner, I would not be writing this sentence right now.

These sentences express, in order, a hypothetical condition, a wish, an order, and another hypothetical condition.

Subjunctive Mood Errors

The vast majority of verb mood errors occur in the use of the subjunctive mood. The following sentence, for example, is incorrect:

✗ It is vital that the students are told what to do in the event of a fire.

This sentence is incorrect. Since the dependent clause "that the students are told what to do in the event of a fire," expresses a recommendation it should be expressed in the subjunctive mood.

The corrected version of this sentence reads,

It is vital that the students be told what to do in the event of a fire.

Another common error made in the subjunctive mood involves the past tense of "is."

✗ If I was tall, I would be a basketball player.

This sentence is incorrect. The correct past tense form of "is" in the subjunctive is "were" not was. "If I was" is never a correct formulation. This thought should be formed as: "If I were."

The corrected version reads,

If I were tall, I would be a basketball player.

Quick Tip

A good indicator that the subjunctive mood will be required for a dependent clause expressing an order or recommendation is the subordinating conjunction "that."



Mood Shifts

Mood shifts, while a natural and relatively constant aspect of any person's inner emotional life, are frowned upon in English grammar. Most of these shifts are similar in nature to the errors discussed above, when through improper verb formation a shift from subjunctive to indicative, or vice-versa, is made within a sentence. Verb mood can also inappropriately shift between complete sentences, if this shift is misleading or arbitrary. Usually, these kinds of errors occur when the mood shifts between the indicative and the imperative clause.

✗ The teacher gave very specific instructions for what to do in a fire. Exit the classroom calmly and in a single file line.

Here, the “specific instructions” are given by “the teacher.” By shifting the verb mood from the indicative to the imperative, the instructions go from being given by the teacher to the students to being given by the writer to the reader.

Corrected, this sentence reads,

The teacher gave very specific instructions for what to do in a fire. She told them to exit the classroom calmly and in a single file line.

Verb Voice

Verb voice is determined by how the agent of action (the “do-er”) is situated in the overall sentence structure. The active voice is used when the agent of action is used as the grammatical subject of the sentence. For example,

Kevin cooked our meal.

The passive voice is used when the agent of action is a grammatical object in the sentence, and the target of the action is used as the subject. For example,

The meal was cooked by Kevin.

There is a common, and unfortunate, myth that has been promulgated by some misguided would-be arbiters of English grammar that it is always incorrect to use the passive voice. While it is true that the active voice is most often the clearer, more succinct choice, there are still some instances in which it is correct to use the passive voice construction (I have used it plenty of times in this very lesson!).

Grammatically speaking, as long as they are used correctly, the two voices are equally grammatically correct; the general preference for the active voice is a matter of style, clarity, and concision.

The passive voice should be used in instances when the agent of action is relatively unimportant contextually, as compared to the recipient or target of the action, or when the agent of the action is unknown.

The important documents were delivered by a delivery guy.

Here, obviously, the “important documents” should be emphasized, as opposed to the agent of action, an anonymous “delivery guy.”



The potential awkwardness of the passive voice is probably clear, but we'll look at one example of an unnecessary and awkward use of the passive voice to make this point clear.

All the mats were cleaned yesterday by Matt, the new P.E. teacher.

That sentence is much more awkward, and less clear in its intent, than this one:

Yesterday, Matt, the new P.E. teacher, cleaned all the mats at the gym.

Another illustrative example of the potential awkwardness is made obvious when the subject of the sentence is also the speaker.

All the mats were cleaned yesterday by me.

Since the subject is recipient of the action here, the speaker (the agent of action in this case) becomes the object, so the objective pronoun case (link) is required, as opposed to the subjective pronoun case, which is much smoother for discussing action.

I cleaned all the mats yesterday.



Practice Problems: Verb Usage Errors

Identify which of the following sentences contain verb errors.

See Correct Answers and Explanations

1. He told me that it is wrong to do what we did.
2. Jerry will finish the report by the end of the day.
3. The bill was passed into law at 2:45 pm on July 3, 2015.
4. It is necessary that a person understands the consequences of their actions if they are to be charged with a crime.
5. If I were a person who enjoyed reading books, I would buy that novel in a heartbeat.
6. Once they were safely on the sidewalk, William smiled at the old woman whom he helped across the street.
7. I was happy to be here.



Punctuation Errors

Punctuation! Of all of the topics covered on the ACT English section, punctuation can be the sneakiest. The crucial distinctions between these tiny markings are extremely significant to sentence meaning and correctness, but can be all too easy to miss, especially when dealing with test-day nerves. Studying common punctuation errors and learning which distinctions to watch out for is a good first step in mastering this aspect of the ACT English section.

Section Summary

[Possessive Apostrophes](#)

- Apostrophes (') can be used to indicate one noun's possession of another noun.

[Apostrophes for Contractions](#)

- Apostrophes can also be used to signify omitted letters in that two words that have been combined as a contraction

[Quotation Mark Errors](#)

- Double quotation marks (" ") are used to convey direct speech or quoted material. Single quotation marks (' ') are used to convey direct speech or quoted material within a quotation. Quotation marks are also used for a few other particular grammatical purposes.

[Colon Errors](#)

- Colons (:) are used to introduce lists of items that follow independent clauses that specifically introduce them. Colons may also be used to introduce and emphasize material that answers an implicit question created by preceding independent clause.

[Semicolon Errors](#)

- Semicolons (;) are used to combine two independent clauses that are related to one another in meaning.

[End-of-Sentence Punctuation Errors](#)

- Sentences can conclude with three punctuation marks. Declarative sentences conclude with periods (.). Interrogative sentences, or questions, conclude with question marks (?). Exclamatory sentences conclude with exclamation marks (!).

[Hyphen and Dash Errors](#)

- Hyphens (-) connect two words that together function as a compound word, often a compound noun or a compound adjective.
- Dashes (—) may either be used alone to introduce material with emphasis, or in pairs like parentheses (()) to set apart material from the main body of a sentence as an “aside” to the reader.

Looking for comma errors? We understand that commas are particularly tricky, so we've devoted an entire subsection to comma usage. [Check it out here!](#)



Possessive Apostrophes

To make a noun possessive, you need to consider whether the noun is singular or plural, and, if plural, whether or not it ends in an “s.”

Singular nouns: Add an apostrophe followed by an “s,” e.g. Cat → Cat’s

Note that this applies to singular nouns ending in “s,” e.g. Mitosis → Mitosis’s

Plural nouns not ending in “s”: Add an apostrophe followed by an “s,” e.g. Mice → Mice’s

Many of the nouns that fall into this category will be irregular nouns, as they won’t simply add an “s” to the singular form to form the plural.

Plural nouns ending in “s”: Add an apostrophe only, e.g. Spas → Spas’

Compound nouns follow this rule, but the apostrophe or apostrophe and “s” need to be placed at the end of the entire compound noun, not at the end of the single-word noun,

mother-in-law → mother-in-law’s

train station → train station’s

Noun phrases involving two or more listed subjects follow the rule as well, placing the apostrophe or apostrophe and “s” after the last listed noun to convey group ownership of the object of possession. If both nouns are singular, treat the phrase as singular.

Tea and coffee → “Tea and coffee’s”

If both nouns are plural, make each plural noun possessive independently:

The companies and the groups → The companies’ and the groups’

If one of the nouns is plural and the other is singular, make them possessive independently:

My cat and panthers → My cat’s and panthers’

Wolves and my dog → Wolves’ and my dog’s



Practice Problems: Possessive Apostrophes

Identify which of the following sentences contain comma errors related to the use of apostrophes.

See Correct Answers and Explanations

1. Alyssa collected hundreds of childrens' hats, scarves, gloves, and coats for the winter clothing drive.
2. The superhero's cape fluttered in the slight breeze.
3. The supervisor constantly checked on things like the pastries' consistency and the loafs' quality.
4. Four different travel groups's bags were taken from the airport on the same day.
5. My work involves studying the common starlings prevalence in various urban and suburban habitats.
6. Athlete's popularity in high school is a profound feature of the American educational system.
7. Under the terms of the contract, the company had to pay the fired employee six months salary.
8. The leaves' bright colors and the air's crispness are two reasons why autumn is Tia's favorite season.



Apostrophes for Contractions

“Its” vs. “It’s”

This distinction can be tricky to remember because “it’s” looks like a possessive formed in the typical way—by adding an apostrophe and an “s.” Instead, for this specific situation, think of the apostrophe as representing a break between two words.

“Its” = possessive

“It’s” = the contraction of “it is”

It’s a good thing you gave the dog its chew toy before it chewed up all our shoes!

Quick Tip

Instead of thinking of apostrophes as always meaning that the words containing them are possessive, for these particular terms, consider the apostrophe to mark a break between two words.

“They’re” vs. “Their” vs. “There”

The apostrophe in “they’re” represents a break between two words—this form is the contraction. As for “their” vs. “there,” “their” is a possessive pronoun and is the one that contains the letter “i” (also a pronoun).

“They’re” = contraction of “they are”

“Their” = possessive form of the pronoun “they”

“There” = demonstrative adverb conveying location at a distance

They’re really lucky that their luggage was at the hotel when they got there.

“You’re” vs. “Your”

Apostrophe to the rescue again! As usual, it represents a break between two words, meaning that “you’re” is the contraction of “you are” and “your” is the possessive form of “you.”

“You’re” = contraction of “you are”

“Your” = possessive form of the pronoun “you”

You’re probably wondering where all the food in your fridge went.

“We’re” vs. “Were”

“We’re” is the contracted form of “we are,” and “were” is the third-person past tense of “was” (“We were swimming.”) or the first-person past tense subjunctive form of “am” (e.g. “If I were . . .”)

“We’re” = contraction of “we are”

“Were” = past-tense indicative or subjunctive verb, depending on context

We were all just discussing this, and we’re all in agreement that were we to explain where all your food went, you might get a bit frustrated with us.

“Who’s” vs. “Whose”

Once again, the apostrophe forms the distinction between a contraction (two words) and a possessive.

“Who’s” = contraction of “who is”

“Whose” = possessive form of the pronoun “who”

Whose idea was it to toss out the paper that said who’s sitting by whom at dinner?



Rare Apostrophe Pitfalls

“Won’t” vs. “Wont”

“Won’t”: a contracted verb phrase meaning “will not”

“Wont”: an archaic adjective meaning “used to (doing something)”

“I know you’re wont to have coffee with every meal, but won’t you consider trying some tea instead?”

“Can’t” vs. “Cant”

“Can’t”: a contracted verb phrase meaning “cannot”

“Cant”: an archaic noun meaning “hypocritical talk,” “popular phrasing,” or “slang”

“The common cant in the 60s was for everything cool to be “far out,” and you can’t use this phrase today without evoking that era.”

Tip: Both of these distinctions can be drawn easily by answering the question, “Is this word acting as a noun or a verb?” Contractions are verbs, and the alternate spellings and meanings of these terms are both nouns.



Practice Problems: Apostrophes for Contractions

Identify which of the following sentences contain comma errors related to the use of apostrophes.

See Correct Answers and Explanations

1. After viewing the forecast, they're sure that it's going to be a snow day tomorrow.
2. The ancient herd of bison made it's way across the open plains.
3. You're won't to be so rude in public that hardly anyone wants to go out with you.
4. Its extremely difficult to win a game of chess if you don't plan your moves carefully.
5. We're you ever planning on telling me that we're almost out of gas or were you going to wait until the car stopped working?
6. "Whose book is this?" she asked inquisitively, flipping through the novel's tattered pages and hoping to find it's owner's initials.
7. I think this is your jacket—it's definitely not mine.
8. They're six different types of subparticles, and each of their names is pretty amusing.



Quotation Mark Errors**Conveying Speech**

There are two kinds of quotation marks: single quotation marks (') and double quotation marks ("). Each kind is always used as a pair of quotation marks. In standard modern English, double quotation marks are the default. Single quotation marks should only be used within double quotation marks to convey direct reported speech within direct speech.

Ruby said, "Opal said, 'I need to take a study break.'"

Note how quotation marks work just like parentheses or brackets in math class or when coding: they have to be closed, and the single quotation marks must be contained within the double quotation marks.

Ruby said, ((Opal said, [I need to take a study break.]))

The rules about quotation mark usage are relatively clear, but you have to be careful to distinguish between direct speech and indirect speech in situations where someone is reporting what someone else said.

Direct speech: Opal said, "I need to take a study break."

In this situation, we are told that the specific words that came out of Opal's mouth were "I need to take a study break." Double quotation marks are necessary.

Direct reported speech: I heard that Opal said "I need to take a study break."

In this situation, we are hearing from the speaker that this person heard Opal say the exact words, "I need to take a study break." Double quotation marks are necessary.

Indirect speech: Opal said she needed to take a study break.

In this situation, we don't learn exactly what Opal said. She could have actually said, "I'm so tired! I haven't had lunch yet. I'm going to get a slice of pizza and finish studying for this test later"; however, the gist of that is that she needed to take a study break. That is the only thing being reported in the sentence.

Indirect reported speech: I heard that Opal said that she needed to take a study break.

As in the situation above, no quotation marks are needed here because we are not being told the words that came out of Opal's mouth—just the main point of what she said.

Incorrect for the purposes of conveying dialogue:

✗ Opal said I need to take a study break.

✗ I heard that Opal said I need to take a study break.

These sentences are incorrect if the writer's intention was to convey spoken dialogue. If Opal is saying that I need to take a study break, not her, then the sentence is correct.

✗ Opal said she "needed to take a study break."

✗ My friend told me that Opal "needed to take a study break."

These sentences could be correct if "needed to take a study break" is being used euphemistically to mean something else, but if that is not the case and the sentence is just trying to convey dialogue, it is not correct.

Don't Stack Punctuation

Don't stack punctuation one mark after another when dealing with quotations. If a quoted sentence ends in a question mark, you don't need a period immediately after it:

Stan asked Jill, "How is your sister doing?"

Note that when a complete directly reported sentence ends a sentence, you don't need to use two periods—the one in the quoted sentence will suffice.

Stan also said, "Jill told me, "My sister is doing very well."

Connecting Quotations with Dialogue Tags

A dialogue tag is the bit of a sentence containing a direct quotation that is not part of the quotation itself and clues the reader into who is speaking, e.g. "she said." Sentences only need one dialogue tag, but that tag can appear before or after a quotation. When a dialogue tag appears before a quotation, it needs to be separated from the quotation with a comma.

Sally said, "You don't eat enough fruit. You need to take your vitamins."

When a declarative quotation appears before a dialogue tag, it should end in a comma. If it ends in a question mark or exclamation point, leave those punctuation marks alone.

"You don't eat enough fruit. You need to take your vitamins," Sally said.

Dialogue tags can also appear between parts of a quotation. In this scenario, you need to be careful and consider whether the surrounding quotation is a single sentence or multiple sentences. If the dialogue tag is surrounded by independent clauses, end it with a period and capitalize the first letter of the next quotation:

"You don't eat enough fruit," Sally said. "You need to take your vitamins."

If the dialogue tag is interrupting a single sentence, end it with a comma and don't capitalize the first letter of the second half of the quotation:

"You don't eat enough fruit," Sally said, "but you certainly eat enough vegetables."



Other Uses of Quotation Marks

Scare Quotes

Quotations can also be used to call attention to a part of a sentence for the purposes of casting doubt or suspicion on that phrasing or what it represents, or to suggest that it means something entirely else and functions as a euphemism. Have you ever seen someone make a gesture indicating quotation marks by flexing two V's made with their index and middle fingers to draw attention to what they're saying? This usage of quotation marks does the same thing. Phrases like "so-called" or "supposed" preceding a quotation can clue you in that the writer is probably using quotation marks in this way.

Her supposedly "unstoppable" plan was stopped within five minutes of her starting it.

"The self-styled 'Emperor' has been dealt with, Mr. President," said the General.

Discussing Words

Quotation marks can also be used to indicate specific words as words being discussed as words, and not just used as part of the sentence.

The word "nice" has a fascinating history of swinging from one meaning to the opposite.

"The poet's frequent and accurate use of the words 'evolve' and 'species' makes me wonder if she has a scientific background," Francine said.

Referring to Titles

Titles of certain works—comparatively short ones—are also put into quotation marks.

I like Robert Frost's poem "Mending Wall" better than his "Apple Picking."

"My favorite chapter in that textbook is called 'Ecosystem Diversity,'" Josie said.



Practice Problems: Quotation Mark Errors

Identify which of the following sentences contain errors related to the use of quotation marks.

See Correct Answers and Explanations

1. Before Sam left, did you hear him say, “Be right back—I’m going to go meet the President?”
2. “I can’t believe he ate the entire cake,” she said, “Can you?”
3. “The agency specifically asked for “a talented businessperson who speaks at least three languages,” and that describes me,” Eilene declared.
4. “Why does your so-called ‘fact sheet’ list obvious fallacies,” she asked?
5. “Words like prejudice and bigotry are too negative for this publication,” she said.
6. I told my friends that “I was too tired to go out,” but really I just wanted to read.
7. “I was supposed to play the tiger in the school play, but I couldn’t project the line ‘Roar!’ loudly enough, so I was recast as part of the chorus instead,” Carrie bemoaned. “This is terrible!”



Colon Errors

Colons (:) are used for two very specific purposes in standard modern English: introducing lists, and providing emphasis. To be prepared to face colon errors on the ACT English section, you just have to do one thing: remember these two uses!

Colons for Introducing Lists

Colons are used to introduce lists. The part of a sentence that precedes a colon should be an [independent clause](#), whereas the list should simply consist of a phrase made up of items that are each grammatically phrased / conjugated in the same way—in other words, a list should employ [parallel structure](#).

I went to the bakery and bought three things for the party: donuts, pretzels, and a cake.

Independent Clause

List of Items

You really need that independent clause, or the sentence becomes ungrammatical:

✗ That time I went to the bakery and bought: donuts, pretzels, and a cake.

Dependent Clause

List of Items

Subject with No Predicate - Incomplete Sentence

Colons for Emphasis

Colons can also be used to provide introduced information in a way that emphasizes it. The first part of the sentence still needs to form an independent clause for this to be grammatically correct, and the latter part of the sentence can be either a phrase or an independent clause; its relationship to the first part of the sentence is what's key.

The content of the two parts of a sentence in which a colon is used for emphasis need to be very closely related. Make sure that the part of the sentence being introduced by the colon is providing some necessary information or answering some crucial question without which the first part of the sentence wouldn't make much sense.

There was only one thing to do: fight the pirates head-on.

Independent Clause

Verb Phrase

Second part of the sentence answers the question, "What was the only thing to be done?"

We'd attempted a sneak attack with the worst results yet: we'd been forced to make a hasty retreat.

Independent Clause

Independent Clause

Second part of the sentence answers the (implicit) question "What were the worst results yet?"



Practice Problems: Colon Errors

Identify which of the following sentences contain errors related to the use of colons.

See Correct Answers and Explanations

1. It's a pity you can't come to the party, since we need so much help: decorating, setting up seating, and preparing food.
2. The way I see it, the animal shelter has three options: ask the city for more money, host a fundraiser, or employ more volunteer workers.
3. It's been a long, hard winter: everyone in the Alaskan village just wants to see some green grass and feel the sun shining again.
4. Learning to crochet is a difficult task: since there are many stitches and patterns to keep track of, and the yarn often slips off the hook entirely.
5. As a final project, all students must prepare the following: a presentation, a poster, and a paper.
6. One thing prevented Harold from hearing the question: the fact that he was listening to music.
7. Jen is an expert on growing several kinds of trees: elms, chestnuts, and oaks are among her specialties.



Semicolon Errors

Semicolons have two major uses in standard modern English. The first, and much more common, is to combine sentences. Semicolons should be used to combine two independent clauses that are thematically related.

When asked what my favorite kind of sandwich was, I faltered; I have several favorites and didn't want to choose.

Semicolons should not be followed by any sort of conjunction, or the compound sentence will become ungrammatical.

✗ Grilled cheese is one of the best, in my opinion; and roast beef is pretty good as well.

In addition, both the material before a semicolon and the material after a semicolon should be able to stand on its own as a sentence. If either can't, the sentence containing the semicolon is ungrammatical.

✗ My favorite breakfast sandwich, eggs on a biscuit; I often make one and eat it before work.

✗ I thought about what kind of sandwiches to bring to the picnic; tuna salad sandwiches.

The second, and rarer, role semicolons play in standard modern English involves a scenario in which items in a list of three or more items are themselves phrases that involve commas. One common example of this is when listing cities followed by the states or countries in which they are located.

✗ I visited Seattle, Washington, Los Angeles, California, and San Antonio, Texas on my vacation.

See how confusing it is to read that? Without prior knowledge, it's very difficult to keep track of which place names refer to cities and which refer to states. Use semicolons in the place of commas in this specific scenario to avoid that confusion:

I visited Seattle, Washington; Los Angeles, California; and San Antonio, Texas on my vacation.

Note that if a comma appears in one item in a list, it is appropriate to use semicolons for the entire list:

I packed my sleeping bag; a pop up tent, which I had borrowed from a friend and only used a few times; a cast-iron skillet; and some popcorn to bring on the camping trip.



Practice Problems: Semicolon Errors[Table of Contents](#)

Identify which of the following sentences contain errors related to the use of semicolons.

[See Correct Answers and Explanations](#)

1. At that point, we didn't know where Jasmine was; it turns out that she didn't know where she was, either.
2. Some laud wind power as a clean, renewable energy source: others worry about the environmental effect of setting up rows of turbines, or "wind farms," either offshore or on land.
3. I've never been a good chef, I tend to get distracted and forget that there's food on the stove.
4. When Harold went to the dentist, he was given two instructions; to brush his teeth twice daily and to avoid acidic beverages.
5. One of the most well-known jazz songs is "Summertime"; it was first performed in a musical.
6. Pensions are a complicated financial problem for many municipalities; but many cities have made changes to their pension programs in recent years.
7. In the past three years, Jing has lived in Madison, Wisconsin; Ann Arbor, Michigan; and Laramie, Wyoming.
8. The entire party worked to get the candidate elected; but the election still went against him.

- 1. This sentence is correct! It uses a semicolon to join two independent clauses.
- ✗ 2. The sentence is incorrect as it is written because it is using a colon to combine two independent clauses into a compound sentence. When combining two independent clauses into a compound sentence, one should use either a semicolon or a comma followed by a conjunction.
- ✗ 3. A semicolon, not a comma, is the best choice to separate these two independent clauses, since the second clause is elaborating upon an idea introduced in the first clause.
- ✗ 4. Because the phrase "to brush his teeth twice daily and to avoid acidic beverages" is a list and a dependent clause, the appropriate punctuation is a colon, not a semicolon.
- 5. This sentence is correct! Its semicolon joins two independent clauses, and the semicolon is placed outside of quotation marks.
- ✗ 6. The sentence as written as a complex sentence, and it joins its two parts with a semicolon and the conjunction "but"; however, a compound sentence needs to be joined either by just a semicolon or a conjunction and comma, but not a semicolon and a conjunction.
- 7. Because the items in the list are compound (each place name contains a comma), semicolons are needed to separate the items and provide clarity. Semicolons are used correctly, so this sentence is correct!
- ✗ 8. The sentence is written as a compound sentence, one that joins two independent clauses together in order to show their relationship. A compound sentence can only be joined by either a semicolon or a comma followed by a conjunction, but not a semicolon and a conjunction, as in this sentence.



End-of-Sentence Punctuation Errors

Every independent clause needs to end with one of three punctuation marks: a period, a question mark, or an exclamation point.

Periods end declarative sentences, or sentences which simply provide information.

I need to take out the trash.

Question marks end interrogative sentences, or questions.

How did you forget to take out the trash?

Exclamation points end exclamatory sentences, or sentences that are said with a lot of emotion—in other words, sentences that you need to convey would be shouted or at least said with greater intensity, were they to be read.

I can't believe you forgot to take out the trash after I asked you to do so five times!

Imperative sentences, or commands, can end with periods or exclamation points, depending on the tone the writer wishes to convey.

Take out the trash.

or

Take out the trash!

These three punctuation marks are only used to end sentences; they should never appear in the middle of a sentence, or they will create sentence fragments.

You should only ever use one punctuation mark at the end of a sentence; multiple punctuation marks, while perhaps somewhat common in informal writing, are never grammatically correct.

Note that when dealing with direct quotations, this rule may require that a period be implied, as in the following sentence:

I heard that George asked the grocer, "What is the most expensive coffee you carry?"

Here, the question mark ends George's question, which is being repeated to us in the declarative sentence; however, since a period would need to go inside the quotation marks and a question mark is already there, the period is left off entirely and implied. If you want to review how to punctuate quotations, [click here](#).



Hyphen and Dash Errors

Hyphen Errors

Hyphens (-) are punctuation marks used to combine multiple words into a single term. Most compound nouns don't need to be hyphenated:

Have you met my mother in law?

The Commander in Chief will see you now.

Some compound adjectives do need to be hyphenated. This includes possessive constructions:

My mother-in-law's purse is missing.

The Commander-in-Chief's reports are always secured.

It also includes constructions in which multiple terms function as a single adjective:

The seventy-year-old's zeal for baking lasagna was astounding.

The high-security files were transported by trained guards.

A general rule of thumb is that adverbs do not need to be hyphenated when working with adjectives, because the adverb is affecting the meaning of the adjective, which is affecting the meaning of something else; this means that they are not grammatically equal concepts being combined into a compound adjective.

The hastily brought pudding pleased the diners greatly.

Single Dashes

You can think of a dashes by itself as functioning something like a blend between a comma and a colon. Like comma, a dash provides a pause in a sentence, but like a colon, its use conveys that the material to follow will answer some crucial question, or at least dramatically emphasizes it.

Answering some crucial question:

There's only one thing we can do now—run!

She offered me a choice between cherry or mint ice cream—my two least favorite flavors.

Adding emphasis to the material that follows:

Camping is my family's favorite pastime—but not mine.

I was ordered to go defeat the dragon in the dungeon—or else.

Context is very important when deciding whether to use a comma, a semicolon, a colon, or a dash. While several can be grammatically correct in a given sentence, each one conveys a subtly different meaning.

Pairs of Dashes

Dashes are used in pairs to set unnecessary material apart from the rest of the sentence. In plays, characters sometimes have "asides," during which they directly address the audience as if no other characters on stage can hear them. Dashes are often used to convey extraneous material in a manner much like an aside. Dashes should never be used to separate essential information from the rest of a sentence. When used in pairs this way, dashes function something like parentheses, but in contrast to parentheses, they emphasize the material being set apart instead of downplaying its importance.

They were waiting for a response—anything would have sufficed—but I couldn't get a word out.

I read my favorite poem—Keats's "Ode on a Grecian Urn"—but no one seemed to get it.

Amidst the group's various complaints, including concern about a sprained ankle and the request for a bandage, mine—"How did that get on my shoe? Ugh!"—seemed relatively unimportant.



Practice Problems: Hyphen and Dash Errors

Identify which of the following sentences contain errors related to the use of dashes.

See Correct Answers and Explanations

1. The seven year old dog still acted like a puppy.
2. If I don't use my frequent flier-miles soon, they will expire.
3. The dress's color—a neon yellow made her stand out from the rest of the guests, who were clad in darker colors.
4. As my relatives chatted around the table, I returned to a favorite pastime—reading.
5. The lowest quality meat processed by that factory ends up in dog food.
6. I was introduced to the director and her assistant, a woman who was effectively second in command when in came to making sure the entire production ran smoothly.
7. The leather-bound book sat on the coffee table.
8. I was told to do one of two things for my final project compose a story or write a research paper.



Comma Errors

Of all the punctuation tested on the ACT English section, commas may be the most difficult because of the wide range of situations in which they can be used at the writer's discretion, and an equally wide range of situations in which they must be used for grammatical correctness. Reviewing the different applications of commas with an eye for when they are needed can help you prepare to tackle any comma that gives you pause on test day.

Section Summary

[Commas in Lists](#)

- Commas are used to separate each item in a list of three or more items.
- The last comma in a list is called the “Oxford comma”; omitting it can cause ambiguity problems in certain sentences.
- Semicolons can also be used to punctuate lists if the items in the list use commas.

[Comma Splices](#)

- A “comma splice” is a type of error in which two independent clauses are incorrectly combined using a comma without a conjunction.

[Appositives](#)

- Appositives are noun phrases that rename a noun already mentioned in the sentence to provide more information about it.
- Appositives that are grammatically unnecessary to the meaning of the rest of the sentence should be surrounded by commas.
- Appositives that are grammatically necessary to the meaning of the rest of the sentence should not be surrounded by commas.

[Commas for Phrases and Clauses](#)

- Dependent clauses that start sentences are separated from the sentence by a comma.
- Dependent clauses that appear in the middle of a sentence are set apart from the sentence by a pair of commas.
- Dependent clauses that conclude a sentence are not separated from the rest of the sentence.
- Relative clauses that are inessential to a sentence are set apart from it by commas.
- Relative clauses that are essential to a sentence are not set apart from it by commas.



Commas in Lists

Of all the many ways in which commas are used, one of the most common is to separate the distinct items in a list. Not all lists need commas, though: only lists with three or more items use commas. Lists of two items don't need commas. Lists with three or more items also need to insert a conjunction (e.g. "and") before the last item.

I bought apples and bananas.

I bought apples, bananas, and pears.

I bought apples, bananas, pears, oranges, coconuts, strawberries, and a pineapple.

These are grammatically incorrect sentences messing up this comma rule:

✗ I bought apples, and bananas.

✗ I bought apples bananas and pears.

These rules hold true for much more complex sentences:

I swept the floor so that all of the dust bunnies ran for cover, dusted them out from underneath the furniture, gathered them all into a big pile, decided I was too lazy to wrangle them into the trash can, and brushed them all under the rug.

That sentence may look imposing, but it's still a list. To make its structure more obvious, try writing out the items in the list:

- I . . .
1. swept the floor so that all of the dust bunnies ran for cover,
 2. dusted them out from underneath the furniture,
 3. gathered them all into a big pile,
 4. decided I was too lazy to wrangle them into the trash can,
 - and
 5. brushed them all under the rug.

There's a comma after each item except the last one, and "and" is immediately before that. This sentence is correct!

The Oxford Comma

The "Oxford comma" is a specific use of a comma to distinguish the second-to-last item in a list from the last item in a list. Some people don't think it's absolutely necessary, and some people do. Why use it? Well, in certain sentences, if you don't use it, you create a lot of confusion:

✗ Some of my favorite foods are muffins, ravioli, ice cream and anchovies.

A reader could potentially interpret "ice cream and anchovies" as being a single item, and we can't be sure whether the writer meant to indicate "ice cream" and "anchovies" as two of his or her favorite foods or "ice cream and anchovies" as one of them. We recommend always using the Oxford comma so you don't have to worry about the possibility of phrasing a list ambiguously:

Some of my favorite foods are muffins, ravioli, ice cream, and anchovies.

Because people are still arguing about the Oxford comma, the ACT English section won't test it in unambiguous sentences. The test covers ambiguity, though, so be on your toes for any confusing lists that can be fixed by a well-placed Oxford comma!

Semicolons in Lists?

Lists can also be divided by semicolons when the items in the list themselves involve commas—for example, if you're listing names of places you've visited followed immediately by their states or countries. Otherwise, all of those commas get confusing! [Click here to learn about this specific use of semicolons.](#)



Practice Problems: Commas in Lists

Identify which of the following sentences contain errors in their use of commas in lists. Consider ambiguity to constitute an error.

See Correct Answers and Explanations

1. Tripping on the sidewalk, breaking down in a public place, spilling a drink on oneself, having something caught between one's teeth—all are humiliating moments for a person.
2. The large fish, the turtles, and the small fish, all got along well in the same tank.
3. On her summer road trip, Kayli visited Boston, and New York for a week each, and she stayed in Baltimore for three days.
4. For the picnic on the White House lawn, Luke's father packed salads, sandwiches with peanut butter and pickles for the children, but unfortunately, he forgot to pack water.
5. The *Día de los Muertos* event at school featured a dinner consisting of a burrito bar, a taco station, and three varieties of flan.
6. Much of the time, only the most discerning animal trainer cannot tell the difference between thoroughbreds Arabians and saddlebred horses.
7. The typical advice to the screenwriter is to keep an audience engaged alert and interested.



Comma Splices

What is a “Comma Splice?”

The word “splice” means join or connect, and a “comma splice” is a specific type of grammatical error in which two independent clauses are incorrectly connected by a comma. Independent clauses are parts of a sentence that contain a subject and a predicate and could stand on their own as complete sentences. Commas can be used to connect dependent clauses to independent clauses, but commas by themselves cannot be used to combine two independent clauses.

Examples of Comma Splices

- ✗ The cat is meowing, I think it is hungry.
- ✗ I saw her on my way to the store, I would have said hi, but she was on the phone.
- ✗ The old car rumbled down the road, everyone for miles around could hear it.

The third example might be the most obvious to you because “everyone for miles around could hear it” could be a complete sentence on its own, yet “everyone” is not capitalized in the sentence. Comma splice sentences in which a word that is typically capitalized (e.g. proper nouns like “I”) immediately follows the incorrect comma might be harder to spot, so keep this in mind!

How to Spot Comma Splices

If you’re ever unsure if you’re looking at a comma splice, simply break apart the sentence at where the comma divides it. Can the resulting two sentences stand on their own? If they can, they are independent clauses, and that comma is a grammatical error. For example:

- ✗ The cat is meowing, I think it is hungry.
The cat is meowing. I think it is hungry.

“The cat is meowing” and “I think it is hungry” are both complete sentences, so the original sentence’s comma constitutes a comma splice. The other two example sentences can be identified using the same technique:

- ✗ I saw her on my way to the store, I would have said hi, but she was on the phone.
I saw her on my way to the store. I would have said hi, but she was on the phone.
- ✗ The old car rumbled down the road, everyone for miles around could hear it.
The old car rumbled down the road. Everyone for miles around could hear it.

Not all sentences are comma splices, though; if a sentence’s comma isn’t forming a comma splice, breaking the sentence apart at that comma will yield one incomplete sentence:

- The car rumbled down the road, sputtering and rattling.
The car rumbled down the road. Sputtering and rattling.

“Sputtering and rattling” is not a complete sentence—what is doing the “sputtering and rattling?” Or what about “sputtering and rattling” are we being told? We can’t answer either of those questions, so the original sentence’s comma does not constitute a comma splice.

If you see a conjunction after a comma, like “and” or “but,” it is very unlikely to be a comma splice, but you should make sure that the conjunction makes sense in the sentence.



Practice Problems: Identifying Comma Splices[See Correct Answers and Explanations](#)

Identify which of the following sentences are incorrect because they contain comma splices.

1. The ocean is cold today, but I think I'll still go swimming.
2. After all, this is the last day of my vacation!
3. I asked my family if they wanted to go to the beach with me, everyone wanted to go along.
4. The last time I was at the beach, I lost the sandwich I'd packed for lunch.
5. A seagull snuck up on me while I was asleep, it stole my sandwich out of my tote bag.
6. This time, I'm packing my sandwich at the very bottom of the bag, underneath my beach towel and far from any thieving seagulls.
7. My dermatologist told me to always wear sunscreen at the beach, that is good advice.
8. I have a bottle of sunscreen in my bag from my last trip, I hope a seagull doesn't try to eat that, too.



Appositives

Appositives are nouns or noun phrases that immediately follow and rename a given noun or noun phrase in a sentence. Appositives that contain grammatically inessential information must be set apart from the rest of the sentence by commas; appositives that contain grammatically essential information must not be surrounded by commas. You may be thinking “Ok, but isn’t all information in a sentence ‘essential’?” You may be surprised to discover that it’s not. Learning to distinguish between grammatically essential and grammatically extraneous information can help make working with appositives a breeze.

Examples of Appositives

Consider the following two sentences, each of which contains an appositive:

My cat, Bill, likes to eat biscuits.

The word “Bill” is set apart from the rest of the first sentence by commas because it is an appositive. An appositive is a type of noun phrase that renames its subject; here “Bill” is renaming “my cat.” Appositives must be surrounded by commas if they consist of “extraneous” or “grammatically inessential information.” This is the case here.

Appositives can also appear at the beginning or the end of a sentence, in which case they only require a single comma to be correctly set apart:

I never seem to stock enough of his favorite kind, slightly burnt biscuits.

Strange behavior for a cat, Bill’s biscuit-eating habit prompted a trip to the vet.

Essential vs. Inessential

You can test whether an appositive contains necessary or unnecessary information by removing it from the sentence and seeing whether what is left makes sense and doesn’t change the sentence’s initial meaning. That sounds complex, but is easier to see in action:

Bill, my cat, likes to eat biscuits.

Bill likes to eat biscuits.

In this sentence, “my cat” is extraneous, and so must be surrounded by commas. You don’t “lose” any specificity contained in “Bill likes to eat biscuits” if you don’t know that Bill is my cat.

Louis Armstrong, my favorite musician, was a trumpet virtuoso.

Louis Armstrong was a trumpet virtuoso.

The same thing happens here: you may not know that Louis Armstrong is my favorite musician, but not knowing that doesn’t change the specificity of “Louis Armstrong was a trumpet virtuoso.”

Let’s take a look at essential information, and what happens when you try to remove it:

The science fiction writer Isaac Asimov is perhaps best known for his *Foundation* series.

✗ The science fiction writer is perhaps best known for his *Foundation* series.

Removing the appositive drastically changes this sentence’s meaning, and introduces confusion by omitting necessary information. Readers are left wondering, “Which science fiction writer are you talking about?” If you’re left with a similar question after removing an appositive, chances are it is essential information. Leave it in the sentence and avoid surrounding it with commas.



Practice Problems: Appositives

Identify which of the following sentences contain comma errors related to the use of appositives.

See Correct Answers and Explanations

1. My neighbor's dog Tiny Tim is a Great Dane the size of a small horse.
2. The great mathematician Archimedes is credited with shouting "Eureka!" upon solving a tough problem.
3. Despite disinterest in school elections, the majority of the students voted Shindra the treasurer, president of the student government.
4. Everyone was talking too loudly for me to hear the speaker, a soft-spoken environmental activist with a strong legal background.
5. Most schoolchildren have heard the tale of Robin Hood the medieval outlaw who stole from the rich and gave to the poor.
6. The speaker, a long-time civic activist, urged the crowd to take a grassroots approach to politics.
7. The immediate reaction to the announcement, complete chaos overtook the newsroom.



Commas for Phrases and Clauses

Prepositional Phrases: One Rule to Remember

Prepositional phrases that begin sentences need to be followed by a comma.

After we look around that entire antique store, we'll probably be tired.

Those that appear in the middle or at the end of a sentence do not need to be preceded by a comma.

I'd like to stop and get some lunch before we start browsing.

A grilled cheese sandwich with tomato soup sounds delicious.

Subordinate Clauses: Location, Location, Location

Commas are used to separate subordinate clauses from the independent clauses to which they attach. Subordinate clauses can appear at the beginning of a sentence, in the middle, or at the end, and comma rules for subordinate clauses depend on the position of the clause in the sentence.

For a subordinate clause that starts a sentence, separate it from the sentence with a comma.

Whenever I pass by that store, I want to go in.

For a subordinate clause in the middle of a sentence, surround it with commas.

I think that its assorted wares, which are all antiques, are fun to examine.

For a subordinate clause at the end of the sentence, don't add a comma.

I think it's fun to sift through old items even if I don't end up buying anything.

Relative Clauses: Essential or Not?

Things get a little trickier when a subordinate clause starts with a word like "who," "which," or "whatever"—a relative pronoun. We call these clauses "relative clauses." Comma usage here depends on whether or not the clause provides information essential to the understanding of the rest of the sentence.

Sarah, who is a piccolo player, practices her music at 5 am and wakes the entire neighborhood up.

Does the information that Sarah is a piccolo player change your understanding of the information that she practices at 5 am and wakes the entire neighborhood up? No. You can check this by removing the relative clause to make sure the sentence still makes sense without it:

Sarah practices her music at 5 am and wakes the entire neighborhood up.

This is an example of an inessential relative clause. While it may add information to the sentence, the information that it adds is not vital to readers' understanding of the rest of the sentence. Inessential relative clauses should be surrounded by commas.

The piccolo player who practices at 4 am is perhaps the neighborhood's bigger problem.

In this sentence, the information conveyed by "who practices at 4 am" is required by the reader to understand the rest of the sentence. Removing it causes confusion:

✗ The piccolo player is perhaps the neighborhood's bigger problem.

Readers are left wondering, "Which piccolo player do you mean?" After all, a comparison is being made with "bigger" that isn't clarified whatsoever. This sentence's relative clause is thus an essential relative clause. Essential relative clauses should not be surrounded by commas.

Relative Pronouns

That
When
Which
Whichever
Whichsoever
Who
Whoever
Whosoever
Whom
Whomever
Whomsoever
Whose
Whosoever
Whatever
Whatsoever



Practice Problems: Commas for Phrases and Clauses

Identify which of the following sentences contain comma errors related to the use of dependent clauses.

[See Correct Answers and Explanations](#)

1. Although he initially refused to go to the cinema, James finally saw the movie, that his friends had recommended.
2. If you decide to go to the beach make sure to wear sunscreen to protect your skin!
3. Istanbul, despite being an old city has recently been the site of major commercial developments.
4. Sally was able to fix the display problem within two minutes due to her knowledge of computers.
5. The restaurant that you recommended was busy, because its food is known for being excellent.
6. I tossed the ball to Sam, who caught it.
7. Whatever dish you want to order for us just make sure it doesn't have peanuts in it.



Agreement Errors

In English grammar, certain elements of each sentence must remain consistent—i.e. agree with one another—throughout the sentence. These elements “agree” in that they take a consistent form with their matching grammatical item. There are four elements of a sentence that must agree throughout the sentences in which they appear: modifiers and the words they modify, pronouns with antecedents, nouns with nouns, and subjects with verbs.

Examples of Correct Agreement

We need to be careful with all of these sharks around.

Here, the “all of these” modifies “sharks” in that it specifies which sharks are being referenced. Since “sharks” is a plural noun, a plural pronoun modifier is required. If the sentence mentioned were just one shark, it would read, “We need to be careful with this shark around.”

Kelly and Peter have spent thousands of dollars trying to fix their car.

This sentence correctly maintains agreement between the plural subject (“Kelly and Peter”) and the plural possessive pronoun used to refer to that subject later (“their”).

All four of my sisters have been to college.

Because a number exceeding one is specified, we know that the noun “sisters” will need to be plural, which, in this case, it is.

One of his books is great!

Subjects must agree, in number, with their verbs. In this case, since “one of his books” is the subject the verb must be singular, which, in this case, it is. Note that if the sentence began with “all of his books,” then “are” would be the correct, agreeing verb form.

Modifier-Word Modified Agreement Errors

Anytime you use a modifier to provide context for or information about an element of the sentence, the modifier and the word modified must agree in number and form.

✗ Jimmy is a man of innumerable talent.

While the subject of this sentence (“Jimmy”) and verb construction of this sentence are both (correctly) singular, the object of the sentence “talent” should be plural, given that the word “innumerable” is used to modify it. Any word modified by “innumerable” will have to be plural. The correct version of this sentence reads:

Jimmy is a man of innumerable talents.

Let’s not forget that there are many things left to be modified other than the mere number or an item. Many modifiers clarify the manner in which something was done, or the qualities something possesses.

✗ Jimmy’s first book is real good.

In this case, the word being modified is an adjective (“good”), so the modifier must agree by taking the form of an adverb. The correct version of this sentence reads:

Jimmy’s first book is really good.



Pronoun-Antecedent Agreement Errors

A good thing to keep in mind when using personal pronouns is that they are shorthand substitutions for proper nouns. Anytime you use a personal pronoun, that personal pronoun should be replaceable with a name, or list of names.

Singular personal pronouns (“he” and “she”) replace individual proper nouns, while plural personal pronouns replace plural proper nouns (“they” and “them”). Anytime a pronoun is used, that pronoun must agree in number and gender with the antecedent to which it refers.

✗ The Markus brothers followed me all the way to the store; I was so furious with him.

Here, the pronoun “him” has no clear antecedent, since it does not correctly refer to the plural antecedent “the Markus brothers.” Since we know that there are at least two Markus brothers, and the two clauses are connected into a single compound sentence through the use of a [semicolon](#), the pronoun here would either have to specify which brother the speaker was angry with, or correctly agree with the plural antecedent.

The corrected version of this sentence reads:

The Markus brothers followed me all the way to the store; I was so furious with them.

Noun-Noun Agreement Errors

Related nouns in a sentence should usually agree in number. For instance, it is incorrect to say that:

✗ The doctor promised me an answer to my questions.

Here, since there are many “questions,” so these questions should be matched with more than one “answer.” The correct version of this sentence reads,

The doctor promised me answers to my questions.

An exception to this rule comes with certain abstract nouns, which should usually remain singular, even if related to a plural noun.

✗ His braveries are amply evidenced by the medals and citations on his wall.

This sentence obviously reads quite strangely. The abstract noun “bravery” should remain a singular noun, and the verb should agree with it, even while the plural nouns making up the subject of this [passive voice](#) remain plural.

His bravery is amply evidenced by the medals and citations on his wall.

Subject-Verb Agreement Errors

A noun acting as the subject of a sentence must always agree with the verb in number. This is most easily demonstrated using “be” verbs.

✗ Those kids is always talking during class.

This sentence is clearly incorrect, because there are many “kids” but the singular form “is” follows. The correction here is simply to change the form of the verb.

Those kids are always talking during class.



Usually the subject will precede the verb, but there are some exceptions. For an example of such an exception let's look at the following sentence.

✗ There is many difficulties inherent to the practice of baking near-perfect cookies.

Sentences beginning with “there is” or “there are” always have their subjects following their verbs. Here, the subject is “difficulties,” so the sentence should read,

There are many difficulties inherent to the practice of baking near-perfect cookies.

Another thing to watch out for is collective nouns. Collective nouns, although they refer to groups of people or things, are singular subjects, grammatically speaking. If a collective noun is the subject of a sentence the verb should remain in a singular form.

✗ The board of directors make these kinds of decisions.

This sentence is incorrect because “the board of directors,” although composed of many people, is acting as a singular noun. Since this collective noun is the subject of the sentence the verb must take a singular form.

The board of directors makes these kinds of decisions.

Anytime the coordinating conjunction “or” is used to connect two subjects in a sentence, those subjects remain singular. When “and” is used in this fashion the two subjects are joined together into a plural; when “or” is used the two subjects are grammatically coordinated but kept unique, and thus singular.

✗ The book or the pen are in the drawer.

Since this sentence uses “or,” each of the two items discussed remains grammatically singular. The corrected version of this sentence reads,

The book or the pen is in the drawer.

Or, alternatively, a corrected version of this sentence could read,

The book and the pen are in the drawer.

A tricky situation arises in verb-agreement when “every” or “each” is used. Both of these words, when used as adjectives or adverbs, will individualize the noun they modify.

✗ After the devastating playoff loss, each member of the team were upset.

This sentence is incorrect as written. Since “each member” is an individual person the verb describing “each member’s” actions must be singular, not plural. Corrected, this sentence reads,

After the devastating playoff loss, each member of the team was upset.

Or, alternatively, since “all” keeps the subject plural, the sentence could read,

After the devastating playoff loss, all members of the team were upset.

It is important to keep track of the subject and verb of a sentence for many reasons, one of these is to make sure that the subject and verb agree in form and number.

✗ The teenagers who enjoy reading this classic myth is few and far between.

Some singular nouns ending with “s”:

- mathematics
- billiards
- measles
- cards
- darts



Here, the phrase “who enjoy reading this classic myth” comes between the subject, “the teenagers,” and the verb, “is.” Remove this intervening descriptive phrase, and the error becomes clear. You would never say, “the teenagers is few and far between.”

The corrected version of this sentence reads,

The teenagers who enjoy reading this classic myth are few and far between.

Keep in mind that some singular nouns end in “s.” If such a word is the subject of a sentence, the verb must agree with the singular subject. Most commonly these non-countable noun ending with “s” refer to games, fields of study, activities, or illnesses.

✗ Rabies are terrible diseases.

“Rabies” refers to one specific disease, so it is incorrect to use a plural for the verb “is” to agree with it. Also, in such cases one must make sure that any articles also agree with the singular subject. The correct version of this sentence reads,

Rabies is a terrible disease.



Practice Problems: Identifying Agreement Errors

Identify which of the following sentences contain agreement errors.

1. William and Gary need to get his affairs in order.
2. Each member of the team needs to keep themselves in good physical condition.
3. The inarticulate man at the coffee shop told me he took it real personal that I had accidentally knocked his drink over.
4. For all the settlement offers they might come up with, we'll have a counter offer ready.
5. Numismatics is the study of coins that have lost their monetary value.
6. A giraffe and his ball, if united in a loving and possessive embrace, is not easily parted.
7. Either of those answers is correct.
8. This pants is split.
9. The schools have long histories of grade inflation.

[See Correct Answers and Explanations](#)



Phrase and Clause Errors

Many errors encountered on the ACT English exam stem from disagreement within phrases and clauses. Whether dealing with illogical conjunctions, dangling modifiers, or faulty comparisons, these errors can be subtle but egregious, so practicing your ability to spot them well before test day can help you develop the skills you need for success.

Section Summary

[Conjunction Errors](#)

- [Coordinating Conjunction Errors](#)
- [Subordinating Conjunction Errors](#)
- [Correlative Conjunction Errors](#)
- [Conjunction Logic Errors](#)

[Modifier Placement Errors](#)

- [Dangling Modifiers](#)
- [Ambiguous Modifiers](#)
- [Misplaced Modifiers](#)

[Parallel Structure Errors](#)

- [Parallel Structure of Lists](#)
- [Parallel Structure of Comparisons](#)



Conjunction Errors

Conjunctions are basic words used to join words, phrases, or clauses together. While conjunctions are simple words, they also have a variety of uses, and rules inherent to those uses. Conjunctions can open many wonderful doors to you in your writing, but you must know what is behind each door before you open it. There are no universal rules that govern every situation, but there are consistent rules for each of conjunctions' various uses. In grammar, you never get something for nothing, each tool must be mastered, and in order to be mastered, it must be understood. Conjunctions do not just connect elements of written discourse—through this functional role, they also clarify various relationships between such elements. In writing, as in life, it is just as important to understand how a thing relates to other things as it is to understand the thing itself.

Coordinating Conjunction Errors

Coordinating conjunctions are used to connect words, phrases, and clauses (both dependent and independent). The coordinating conjunction is the most commonly used, and misused, form of conjunction. A key thing to remember about coordinating conjunctions is that, unlike subordinate conjunctions, they connect ideas of equal grammatical rank.

Quick Tip

The seven coordinating conjunctions are “and,” “but,” “for,” “yet,” “so,” “or,” and “nor.”

✗ Jerry is an accomplished writer and he travels to Ireland for residencies quite often.

Coordinating conjunctions can be used to connect two independent clauses into one compound sentence ([semicolons](#) can fulfill the same function, but note that semicolons can never be followed by coordinating conjunctions). Since both clauses could stand on their own as grammatically complete sentences, they require a comma followed by a coordinating conjunctions to be connected into one compound sentence. The sentence above makes an error of punctuation, omitting a necessary comma before the coordinating conjunction “and.”

Jerry is an accomplished writer, and he travels to Ireland for residencies quite often.

Coordinating conjunctions can also be used to directly connect two grammatical items in a sentence.

✗ William, and I are best friends.

The sentence above makes a common error. Many people, in search of the comfort that so often accompanies solidity, make the mistake of always inserting a comma before a coordinating conjunction. Coordinating conjunctions need to be preceded by a comma if they are connection two independent clauses. When they are directly connecting two grammatical items of equal rank, it is incorrect to place a comma before the conjunction.

William and I are best friends.

A brief note on conjunctions at the start of sentences: strict, conservative grammarians will say that it is incorrect to begin a sentence with a conjunction. At this point in English grammar, it is considered, on occasion and with care, allowable to begin a sentence with a conjunction (usually “and” or “but”) followed immediately by a comma; however, this practice should not be overused. It is often unnecessary, if not overtly incorrect, to begin a sentence with a conjunction.



Subordinating Conjunction Errors

As we all know far too well, not all relationships are equal; this is as solid a truth on the field of grammar as it is on the sports field. Subordinate clauses transition between two ideas of unequal grammatical standing in a sentence, and in so doing they assert and express the inequality of the grammatical relationship they are facilitating. The transition facilitated by a subordinate conjunction will point to the causal, temporal, and spatial relationship of the two items being connected.

Coordinating conjunctions join items of equal grammatical rank, like one word to another, or one independent clause to another. Subordinating conjunctions connect a lower-ranked word, clause, or phrase to a higher-ranked one, but how are we ranking these grammatical items?

In this case, the most obvious and illustrative unequal grammatical relationship is that of a [dependent clause](#) to an [independent clause](#). An independent clause is considered a higher-ranking grammatical unit than is a dependent clause, which cannot stand on itself as a sentence and makes grammatical sense only in its relationship to a main clause (also known as an independent clause).

✗ Whenever I go to the store.

This sentence is a fragment; it is a dependent clause connected to nothing. Now, we see how subordinating conjunctions not only signal, but also help determine the grammatical status of a sentence. “I go to the store” is a grammatically complete and correct sentence, but “whenever I go to the store” is a dependent clause. The addition of the subordinating conjunction “whenever” necessitates a main clause explaining what the speaker does “whenever” he or she is at the store. Adding “whenever” has made this phrase into an introductory phrase.

A possible correct version of this sentence is,

Whenever I go to the store, I spend at least twenty dollars.

Correlative Conjunctions

Certain conjunctions require a second, correctly paired conjunction in order to be complete. These are called correlative conjunctions. The rules governing correlative conjunctions are probably the simplest of any of the conjunction rules. Basically, a correlative conjunction will have an accompanying conjunction that it requires in order to be correct.

✗ Neither you or your brother are allowed back in my steak house, if that’s how you talk!

The error here is simple. The correct correlative conjunction for “neither” is “nor.” Anytime you see “neither,” you must see “nor” connecting the two items being discussed or the sentence will be correct. The table to the right contains a list of some of the most commonly used correlative conjunctions.

As . . . as
Hardly . . . when
Either . . . or
Neither . . . nor
What with . . . and
Just as . . . so
Both . . . and
If . . . then



Conjunction Logic Errors

Conjunctions are not just words used to connect grammatical items, they are words themselves, and as such they carry meaning and impose logic on the sentences to which they are added. No two conjunctions are interchangeable; they each carry their own meaning and signal different types of relationships.

Here is an example of such an error:

✗ Although William is a large, strong man, he was able to lift the bag of flour easily.

Here, the subordinating conjunction “although” goes against the meaning and logic of the rest of the sentence. “Although” is used to suggest that in spite of whatever dynamic or fact is being stated, a result that runs counter to what that fact would suggest is true. The fact here is that “William is a large, strong man,” which, in this case lines up perfectly with the result of him being “able to lift the bag of flour easily.”

A correct version of this sentence reads,

Because William is a large, strong man he was able to lift the bag of flour easily.

Since “because” is a straight-line indicator of causality, it makes sense here to explain that William’s size and strength allow him to be “able to lift” heavy things.

If you are ever in doubt about the logic of a sentence as it relates to conjunction, first look at the subordinate conjunctions in the sentence, and decide if they signal a temporal (or order of events), cause-and-effect, or a spatial relationship. If the kind of relationship signaled by the conjunction is appropriate to the sentence, you should then look at the rest of the sentence and decide in what way the dependent clause logically relates to the main clause. Sometimes this relationship will be counterintuitive to the results (signaled by “although,” “even if” etc.), and sometimes the relationship will be consistent with, or suggestive of, the results (“because,” “as,” “provided that,” etc.). Errors of conjunction logic will mostly be a thing to look out for in cases where the relationship is related to causality (illogical temporal relations are usually fairly obvious).

✗ Because I ate so much steak, I was still hungry a mere hour later.

Here, we have an illogical logical relationship. Steak is a protein-heavy food, and quite unlikely to actually cause further hunger. But, hey, everyone has a different metabolism, so let’s look for a grammatical suggestion of the illogic of this sentence. “Because” is a one-and-done coordinating conjunction, it does not require a correlative conjunction, and especially should never be followed by a correlative conjunction suggestive of a contradictory relationship (like “still”). If you see because in the dependent clause, you should never see a conjunction like “still” in the main clause.

A correct version of this sentence reads,

Even though I ate so much steak, I was still hungry a mere hour later.



Practice Problems: Conjunction Errors

Identify which of the following sentences contain errors related to the use of conjunctions.

See Correct Answers and Explanations

1. Though Kenny is athletic but he does not enjoy sports.
2. Even if Kenny is naturally talented at sports, he can still hold his own on the basketball court.
3. Because Kenny was so content to stay home, therefore he would never push himself to succeed.
4. The shooting stars flashed overhead; but Kenny simply stared despondently at the asphalt beneath his feet.
5. Kenny is a fun guy however a bit of a downer tonight.
6. Kenny was definitely having a really tough time for a while, but he seems happy, and healthy now.
7. Provided that Kenny stays on the straight and narrow and he has a good chance of being drafted.
8. After he was drafted, Kenny bought both his mother and his brother condominiums.



Modifier Placement Errors

What are Modifiers?

In writing, modifiers are words or phrases that alter, clarify, or place limits/restrictions on a particular word within a sentence. Modifiers are an important foothold in the long, forever steep climb to the peak of fluent and advanced prose writing. By providing detail about, and context for, basic elements of a sentence, modifiers are a great way to elevate your sentences beyond a basic [Subject-Verb-Object](#) structure.

An Example Illustrating the Necessity of Modifiers

Consider the following sentence:

Dmitri took my sandwich and tapped my cheek with the back of his hand.

In this case, all we, as readers, know is what actions Dmitri took.

Dmitri, my best friend and trusted business partner, took my sandwich and tapped my cheek with the back of his hand.

Now, we not only know what Dmitri did, but we also have a better clue as to why he did those things and how the speaker would have felt about his actions. In other words, the modifier here provides us with an opportunity to know not just what happened, but the spirit in which events were made to happen and the way in which they were received.

Now, to solidify this point, let's insert a different modifier in place of "my best friend and trusted business partner":

Dmitri, a known bully, took my sandwich and tapped my cheek with the back of his hand.

Now, thanks simply to a change in the modifier, the meaning, tone, and emotional content of this sentence is fundamentally altered. What we see here, is that modifiers add content to sentences, and when you add content you are making positive change to the meaning of your sentence.

"Dmitri took my sandwich and tapped my cheek with the back of his hand," is a certainly a grammatically complete sentence, but as we can see, there is so much more to this life than contextless, basic correctness. Writing, at its heart, is about expressing thought, and thoughts are rarely simple enough to expressed with just a subject and a verb.

While modifiers are helpful and necessary, they do add complexity to sentences, and anytime you make something more complex (and interesting) you open the door to that ever-willing house guest, error. There are three basic types of errors specific to the addition of a modifier to a sentence, and we'll look at them one at a time.

Dangling Modifiers

Dangling Modifiers occur when a subject is clearly tied to the main clause of a sentence, and is then forgotten (left hanging or, you might even say, "dangling") in a modifying clause. Always keep the basic, fundamental grammatical structure of your sentence in mind. The modifier must always match the subject of the main clause of the sentence.



Spotting Dangling Modifiers

You can spot a dangling modifier by paying close attention to what thing or object the introductory phrase refers to, and how it interacts with the noun that immediately follows it. Consider the following sentence:

✗ Having finished his book, the lights were turned out.

In this example, “having finished his book” seems to describe “the lights.” This doesn’t make any sense—lights don’t read books! They also aren’t assigned genders in English, so the sentence’s use of “his” provides a major clue that something is grammatically incorrect. Consider another example:

✗ After reading the book, the movie seemed shallow and simple-minded.

In the second sentence, “After reading the book” appears to describe “the movie.” Again, it doesn’t make sense for a movie to read a book, so this sentence also has the potential to confuse readers.

Keep in mind that dangling modifiers don’t have to be phrases: they can consist of single misplaced words:

✗ Burnt, the campers ate the marshmallow.

In this example, it is unclear whether “burnt” refers to the campers (ouch!) or the marshmallow.

Fixing Dangling Modifiers

On the ACT English section, you might be asked to fix dangling modifiers by choosing the best way to rephrase a sentence containing one, so let’s talk about some strategies to fix these errors. In both cases, the main clauses come after the comma.

One option is to name the subject of the sentence in the first clause.

After Kevin had finished his book, he turned the lights out.

After I had read the book, the movie seemed shallow and simple-minded.

Another option is to add the appropriate subject to the main clause.

Having finished his book, Kevin turned the lights out.

After reading the book, I found the movie shallow and simple-minded.

This also makes the sentence smoother by removing an unnecessary and confusing use of [the passive voice](#).

A final, probably less common option, is to simply combine the two clauses into one.

Kevin finished his book and turned out the light.

Note that in this case the modifier has simply been re-formed as a second verb in the sentence.



Ambiguous Modifiers

Ambiguous modifiers are errors that occur when a modifier could logically refer to more than one element of the sentence. Modifiers must always clearly refer to one, and only one, specific element of the sentence. Whereas dangling modifiers tend to occur when modifying phrases are used, ambiguous modifiers often occur when one word modifiers are used to describe an element of the sentence. This is not an absolute rule for either case; one-word dangling modifiers exist, as do ambiguous modifying phrases, but one-word ambiguous modifiers can often be the most difficult to spot.

Ambiguous modifiers are errors because they confuse the reader, rather than helping clarify the situation described in the sentence for the reader.

Examples of Ambiguous Modifiers

- ✗ Kevin's ophthalmologist told him to strictly read during the day.
- ✗ People who enter a movie theater with low expectations often are pleasantly surprised.

First, let's identify the modifiers in each of these sentences, and the parts of the sentences to which they refer. In the first sentence "strictly" modifies either the manner in which Kevin's ophthalmologist instructed him (hence the ambiguity). In the second sentence the modifying phrase "with low expectations" refers to either the people or the theater.

Quick Tip

As a general rule, it is best to place a modifier as close as possible to the subject it modifies in order to avoid confusion.

To fix these errors, we simply need to move the modifiers so that they are as close as possible to the grammatical item to which they refer.

Kevin's ophthalmologist strictly told him to read during the day.

Kevin's ophthalmologist told him to read strictly during the day.

People who enter a movie theater with low expectations are often pleasantly surprised.

People who often enter a movie theater with low expectations are pleasantly surprised.

Note that since these modifiers were ambiguous, there are two grammatically correct options to change the sentence. Whether one option is more logical is another question.

A Note on Limiting Modifiers

Be especially careful with the placement of limiting modifiers ("only," "nearly," "just," etc.). Limiting modifiers can often be ambiguous, as they can correctly refer to many parts of a sentence, so they should be placed directly in front of the thing they are placing a limit on.

For example, notice the difference in meaning between these two sentences:

Only Jim hates that movie.

Jim hates only that movie.

The first sentence means that Jim and Jim alone "hates that movie." The second sentence means that Jim does not hate any other movie other than the one being discussed. The entire meaning of a sentence can be changed simply by moving a limiting modifier. Both these sentence are correct, grammatically, but if you intended one meaning and ended up with the other it would be a completely misleading communication of your intended meaning.



Misplaced Modifiers

Misplaced Modifiers are similar in nature to Ambiguous Modifiers. They are modifiers that are incorrectly placed in a sentence; however, instead of possibly referring to two items in the sentence, through their inaccurate placement misplaced modifiers do not correctly modify anything in the sentence. Consider the following example of a misplaced modifier:

✗ In the locker room, I found a silver man's tie clip.

This sentence is clearly incorrect, both grammatically and in terms of its relation to reality. At this particular juncture of history, humans are not made of silver; they are flesh and bone. Some human men do, however, own "silver tip clip[s]." Also, note that in this case, if you were reporting on finding a tie-clip and wished to categorize the department in which the tie clip would be sold, the correct wording would be "a silver men's tie-clip."



Practice Problems: Modifier Errors

Identify which of the following sentences contain errors or ambiguity related to the use of modifiers.

1. Seeing how rampant the corruption was, a citywide inquest was called.
2. I'm writing a biography of Thomas Love Peacock in my graduate seminar.
3. After having been humiliated in the three-legged race, Kevin and Jane quietly walked back to their car.
4. We served delightful smoothies to our guests in tall, flimsy paper cups.
5. On the way home from school, I saw my best friend Juan buying a cup of gelato.
6. Only my boss paid me six dollars an hour to do his taxes for him.
7. He nearly ran 10 kilometers every day.

[See Correct Answers and Explanations](#)

Parallel Structure Errors

What is Parallel Structure?

Parallel sentence structures occur when a matching pattern of words is used. You use a parallel structure to suggest similarity, especially a similarity in terms of importance, between two or more items in a sentence. Parallel structures will often include the conjunctions “or” or “and.” As you may guess from that feature, parallel structure is an important component of all comparisons. Parallel structures can occur within phrases, between two words, or within entire clauses.

Examples of Correct Parallel Structure

I went to the greenhouse and purchased petunias, tulips, and chrysanthemums for the garden.

This sentence uses correct parallel structure because each of the items in its list is a plural noun.

My favorite sports are hockey and swimming.

This sentence’s parallel structure is also correct because the two favorite sports listed are both nouns. (Though “swimming” is a gerund, it is acceptable because there is no noun naming the sport in common parlance that is not a gerund.)

I think that my grandmother’s cookies are better than anyone else’s.

This sentence may look a bit tricky, but it also demonstrates a correct use of parallel structure because “anyone else’s” implies the meaning “anyone else’s cookies.” That means that two types of cookies are being compared: those that the speaker’s grandmother makes and those that anyone else makes.

Parallel Structure Errors

Parallel structures can be useful in suggesting parity between things and in formatting lists of three or more items, but it is important to keep in mind that once you begin using a parallel structure, you are committed! Parallel structures must remain consistent throughout a sentence.

Examples of Parallel Structure Errors

- ✗ You must get results quickly, efficiently, and at a low cost.
- ✗ I enjoy reading novels, playing poker, and to ride my bicycle along the Olentangy River.
- ✗ He likes to eat but not cooking.

In the first sentence, we see parallel structure set up with the adverbs “quickly” and “efficiently”; therefore, the third item in the list should also be an adverb that refers to the verb “get.” A good option would be “cheaply.” If the first two items in a list are adverbs, the third item should also take an adverbial form.

You must get results quickly, efficiently, and cheaply.

In the second sentence, a parallel structure is set up with the use of the gerundive forms of “reading novels” and “playing poker,” so the third item needs to match this form. “Riding my bicycle” would correct this error.

I enjoy reading novels, playing poker, and to riding my bicycle along the Olentangy River.

The third sentence highlights an important rule for parallel structures. When two items are connecting



with a coordinating conjunction (these items can be words, clauses, or phrases) the structure of the items must be parallel.

He likes to eat but not to cook.

A Note on Lists

Lists of three or more items should be framed in parallel structures. Note that the rules surrounding lists of three or more items remain consistent whether or not the list is preceded by a colon.

✗ Max loved only three things: model airplanes, fast cars, and going fishing

This sentence is incorrect, since the third “thing” is not a “thing” like the first two, but is instead an activity. Since “going fishing” is fundamentally a different kind of “thing” than the other two, it would be best to split this sentence in two.

Max loved model airplanes and cars. He also loved to go fishing.

Comparisons

Parallel structures must also be used when making direct comparisons. Comparative adjectives such as “more,” “better,” “less,” and “worse” are good indicators that a parallel structure will be needed.

✗ It takes much less time to drive there than it does flying.

✗ My physical abilities pale in comparison to how smart I am on the baseball field.

Both of these sentences create faulty parallelism. The first sentence needs to maintain the same verb form for both items being compared. The second sentence makes a similar error, but this time the faulty parallelism exists between nouns in the sentence.

The sentences can be corrected as follows:

It takes much less time to drive there than it does to fly

My physical abilities pale in comparison to my intelligence on the baseball field.

Comparisons can also house a different type of parallel structure error in which two nouns are being compared, but they are two different types of things, resulting in a sentence that just doesn’t make sense.

Consider the following incorrect example:

✗ I think that Sheila’s painting is much more original than Bob.

While “Sheila’s painting” and “Bob” are both nouns, we are comparing a painting to a person, and we need to compare a painting to a painting. Editing “Bob” to read “Bob’s painting” fixes this error:

I think that Sheila’s painting is much more original than Bob’s painting.



Practice Problems: Parallel Structure Errors

Identify which of the following sentences contain errors related to the use of parallel structure.

1. I've always felt that my work ethic was more valuable than how much schooling I had.
2. There are so many different kinds of great chefs: self-taught chefs, classically trained chefs, restaurant-trained chefs, and people who just go in there and cook.
3. I've always enjoyed going to the library far more than I've enjoyed going to the gym.
4. The three rescue dogs Jeremy is thinking about adopting are, a greyhound, a golden retriever, and this other weird kind I've never heard of.
5. My sisters all have their good traits: Angela refuses to take no for an answer, Mary is loyal, Jenny is brave, and Camilla is gentle.
6. Either you can drive me to the airport or you will be a jerk.

[See Correct Answers and Explanations](#)

Sentence Completeness Errors

To be grammatically correct in standard modern formal English, a sentence must be correct. Casual conversations and their representations in writing follow a similar but more lax set of rules that provide a few key exceptions. Working with sentence fragments—errors that appear when a sentence doesn't contain a subject and a verb (and isn't one of those exceptions)—as well as run-on sentences and other errors encountered when combining multiple sentences together can help prepare you for dealing with sentence completeness errors on test day.

Section Summary

[Sentence Fragment Errors](#)

- Sentence fragments are ungrammatical constructions in which a subject or predicate is missing from an independent clause.

[Sentence Combination Errors](#)

- A “run-on” sentence is a sentence in which multiple independent clauses are incorrectly connected into a single sentence.



Sentence Fragment Errors

Almost* any sentence that doesn't contain a subject and a predicate is a sentence fragment and grammatically incorrect. Sometimes these sentence fragments are easy to spot:

- ✗ The manatee, also know as a "sea cow."
- ✗ Swimming along peacefully.

*Three exceptions exist, they are not likely to show up on the ACT English section as each one is limited to being used specifically in conversation, and so would need to be represented as speech or dialogue.

- 1.) Commands don't have a subject in the sentence, but can be read as having an understood or implied subject of "you," the person being addressed:

(You) Come look out the window right now!

- 2.) It's permissible in standard formal English for certain exclamations or verbal expressions to lack a subject or a predicate. These are often single-word expressions or extremely short ones.

"Stop! Thief!"

"What?"

"Ouch!"

- 3.) Also, in written dialogue or when speaking, when providing the answer to a question that someone has asked, it's completely permissible to leave out the subject, as long as it is understood.

"Where's the pizza?"

"Over there." (Meaning "The pizza is over there." "The pizza" is the understood subject.)

While informal communication is relatively lax about the demand for both a subject and a verb in each non-imperative sentence, the ACT English section is testing your understanding of formal English, so if you see a sentence fragment, make sure and correct it, no matter how casual the tone of the passage.

- ✗ When Amy got back and found her car was missing, she was flabbergasted. And mad!

If you want to keep the emphasis on information conveyed in a sentence fragment that appears after a complete sentence to emphasize some aspect of what's being discussed, you can use a hyphen to set off the emphasis material in a way that results in a grammatically correct, but just as emphasized, construction:

When Amy got back and found her car was missing, she was flabbergasted—and mad.

Questions and certain constructions like those that use the subjunctive may require a bit of hypothetical rearranging into a more declarative-like order in order to confirm that they contain a subject and a verb.

Did you not anticipate that many guests? --> (You did not anticipate that many guests.)

You saw the balloon floating away, too? --> (You saw the balloon floating away, too.)

- ✗ If only a double batch of cookies!
(If only we had made a double batch of cookies!)

Even if a sentence looks like it contains a verb, the word that looks like a verb might be a participle, a verb functioning as an adjective, which would leave the sentence without an actual verb.

- ✗ The general dancing a lively polka with his archnemesis.

This is an incomplete sentence that it might be easy to miss due to its use of a participle. It leaves the reader asking, "What did 'the general dancing a lively polka with his archnemesis' do?" It can be fixed by adding a helping verb to create a present progressive verb or by changing "dancing" to another form of the verb "dance." Of course, you can always add a predicate to the sentence as well to complete it.

The general is dancing a lively polka with his archnemesis.

The general danced a lively polka with his archnemesis.

The general, dancing a lively polka with his archnemesis, certainly appeared to be having fun.



Practice Problems: Sentence Fragment Errors

Identify which of the following sentences or pairs of sentences are ungrammatical fragments.

See Correct Answers and Explanations

1. Were we to purchase more food.
2. We were to purchase more food.
3. Natalie is busy buying vegetables. Or, more likely, more chickens nuggets, her favorite food.
4. As soon as we learned the news that the test had been rescheduled, the hush that fell over the entire study group.
5. The singer belting out the refrain on stage while remaining perfectly on-key.
6. Do you know where I can dry clean some socks? Because mine are damp and gross.
7. Because it was raining outside, I decided to bring an umbrella.



Sentence Combination Errors

“Run-On” Sentences

There is a wide variety of ways in which two sentences can be incorrectly combined, but in many of them, the infamous “run-on” sentence rears its head. Whereas many people may understand “run-on” to mean *potentially grammatically correct but just way too long and unwieldy for readers to easily understand*, we’re going to use “run-on” to specifically indicate an ungrammatical combination of sentences that needs a comma to be corrected. Grammatically correct sentences can be difficult to read due to being too long and benefit from trimming, but in this case, we’re going to restrict our use of “run-on” to describing the grammatical error, not the stylistic one.

Combining Sentences Correctly

Consider the following:

Whenever I wear my favorite shoes, it rains. They get wet no matter how hard I try to avoid puddles.

There are two ways in which two declarative sentences can be combined into a complex sentence:

1. Connect the sentences with a semicolon and no conjunction

Whenever I wear my favorite shoes, it rains; they get wet no matter how hard I try to avoid puddles.

2. Connect the sentences with a comma followed by a conjunction

Whenever I wear my favorite shoes, it rains, and they get wet no matter how hard I try to avoid puddles.

There’s also a third option to consider when dealing with run-on sentences:

3. Separate the sentences into distinct independent clauses, end the first with a period, and capitalize the first word of the second. In other words, make them two distinct sentences. Be careful where you break the run-on apart, though! Each new sentence needs a subject and a verb!

✗ Whenever I wear my favorite shoes, it rains and they get wet. No matter how hard I try to avoid puddles.

Sentence Combination Errors

Sentences are combined incorrectly if a conjunction is used after a semicolon, or if no conjunction is used after a comma. (That latter error is known as a “comma splice” and is treated in detail [here](#).)

✗ Whenever I wear my favorite shoes, it rains; and they get wet no matter how hard I try to avoid puddles.

✗ Whenever I wear my favorite shoes, it rains, they get wet no matter how hard I try to avoid puddles.

In addition, it is incorrect to combine two independent clauses with a conjunction alone.

✗ Whenever I wear my favorite shoes, it rains and they get wet no matter how hard I try to avoid puddles.

Of course, it’s also (rather blatantly) incorrect to just jam two sentences together.

✗ Whenever I wear my favorite shoes, it rains they get wet no matter how hard I try to avoid puddles.



Making and Analyzing Editing Decisions

Certainly the broadest in scope, ACT English questions that test your understanding of how paragraphs work together and how the entire passage works as a unit are also perhaps the test's most challenging. These questions look beyond discrete black-and-white grammatical errors and instead ask questions that require you to pick up on subtler shades of meaning and to consider the passage's elements from the perspective of a hypothetical reader as you weigh in on which content should stay, which should go, and which should be rephrased. Needless to say, practice is crucial in preparing for these sparse but challenging questions, and the more you can hone your broad-range editing skills, the more comfortable you can be with this material on test day.

Section Summary

[Introduction: Editing in Context vs. Error Identification](#)

[What's "Context?"](#)

[Connotation vs. Denotation](#)

[Direct Editing Decisions: Which Option is Best?](#)

[Adding Content](#)

[Choosing the Best Sentence for a Spot](#)

[Choosing the Best Spot for a Sentence](#)

[Rearranging Content](#)

[Moving Sentences](#)

[Paragraph Breaks](#)

[Rephrasing for a Particular Effect](#)

[To Edit Content or Not—and Why](#)

[To Add Content or Not—and Why](#)

[To Remove Content or Not—and Why](#)

[To Rephrase or Not—and Why](#)

[Identifying the Function of Passage Elements](#)

[Identifying the Function of Punctuation](#)

[Identifying the Function of a Word](#)

[Identifying the Function of a Phrases or Clause](#)

[Identifying the Function of a Sentence](#)

[Identifying the Effects of Editing](#)

[Identifying the Effects of Adding Content](#)

[Identifying the Effects of Removing Content](#)

[Identifying the Effects of Rephrasing Content](#)

[Analyzing Passage Purpose](#)



Editing in Context vs. Error Identification

Editing and passage-based question types and the skills they test are vastly different from the “find-the-grammatical-error-if-there-is-one” questions of the rest of the section. Those present little difficulty in the way of timing because all of the information you need to answer the question is contained in the underlined section or the sentences surrounding that selection. The same is not true for paragraph-level questions, though. First of all, the answer options you are presented with are all grammatically correct, which may throw some students off if they are in error-seeking mode. What is being tested here is not error-finding skills but the ability to pick up on far subtler things than comma splices and semicolons—the shades of meaning conveyed to readers in choices of authorial tone, passage purpose, sequence of information, and topic material.

What’s “Context?”

You may have heard the word “context” used before in phrases like “reading in context” or when someone is talking about a statement “taken out of context.” “Context” is just a fancy term for all of the information around the sentence, word, or grammatical item that you’re considering in a text. For instance, the word “blue” can refer to *the color of the sky*, or it can mean *sad and melancholy*. Both are completely valid meanings, so readers have to use context clues every time they see the word “blue” in order to figure out which meaning is being used in a given work. Consider the following sentence:

He was feeling blue and almost didn’t attend the party, which featured blue streamers.

This sentence uses both meanings of the word “blue,” but you can tell which is which by looking at what else is going on in the sentence—the “context.” The first use of “blue” is used in the context of someone’s emotion—the person being discussed is “feeling blue.” It doesn’t make sense for this use of “blue” to refer to a color, but it does make sense for it to refer to an emotion. This use of blue must mean *sad and melancholy*. As for the second use of blue, this one is referring to “blue streamers.” Streamers are inanimate objects that can’t feel emotions, so it doesn’t make sense for this use of blue to mean *sad and melancholy*. This use of “blue” must be referring to the color blue.

Understanding what clues the context of a term or sentence within a paragraph offers is a valuable skill on the ACT. In the ACT Reading section, contextual understanding is placed in the spotlight along with other aspects of reading comprehension prowess, but it also plays a role in certain questions on the ACT English section, those which ask test-takers to consider entire passage and weigh the pros and cons of multiple potential adjustments to the text.

Now that we’ve a clear understanding of what “context” means, let’s consider two types of questions that concern picking additional content to add to an ACT English passage. There are two factors in play for these question types: the sentence to be added and where it will be added. Which one you are provided and which you are asked to choose creates two distinct question types.

Connotation vs. Denotation

The meaning of a word can be defined in different ways; two of those ways are through its connotation and its denotation. A word’s denotation is the meaning that you would expect to find if you looked it up in the dictionary; it’s denotation is what concrete idea it conveys. In contrast, a word’s connotation is the set of emotions or feelings it prompts in readers. Certain words can have near-identical denotations but differ wildly in the connotations they carry.



Words are often discussed in terms of having “positive” or “negative” connotations. For instance, consider the following words:

- To grace (a location) with one’s presence
- To visit
- To infest
- To plague

Each of these terms describes being in a location, but “to grace (a location) with one’s presence” has positive connotations about the person or thing in the location, whereas “to plague (a location)” has negative connotations. These connotations form part of the meaning of the word that can become the deciding factor when picking between two similar answer choices, so it’s important to be aware of these subtle shades of meaning.



Direct Editing Decisions: Which Option is Best?

On the ACT English section, some passage-editing questions present options for rephrasing a specified part of the passage. Just like how error-based questions don't ask you to specify the type of error you're fixing, these questions don't ask you why you're picking a particular answer—they just want you to indicate which best suits the paragraph. Whether you're choosing the best spot for a provided sentence, picking out the best point for a paragraph break, or rephrasing a sentence to achieve a particular effect, your writing and editing skills are sure to be challenged. These questions may be the most straightforward of the ACT English section's passage-level problems, but that doesn't mean they're easy!

Section Summary

[Adding Content](#)

Practice Passage: "The Big Impact of Monet's *Impression, Sunrise*"

[Choosing the Best Sentence for a Spot](#)

- These questions point to a particular spot in the passage and ask which of the four provided sentence options best works when inserted into that specific spot.
- Consider how variation in the answer choices works with the sentences immediately before and after the indicated location.

[Choosing the Best Spot for a Sentence](#)

- These questions provide a sentence and ask you where to put it in the passage.
- Pay attention to sentence transitions around the designated location and how inserting the provided sentence affects the order in which information is presented in the passage.

[Rearranging Content](#)

Practice Passage: "National Parks and Monuments"

[Moving Sentences](#)

- These questions ask if a sentence in the passage should be moved elsewhere in the passage or left where it is.
- Consider how the indicated sentence would interact with the sentences that would precede and follow it.

[Paragraph Breaks](#)

- These questions ask where the writer should break apart one long paragraph into two distinct paragraphs.
- Consider if two ideas are discussed, and try to pinpoint the exact moment between two sentences where the writer stops talking about one idea and starts talking about another. That is likely the best place for a new paragraph to start.
- Remember, when dealing with dialogue, convention says that if a new person is speaking, their dialogue should begin a new paragraph.

[Rephrasing for a Particular Effect](#)

Practice Passage: "Tourism vs. Preservation in the Lascaux Caves"

- These questions ask which of the presented options best fulfills a stated goal or objective.
- None of the answer choices will contain grammatical errors, and they all may seem like great options to use in the passage. It's crucial that you focus on the described effect the author wants to achieve with the indicated text when deciding on the best option.



Adding Content

Questions that add content test your sensitivity to issues of paragraph flow and the order in which information is introduced. They may indicate a spot in the passage and ask which of the answer options would work best in the passage if added in that location, or they may present a sentence and ask in which of several indicated locations it should be placed. Other paragraph-editing ACT English questions may ask you to deal with content already in the passage—rephrasing it, indicating its effect, or considering if it should be moved to a different location. Adding content questions are set apart in that they ask you to deal with content that is completely distinct from the presented passage. Learning to discern associations between the presented options and the text can help you face these problems with confidence.

Choosing the Best Sentence for a Spot

Picking the correct sentence to add to an ACT English passage can require a great deal of skill. Not only do you have to understand all the details of context, but tone and sequence also become relevant in deciding which of the presented options best fits the passage at a particular point. Whether you are asked to choose an introduction, a transition, a body paragraph sentence, or a conclusion, the premise remains the same: take your time and pick up on every detail you can in order to determine which sentence not only makes sense, but doesn't disrupt the flow, tone, or topic of the passage.

Practice Passage: The Big Impact of Monet's *Impression, Sunrise*

- [1] When Monet's painting of his hometown's harbor titled *Impression, Sunrise* debuted in a Paris art show, critics had no idea what to make of it. [2] Despite the qualms and complaints of art critics, Monet's piece became famous.
- [3] Many artists began to mimic the style of Monet's painting, which departed from the carefully realistic portraiture and history paintings of previous works and instead aimed to present the suggestion of a subject, paying special attention to the interaction of light with the subject. This style eventually developed into an entire artistic movement that took its name from Monet's painting and came to be called "Impressionism." [4]

Choosing an Introductory Sentence

When choosing an introductory sentence for a passage, even if this is the first question of the set, don't answer it without first reading the entire passage! Introductory sentence options can differ based on their varying ability to foreshadow or reference material that is mentioned later in the passage, starting it by pointing it in the direction of topics later discussed. Discerning between these answer options obviously requires you to know what the passage talks about in each of its sections, so read the entire thing first before picking the best opening line.



Sample Question

Which of the following would best function as the first sentence of the passage, e.g. where it currently says “[1]?”

- A. Claude Monet’s most famous painting is without a doubt *Impression, Sunrise*, though critics hated it when it first debuted.
- B. Claude Monet is famous in art history for producing the first work in a famous and important movement in painting that also gave that movement its name.
- C. Artists like Claude Monet have proceeded despite criticism, providing a great example of why artists shouldn’t be discouraged by critical disparagement.
- D. Claude Monet is a painter who is known for his beautiful landscapes, which many people love for their bright colors.

A few factors determine this question’s correct answer: sequence, type of passage, and scope.

Sequence: When picking a sentence to add to an ACT English passage, it’s important to consider the sequence of the resulting work. Certain information can be presented too early, or it might not make sense given the context of other lines. This is the case in this sample question. Answer choice A, “Claude Monet’s most famous painting is without a doubt *Impression, Sunrise*, though critics hated it when it first debuted,” might seem like a perfect choice for the passage given that it mentions the specific painting that is discussed and the initial critical reaction to it; however, take a closer look of how this topic sentence would actually function in the first paragraph:

Claude Monet’s most famous painting is without a doubt *Impression, Sunrise*, though critics hated it when it first debuted. When Monet’s painting of his hometown’s harbor titled *Impression, Sunrise* debuted in a Paris art show, critics had no idea what to make of it.

Getting déjà vu? This topic sentence tells us much of the information that the paragraph’s second sentence provides, creating a bit of confusion for the reader as they encounter details like the painting’s name twice. Plus, we’re told in the first sentence that “critics hated [Monet’s painting] when it first debuted”—a very specific reaction—before the second sentence provides a more general description of their reaction: “critics had no idea what to make of it.” This also creates confusion for the reader, as the passage doesn’t read as if a more specific description of the critics’ reaction was given in the first sentence. The sequence problems created by this answer choice mean that it isn’t the best topic sentence for the passage after all.

Type of Passage: This is not an argumentative passage. The writer is not attempting to convince us of his or her opinion on anything by providing evidence; he or she is simply providing facts. Once you’ve picked up on the general purpose of the passage, any answer choices that don’t jive with that purpose, such as answer choice C, stick out like a sore thumb: “Artists like Claude Monet have proceeded despite criticism, providing a great example of why artists shouldn’t be discouraged by critical disparagement.” Why do we need “a great example of why artists shouldn’t be discouraged by critical disparagement?” In this passage, we clearly don’t. At no point in the passage do we hear the author’s opinion and voice as strongly as it comes through in this answer choice. We don’t want to begin an informative passage with an exhortatory sentence; this is an incorrect answer.



Scope: The scope of a passage is a crucial aspect of any answer choice. It can easily be the determining factor that separates the correct answer from incorrect options. Any correct answer needs to introduce the content of the passage—but think of it as “just the content, and nothing but the content.” Introductory sentences that overreach and talk about other things, or that focus on only one detail of many discussed in the passage should immediately catch your eye as potentially incorrect, as issues of scope are major red flags.

Answer choice D is incorrect because its scope does not match that of the rest of the passage. “Claude Monet is a painter who is known for his beautiful landscapes, which many people love for their bright colors” could be a great topic sentence for a different passage, but not for this one, since the focus is just on one of Monet’s works—*Impression, Sunrise*—the critical reaction to it, and its lasting effects on the art world. The difference in scope is emphasized if you consider how the passage would read with this sentence as its opener:

Claude Monet is a painter who is known for his beautiful landscapes, which many people love for their bright colors. When Monet’s painting of his hometown’s harbor titled *Impression, Sunrise* debuted in a Paris art show, critics had no idea what to make of it.

While the sentence does introduce Claude Monet, it focuses on his landscapes and doesn’t mention anything about the critical reaction to his first painting or how he changed the art world. There are better answer options for this passage’s opening line.

Eliminating those three answer choices leaves us with the correct answer, B: “Claude Monet is famous in art history for producing the first work in a famous and important movement in painting that also gave that movement its name.” While this information is presented later in the passage, it is presented much later, in its second paragraph, and in greater detail. It isn’t a shock to the reader to hear about Monet’s relationship to Impressionism in both the topic sentence and in later body paragraphs. In fact, this is helpful: it assists the reader in knowing what to expect from the rest of the passage. In addition, it flows nicely into the passage’s second line:

Claude Monet is famous in art history for producing the first work in a famous and important movement in painting that also gave that movement its name. When Monet’s painting of his hometown’s harbor titled *Impression, Sunrise* debuted in a Paris art show, critics had no idea what to make of it.

This answer choice flows nicely and foreshadows the rest of the passage’s content. It is the best option presented.

Choosing a Sentence for the Middle of a Passage

After considering questions that ask you to pick a topic sentence for the paragraph, you may think that questions asking you to choose a sentence for a location in the middle of a body paragraph have to be easier. Well, yes and no: no longer are you being asked to discern the passage’s main idea and express it in a way consistent with the rest of the text. You’re being given a different challenge in these questions, though: now the details you need to be concerned with most likely lie in the immediate vicinity of the designated spot. Be sure to read a few sentences before and a few sentences after before picking an answer.



Sample Question

Which of the following would best function if inserted in place of “[2]?”

- A. It was completely unlike anything the critics had ever seen, and they openly criticized it.
- B. Critics provided mixed reviews about the work: some liked it, others not so much.
- C. Monet was a master artist, so he shouldn’t have cared what the critics thought of his work anyway.
- D. Every critic adored the new style of the painting except for one.

Consider the surrounding context of this location in the text:

... When Monet’s painting of his hometown’s harbor titled *Impression, Sunrise* debuted in a Paris art show, critics had no idea what to make of it. [2] Despite the qualms and complaints of art critics, Monet’s piece became famous.

We’re being given a great deal of information in the context of this passage. The sentence before the spot tells us that we’re talking about the critical reaction to *Impression, Sunrise*, and the sentence after the spot tell us that the work became famous “Despite the qualms and complaints of art critics.” So, we need to pick out a sentence that bridges the ideas of the critics being confused (“had no idea what to make of it”) and of them not liking the painting (“qualms and complaints of art critics”).

With that in mind, let’s consider our answer options. We can immediately discard D, “Every critic adored the new style of the painting except for one,” as a potential correct answer. The passage gives no indication that every critic but one liked Monet’s work—the opposite is true, in fact; the sentence that follows the indicated spot suggests that a lot of critics, perhaps most of them, didn’t like *Impression, Sunrise*.

Answer choice C, “Monet was a master artist, so he shouldn’t have cared what the critics thought of his work anyway,” is simply off topic in terms of passage voice. We’re getting a strong opinion from the writer in this answer option that doesn’t match the voice of the rest of the passage, which simply conveys information about Monet and art history. This answer choice isn’t correct either.

This leaves us with two remaining answer choices: A, “It was completely unlike anything the critics had ever seen, and they openly criticized it,” and B, “Critics provided mixed reviews about the work: some liked it, others not so much. Here’s where context comes into play as the deciding factor. We need a body sentence that bridges the ideas of confusion and complaining. Nothing in the passage suggests that any critics liked the work; in fact, from the sentence that follows the designated spot, it seems like the overwhelming majority of them didn’t like Monet’s painting. A is the better answer choice because it more directly continues the idea of the critics’ confusion (“It was completely unlike anything the critics had ever seen”) and provides a statement of the critics’ complaints (“they openly criticized it”) that the following sentence can then reference nicely in its “despite” subordinate clause:

When Monet’s painting of his hometown’s harbor titled *Impression, Sunrise* debuted in a Paris art show, critics had no idea what to make of it. It was completely unlike anything the critics had ever seen, and they openly criticized it. Despite the qualms and complaints of art critics, Monet’s piece became famous.

Answer choice A is the best option for the designated spot.



Choosing a Transition Sentence

Good, logical transitions are one of the features that can make good writing great, bringing the reader out of the last few details of the concluding body paragraph and helping them realize how those details fit into the “big picture” that the entire passage is trying to convey. Like picking out a body paragraph sentence, picking out a good transition sentence also requires you to pay attention to context clues. Instead of working only at the sentence-level, though, you need to pick the sentence that best bridges ideas conveyed by entire paragraphs as well as those conveyed by the surrounding sentences. This may sound more challenging, and frankly, it can be; however, the increased specificity of having to bridge both details and big-picture ideas can help you in identifying answer choices that don’t meet the necessary criteria.

Sample Question

Which of the following would best function if inserted at the beginning of the second paragraph, where it says “[3]”?

- A. Monet painted other famous works, including a series of works in which he portrayed the water lilies in his pond.
- B. Many critics thought that Monet’s painting didn’t demonstrate any artistic talent whatsoever.
- C. Not only did Monet’s painting become well-known; it also had a large effect on the art world.
- D. Like the critics, artists of Monet’s era didn’t know what to make of *Impression, Sunrise* either.

Let’s consider the specified spot in the passage:

.. Despite the qualms and complaints of art critics, Monet’s piece became famous.

[3] Many artists began to mimic the style of Monet’s painting, which departed from the carefully realistic portraiture and history paintings of previous works and instead aimed to present the suggestion of a subject, paying special attention to the interaction of light with the subject.

We need to pick out an answer choice that helps the reader move from the idea of Monet’s unliked piece becoming famous to the idea that lots of other artists began to paint in his new style. Answer choice A, “Monet painted other famous works, including a series of works in which he portrayed the water lilies in his pond,” can be immediately discarded because discussion of Monet’s other works has nothing to do with *Impression, Sunrise* or the reaction of critics and artists to it. Answer choice B, “Many critics thought that Monet’s painting didn’t demonstrate any artistic talent whatsoever,” works in continuing discussion of the critical reaction to the painting, but it doesn’t flow very nicely into the second sentence of the second paragraph:

Many critics thought that Monet’s painting didn’t demonstrate any artistic talent whatsoever. Many artists began to mimic the style of Monet’s painting, which departed . . .

It’s a bit of a jolt to jump from discussion of critics to discussion of artists with no connecting phrases or sentences. We need to pick out a connecting sentence, and this isn’t it.



Answer choice C, “Like the critics, artists of Monet’s era didn’t know what to make of *Impression, Sunrise* either” looks like a decent choice in that it makes a comparison between the critics discussed in the first paragraph and the artists discussed in the second paragraph; however, it doesn’t make sense to say that “artists didn’t know what to make of *Impression, Sunrise*” because we’re then told that the style of the work became very popular. It sounds like the artists liked the work. This isn’t the best answer option either.

The correct answer choice is D, “Not only did Monet’s painting become well-known; it also had a large effect on the art world.” This sentence helps the reader jump from the final clause of the first paragraph, “Monet’s piece became famous,” to the idea that the painting affected the art world in that artists started mimicking its style.

Choosing a Concluding Sentence

Choosing a sentence to end a passage is much like choosing one to begin it, except instead of taking into account what’s going to be said, you have to take into account what has been said. Both question types require understanding of the passage as a whole unit.

Sample Question

Which of the following would best function as a concluding sentence for this passage, where it says “[4]?”

- A. If only Claude Monet could see the effect that his painting had on the art world!
- B. Artists should study great masters like Monet in order to learn how to produce truly original work.
- C. Impressionism is just as popular today as other famous artistic styles such as Post-Impressionism and Cubism.
- D. Despite the art world’s initial resistance to the style, Impressionism has become a beloved style of painting, all thanks to Claude Monet and his *Impression, Sunrise*.

Let’s consider what the passage discusses. The first paragraph introduces Monet’s work *Impression, Sunrise* and talks about its debut. The second paragraph tells us that artists began copying the style of the painting, and that the title of the Impressionist artistic movement was derived from its name. The sentence before the concluding one we need to pick reads as follows:

This style eventually developed into an entire artistic movement that took its name from Monet’s painting and came to be called “Impressionism.” [4]

Voice: Considering the answer choices, we can discard B because it doesn’t match the voice of the rest of the passage. This is an opinionated answer choice that tells readers what the writer thinks artists “should” do; the rest of the passage doesn’t sound like that! It provides information, not the writer’s opinion, so this answer choice is incorrect.

Scope: Answer choice C is also incorrect, because instead of concluding the passage by remaining focused on its topic of Impressionism, Monet, and *Impression, Sunrise*, it introduces two artistic styles that haven’t been mentioned in the passage: Post-Impressionism and Cubism. This makes the concluding sentence off-topic in comparison to the rest of the passage and this answer choice incorrect.



This leaves us to choose between two answer choices: A and D. What sticks out about each answer choice? Well, A is exclamatory. The rest of the passage doesn't include any exclamatory statements, but it very well could conclude with one. It's discussing a hypothetical: "If only . . ." The effect of this hypothetical is to emphasize the effect of Monet's work on the art world, which is something the passage discusses; however, it does this in an indirect way that doesn't really match the rest of the passage. Answer choice D, on the other hand, directly connects to ideas that have been previously mentioned in the passage: it mentions "the art world's initial resistance to the style" which is discussed in the critics' reactions at the end of the first paragraph, and it again links Impressionism with Monet and *Impression, Sunrise*. This is the better answer choice because it is more directly related to more ideas discussed in the paragraph and matches the rest of the text in terms of tone and style.

Choosing the Best Spot for a Sentence

Sometimes, the ACT English section inverts the previous question formula: instead of presenting you with multiple sentences and asking which one goes best in a specified location, it presents you with a single sentence and asks where that sentence should go in the passage. While the factors have been inverted, the skills being tested are similar. You still need to understand the passage's context, but instead of paying special attention to tone, scope, and voice, you can turn your attention to how the passage progresses line-by-line in moving from one topic to another.

Practice Passage: Japanese Tea Ceremonies

[1] Tea is found in a myriad of different varieties throughout the world and plays an important part in a great many cultures. [2] In traditional Japanese culture, tea ceremonies play a special role in creating a social space in which tea is enjoyed with others. [3] Matcha is a kind of green tea that has been ground into a fine powder. [4] Japanese tea ceremonies vary in their details; there are many different schools that all differ in the details of how one prepares and serves the tea. [5] During a ceremony, the tea is prepared carefully in a manner according to tradition. [6] Whereas some people can down a cup of tea in minutes, tea ceremonies draw out the tea-drinking process. [7] Tea certainly plays a special role in traditional Japanese culture, as tea ceremonies demonstrate. [8]

Sample Question

The sentence "A particular tea is often used in these ceremonies: matcha." would be best inserted into the paragraph in which of the following locations?

- A. Location [2]
- B. Location [3]
- C. Location [4]
- D. Location [7]



Consider the sentence we're handed. It moves from general to specific, specifying one tea that is often used in "these ceremonies." The use of the demonstrative pronoun "these" tells us that at the point in the passage where this sentence makes sense, the Japanese tea ceremonies have already been introduced. Otherwise, it doesn't make much sense to refer to them as "these ceremonies"—readers would ask, "Which ceremonies?" and get terribly confused. This allows us to rule out Location 2, as at that point in the passage, tea ceremonies have yet to be introduced. Location 7 doesn't make much sense either. At that point in the passage, the focus is on the timing of the ceremony, not on any particular kind of tea.

This leaves us with Locations 3 and 4 to choose from. Skimming the passage, you can see that matcha is discussed in only one place: in the sentence, "Matcha is a kind of green tea that has been ground into a fine powder." How does the question's sentence relate to this sentence? The question's sentence seems like it's introducing matcha to the reader for the first time, whereas the sentence in the passage provides a definition of what matcha is. Thus, it makes sense for the question's sentence to precede the sentence in the passage, making Location 3 the right answer:

In traditional Japanese culture, tea ceremonies play a special role in creating a social space in which tea is enjoyed with others. A particular tea is often used in these ceremonies: matcha. Matcha is a kind of green tea that has been ground into a fine powder.

This makes sense because the inserted sentence provides a general introduction to matcha that leads in to the following sentence's more detailed definition.

Sample Question

The sentence "No matter which school is followed, many tea ceremonies involve the consumption of light food along with the tea." would be best inserted into the paragraph in which of the following locations?

- A. Location
- B. Location
- C. Location
- D. Location

In this question's sentence, details are again our friends. Look at how the sentence begins: "No matter which school is followed . . ." In order for a sentence to start this way, a variety of schools needs to have just been introduced in the previous sentence. Which sentence in the passage does that? "Japanese tea ceremonies vary in their details; there are many different schools that all differ in the details of how one prepares and serves the tea." This means that the location that immediately follows this sentence, Location 5, is the correct answer.

Sample Question

The sentence "Ceremonies can span hours." would be best inserted into the paragraph in which of the following locations?

- A. Location
- B. Location
- C. Location
- D. Location



Ok, we don't have any obvious context clues to go on in this sentence. It's talking about details, and specifically emphasizing how long tea ceremonies can last. The passage only discusses the timing of a tea ceremony near its conclusion, so we can discard Location 2 as a potentially correct answer and narrow our focus to the remaining three answer choices, all which fall around the part of the passage where timing is discussed:

[5] During a ceremony, the tea is prepared carefully in a manner according to tradition. [6] Whereas some people can down a cup of tea in minutes, tea ceremonies draw out the tea-drinking process. [7]

We can ignore Location 5, because our sentence provides more detail about the speed of a tea ceremony than does the sentence that begins with "During a ceremony." This means that our sentence comes after this one, leaving us to decide whether it best fits into the passage at Location 6 or Location 7. Again, consider levels of specificity: whereas the sentence in the passage tells us that "tea ceremonies draw out the tea-drinking process," a general statement, our sentence specifies that "Ceremonies can span hours." This sentence thus best fits into the passage after the more general statement, at Location 7.



Practice Problems: Adding Content

[1] While perhaps newer than its counterparts, Southwestern architecture draws on Spanish and Native American influences that are hundreds or thousands of years old. [2] These influences can be seen in the use of native building materials such as adobe and pine timber, both easily found in the Southwest, and in the use of a central plaza or veranda. [3] Traditional building elements include walls with rounded corners, exposed timber beams, flat roofs, and a simple, uncluttered aesthetic. [4] Some houses are built around a central courtyard; while others open onto a patio flanked by ornate grillwork.

[5] Pueblo Revival buildings are dignified by their vigas, which are roof beams that extend beyond the outer edge of walls. [6] Recessed windows and doorways with arched tops are also very common in this style of architecture. [7] Some famous examples of Pueblo Revival buildings include the New Mexico Museum of Art, the Painted Desert Inn, and various edifices at the University of New Mexico in Santa Fe.

1. The sentence “For example, the Southern plantation home is characterized by neoclassical proportions and exterior columns, while the New England saltbox has a long, slanting roof and a flat façade.” would best be inserted between which two sentences?
 - A. Sentences 1 and 2
 - B. Sentences 4 and 5
 - C. Sentences 5 and 6
 - D. Sentences 3 and 4
2. The sentence “One particular category of Southwestern architecture is the Pueblo Revival style, first popularized in the early 1900s.” would fit best before which sentence?
 - E. Sentence 6
 - F. Sentence 3
 - G. Sentence 8
 - H. Sentence 1
3. What sentence would best conclude this passage?
 - A. While mainly restricted to the Arizona, New Mexico, and California, this movement has made valuable contributions to American architecture as we know it today.
 - B. This, however, should not be confused with the Mission Revival Style, which is a different movement altogether.
 - C. Although aesthetically pleasing, buildings made in this style are ecological disasters.
 - D. Recently, architects have been wondering if this style could be adapted to Northeastern climates.
4. Which sentence would best open this passage?
 - A. American architecture has many distinctive styles and signature elements.
 - B. American art, architecture, and music all have their own distinctive flair.
 - C. Native American traditions and legends are a core part of the American Southwest.
 - D. Architecture is the art of designing both functional and aesthetically pleasant buildings.



Answer Key: Practice Problems: Adding Content

1. The sentence “For example, the Southern plantation home is characterized by neoclassical proportions and exterior columns, while the New England saltbox has a long, slanting roof and a flat façade.” would best be inserted between which two sentences?

→ **A.** Sentences 1 and 2
B. Sentences 4 and 5
C. Sentences 5 and 6
D. Sentences 7 and 8

2. The sentence “One particular category of Southwestern architecture is the Pueblo Revival style, first popularized in the early 1900s.” would fit best before which sentence?

→ **E.** Sentence 6
F. Sentence 3
G. Sentence 8
H. Sentence 5

3. What sentence would best conclude this passage?

→ **A.** While mainly restricted to the Arizona, New Mexico, and California, this movement has made valuable contributions to American architecture as we know it today.
B. This, however, should not be confused with the Mission Revival Style, which is a different movement altogether.
C. Although aesthetically pleasing, buildings made in this style are ecological disasters.
D. Recently, architects have been wondering if this style could be adapted to Northeastern climates.

4. Which sentence would best open this passage?

A. American art, architecture, and music all have their own distinctive flair.
→ **B.** American architecture has many distinctive styles and signature elements.
C. Native American traditions and legends are a core part of the American Southwest.
D. Architecture is the art of designing both functional and aesthetically pleasant buildings.

This sentence elaborates upon an idea introduced in Sentence 1 (the variety of American architectural styles) and provides a seamless transition to Sentence 2.

This sentence is an excellent topic sentence for the paragraph’s second passage, which discusses in detail some traits of Pueblo Revival architecture. For this reason, it belongs at the very beginning of the second paragraph.

The sentence that best wraps up the passage and underscores the importance of the author’s discussion is this one: “While mainly restricted to the Arizona, New Mexico, and California, this movement has made valuable contributions to American architecture as we know it today.” All of the other options introduce brand-new ideas, which is a good strategy for beginning but not ending a passage.

American art and music aren’t mentioned in the passage at all, so answer choice A is too general to be the best option. Similarly, while the passage discusses Native American architecture, it does not talk about Native American traditions and legends, so C isn’t the correct answer either. D mentions architecture, but provides an extremely broad definition of the term, whereas B focuses on American architecture. B is the better answer of the two because it provides an opening statement closely related to the content of the passage.



Rearranging Content

Just as certain questions on the ACT English section may ask you about where in the passage to place a new sentence, other questions can ask you where to move a sentence that is already part of the paragraph. The sentences indicated by these questions may stick out to you as you read the paragraph, as there are better spots for them to be placed—places in which they fit perfectly into the order in which the passage presents its information. Still other questions may ask you to consider a lengthy paragraph and deduce where it would be best to break it into two smaller paragraphs. Mastering both of these question types requires you to train a keen eye for the progression of topics in a passage.

Practice Passage: National Parks and National Monuments

[1] National Parks in the United States serve as a way for the government to officially protect pristine natural ecosystems, and each park has characteristic features that make it famous. [2] Yellowstone National Park, the first to be protected, is renowned for its geysers (like the famous “Old Faithful”) and sulfur pools. [3] Other national parks like Yosemite and Glacier are famous for their beautiful mountain vistas, while Sequoia National Park contains some of the oldest and tallest trees in the world. [4] The U.S. also maintains a different category of protected land: national monuments. [5] In contrast to national parks, which are treasured for their natural landscapes, national monuments are protected for their historical significance. [6] National monuments may include such things as landmark sites where historically significant documents were signed, battlefields, and homes of individuals famous in U.S. history. [7] There are currently 59 national parks in the U.S.—at least one in each state—and millions of people visit them every year. [8] The next time you pass a federally protected site, whether it’s a national park or a national monument, be sure to visit and enjoy it yourself!

Moving Sentences

ACT English questions that ask you to consider rearranging content may ask about potentially moving one of the sentences in the passage to a different location. These questions are likely to focus on issues of sequence and logic, since shifting a sentence in this way alters not the information the passage presents, but the order in which that information is presented.

Sample Question

In order for the paragraph to make the most logical sense, Sentence 7 should be placed at which of the following locations?

- A. After Sentence 1
- B. After Sentence 2
- C. After Sentence 3
- D. It should be left where it is in the passage now.



First things first: consider the sentence in question, Sentence 7: “There are currently 59 national parks in the U.S.—at least one in each state—and millions of people visit them every year.” Looking over the passage as a whole, we can see that this sentence connects to the main topic of U.S. National Parks. The passage discusses both national parks and national monuments, though, and this allows us to narrow our focus to those sections that discuss national parks, not national monuments. Our sentence will go somewhere in that part of the passage. The passage discusses national parks near the beginning and national monuments near the end, so we can predict that our sentence will go somewhere near the beginning of the passage. This doesn’t do us much good in this question as we’re given the first three sentences of the passage as potential answer choices, but it does allow us to take a critical glance at where the sentence is now: smack-dab in the middle of the part about national monuments. It’s in the wrong spot, so we certainly won’t be leaving it where it is now, and D isn’t the correct answer.

In deciding whether our sentence belongs after Sentences 1, 3, or 3, we need to pay extra special attention to how sentences next to one another work together. If two consecutive sentences are particularly tight-knit in terms of content and transitions, it won’t benefit the passage to break up that flow by inserting a different sentence between them. Let’s consider Sentences 1 and 2, as we’re asked about inserting Sentence 7 after Sentence 1:

[1] National Parks in the United States serve as a way for the government to officially protect pristine natural ecosystems, and each park has characteristic features that make it famous. [2] Yellowstone National Park, the first to be protected, is renowned for its geysers (like the famous “Old Faithful”) and sulfur pools.

If you were to characterize how these sentences relate to one another, how would you do so? Well, you might notice that we’re going from a general introduction of national parks and a foreshadowing of details (“each park has characteristic features that make it famous”) to specific details about Yellowstone that fulfill that foreshadowing by telling us what makes it famous (“Yellowstone . . . is renowned for its geysers”). These sentences directly connect to one another, so we don’t want to insert a new sentence between them. It would make it difficult for the reader to jump from the concept of “there are things that make national parks unique” to “here is an example of what makes a national park unique.” Answer option A isn’t the correct answer.

On to considering Sentences 2 and 3:

[2] Yellowstone National Park, the first to be protected, is renowned for its geysers (like the famous “Old Faithful”) and sulfur pools. [3] Other national parks like Yosemite and Glacier are famous for their beautiful mountain vistas, while Sequoia National Park contains some of the oldest and tallest trees in the world.

These sentences have a lot in common because they are both serving the same purpose in this passage: providing more details and examples about what makes different national parks unique. This is particularly evident in how the passage moves from a simple declarative example about Yellowstone to more examples using the words “Other” and “while”: “Other (example does this), while (yet another example does this).” It wouldn’t make sense to place Sentence 7 between these two lines, so answer choice B is incorrect.

We’ve figured out the correct answer using process of elimination: C, after Sentence 3, is the best spot in which to place Sentence 7. To check our answer, let’s see how the paragraph reads when we do that:



[1] National Parks in the United States serve as a way for the government to officially protect pristine natural ecosystems, and each park has characteristic features that make it famous. [2] Yellowstone National Park, the first to be protected, is renowned for its geysers (like the famous “Old Faithful”) and sulfur pools. [3] Other national parks like Yosemite and Glacier are famous for their beautiful mountain vistas, while Sequoia National Park contains some of the oldest and tallest trees in the world. [7] There are currently 59 national parks in the U.S.—at least one in each state—and millions of people visit them every year. [4] The U.S. also maintains a different category of protected land: national monuments.

This is a great spot for Sentence 7: it doesn’t interrupt the tight-knit construction of the earlier sentences about national parks, and it switches the topic from listing national parks (neatly wrapping up the example-naming by providing an encompassing statistic) to a different fact about them—how popular they are as tourist destinations. The correct answer to this question is C, After Sentence 3.

Paragraph Breaks

Breaking apart a paragraph can be tricky business. The very fact that the ACT English section presents material as a single paragraph can predispose you to assume that the single-paragraph format is the best one when that isn’t the case. As you read through passages, be on the lookout for moments where it seems the author needs to take a pause, but doesn’t—and not in terms of comma placement. A paragraph break inserts a specific kind of moment into the reader’s experience—a short pause in which his or her brain keeps digesting information while his or her eyes move on to the next segment of the text. Think of a paragraph break in a passage as functioning something like a comma in a sentence, but know that whereas commas can break apart sentences regardless of how material ending up on each side of the comma relates, paragraph breaks are always entrenched in what the author is saying. You don’t break up a paragraph in the middle of talking about a point—you start a new paragraph when you’re introducing a new point.

Sample Question

The author wants to separate this paragraph into two paragraphs. Before which of the following sentences would it be most logical for him or her to do this?

- A. After Sentence 2
- B. After Sentence 3
- C. After Sentence 4
- D. After Sentence 5

You can apply the same skill of analyzing the relationships between various sentences to questions that ask you about breaking a paragraph into smaller paragraphs. Reconsidering the passage, we can see that it moves from introducing national parks and foreshadowing details (Sentence 1) to providing details about what makes national parks unique (Sentences 2 and 3) before beginning to talk about national monuments in contrast to national parks (Sentence 4) and giving examples of sites that might be protected as national monuments (Sentence 5). Did you notice that shift—the large-scale idea shift from national parks to



national monuments? That shift occurs in Sentence 4, where national monuments are introduced as a topic of discussion. The best place to break apart the passage would therefore be right before Sentence 4, so Sentence 4 could serve as the introduction to a second paragraph about national monuments.

Let's take a look at how the passage would flow if it was broken into two paragraphs right before Sentence 4:

[1] National Parks in the United States serve as a way for the government to officially protect pristine natural ecosystems, and each park has characteristic features that make it famous. [2] Yellowstone National Park, the first to be protected, is renowned for its geysers (like the famous "Old Faithful") and sulfur pools. [3] Other national parks like Yosemite and Glacier are famous for their beautiful mountain vistas, while Sequoia National Park contains some of the oldest and tallest trees in the world.

[4] The U.S. also maintains a different category of protected land: national monuments. [5] In contrast to national parks, which are treasured for their natural landscapes, national monuments are protected for their historical significance. [6] National monuments may include such things as landmark sites where historically significant documents were signed, battlefields, and homes of individuals famous in U.S. history. [7] There are currently 59 national parks in the U.S.—at least one in each state—and millions of people visit them every year. [8] The next time you pass a federally-protected site, whether it's a national park or a national monument, be sure to visit and enjoy it yourself!

One important thing to keep in mind is that if you should have to deal with both sentence-rearranging questions and paragraph-breaking questions that concern a single passage, do not assume that sentences have actually been moved as you suggested. Treat each question as distinct and as referencing the passage as it is written, not as it would look if the writer took your suggestions and made each change you decided was best in previous questions. In short, does Sentence 7 sound awkward where it is currently placed? Yes. But you have to assume it's still there, and that Sentence 7 hasn't been moved. Regardless of whether Sentence 7 were moved, it is best to break apart the paragraph before Sentence 4. The ACT English test will likely avoid "stacking questions" like this, but always remember to consider the passage as it's written in each question presented to you about it.



Rephrasing for a Particular Effect

The questions we'll consider in this section all combine editing decisions with specific goals. No longer are you being asked to identify the "best" answer choice—the one that best fits the tone, flow, and logic of the passage. Here, you are being asked to pick the rephrasing option that best creates a certain effect. The author might want to play up one particular aspect of a topic, or use a negatively-connoted word to help bias the reader against it. He or she may want to foreshadow a discussion that appears later in the passage, or suddenly shift tones. You may not agree with all of the writer's decisions to change the passage in specific ways, but keep in mind: you're not the one setting the goal. You just need to figure out how to best accomplish it.

These questions can present more trouble than more direct ones that ask you to identify the best revision or why a selection should be revised or not. This is because it is completely possible for each of the answer choices presented for effect-driven questions can sound perfectly good in the passage and match flow, tone, style, logic—none of the typical concerns may throw up red flags. You may be initially confused, being asked to pick from multiple answer choices that sound "correct." It is crucial that you analyze the answer choices not from a perspective of asking which is grammatically correct (they all should be) or even which sounds the best (they all might sound equally good). Use the described effect as a lens through which you can structure your response to the question.

Practice Passage: Tourism vs. Preservation in the Lascaux Caves

Should historical monuments be completely open for the public to examine, or is there greater value in protecting them for everyone? While many people are willing to travel
5
thousands of miles to view ancient works of art and architecture up close and in person, the more visitors an ancient site has, the more quickly it crumbles. The history of the
2 6
Lascaux caves in France attests to this fact. Discovered by accident in 1940, the caves are
6
covered in breathtaking Paleolithic artwork portraying a wide variety of animals in motion. Visitors poured into the site as it got really popular, but such a high rate of tourism has
4
damaged the art, causing chemical changes as well as the introduction of lichen and black
3
mold, as well as other deposits. Overinteraction with the caves has turned them from a fascinating tourist destination into a world wonder desperately in need of help. If you want
1 7
to experience the beauty of the Lascaux caves, visiting the replica site Lascaux II can help
7
you have the experience of exploring the caves without damaging their fragile art.
7



Rephrasing a Single Word

Let's first consider a question that asks about options for changing out one word for another one. Note that it's a completely valid option to say that the passage best achieves the stated goal by leaving the word included in the passage alone. If you find yourself wondering if you have picked "too many" or "too few" of this particular answer choice, approach these questions as if there is a blank in the passage and you need to fill it with whichever option best fulfills the goal. Don't let the fact that the "NO CHANGE" option is already in the passage and accompanied by an all caps phrase sway you in favor of it or away from it, and don't try to base any guesswork that becomes necessary on how many of these particular answers you've picked.

Sample Question #1

Which of the following conveys both that the site's current problems need to be addressed and that future damage needs to be prevented?

- A. preservation
- B. assistance
- C. protection
- D. NO CHANGE ("help")

This question focuses on the subtle variation in meaning between four closely-related words. We need to pick out the one that expresses two things simultaneously: the treatment of current problems and the avoidance of future issues.

Even though a distinct goal is provided, let's look at the sentence to see what contextual information we can glean:

Overinteraction with the caves has turned them from a fascinating tourist destination into a world wonder desperately in need of help.

"Help" and "assistance" seem particularly close in meaning—definitely synonyms, and often interchangeable. Do their meanings skew in particular toward current or future problems? Current. You ask for help or assistance with a current issue; the word doesn't focus much on helping to avoid later problems. While it may have seemed worrisome that two of the listed words were so close in meaning, in this case, it becomes a non-issue, as neither of these answers looks like the best one. Let's consider the other two options: "protection" and "preservation." "Protection" skews in the opposite "direction" of "help" and "assistance"—"protecting" something means helping it avoid future damage in particular, with little to no focus given to remedying current problems. "Preservation" strikes the best balance between these two needs: "preserving" a document, building, work of art, or other object means working to keep it in good condition (thus addressing any current problems) as well as helping it to avoid accruing damage in the future. For example, if historians uncovered a faded painting by a famous artist, preservation work might involve restoring the painting's original colors (addressing a current problem) as well as placing it behind special glass when displaying it in a museum (helping to avoid future problems). The correct answer is A!



Let's consider another single-word-based sample question:

Sample Question #2

Which of the following best encompasses both two-dimensional works of art as well as three-dimensional works of art and architecture?

- A. fades
- B. breaks
- C. deteriorates
- D. NO CHANGE ("crumbles")

Again, we're going to be focusing on shades of meaning, so none of these terms may seem like a bad or incorrect choice initially. Differentiating between them might take some digging, but let's first glance over the sentence at hand:

While many people are willing to travel thousands of miles to view ancient works of art and architecture up close and in person, the more visitors an ancient site has, the more quickly it crumbles.

Ok, the distinction here is one of logical reference: we need to pick out the term that best refers to two-dimensional and three-dimensional works. (Don't let the phrasing of "best" throw you off, here—as you may have guessed, there's only one correct answer choice that refers to both.) "Crumbles," the word in the passage, isn't cutting it—a painting can't really "crumble." The paint could potentially crumble off of the painting, but the work itself wouldn't likely be described as "crumbling." Using similar logic in the opposite "direction," paintings can "fade," but buildings can't. We've knocked out two answer choices! Now we're down to "breaks" and "deteriorates," both relatively general terms; however, "breaks" conveys a physicality that "deteriorates" does not, and that physicality only really jives with three-dimensional works. Think about it: when was the last time you heard someone refer to "a broken painting?" That might be the first time you've heard or considered that phrase. "Deteriorates" is the best answer. A painting can "deteriorate" just as a building can because the word is so general in its meaning.

Let's look at one more word-based question, this time one that asks you to consider [connotation](#) instead of [denotation](#).

Sample Question #3

Which of the following best conveys the author's distaste for the lichen and black mold?

- A. colonies
- B. infestations
- C. appearances
- D. NO CHANGE ("the introduction")

Here is the sentence in question:

Visitors poured into the site as it got really popular, but such a high rate of tourism has damaged the art, causing chemical changes as well as the introduction of lichen and black mold, as well as other deposits.



This is a perfectly good sentence; “the introduction” doesn’t clash with the passage in terms of tone or logic, and it doesn’t interrupt the flow of the text. But don’t let that sway you: one of the other presented options might better meet the stated goal. We’re looking for the answer choice that best demonstrates “the author’s distaste for the lichen and black mold,” so we’re most likely going to be looking for a negative word. Make sure to distinguish between the connotation of the passage’s content (mold in Lascaux is clearly a bad thing) and the connotation of the words themselves. Try considering how the words would sound when used to describe other subjects—neutral ones and good ones. If the words have negative connotations, they’ll probably be thrown into sharp relief. “The introduction” is neutral—not particularly positive or negative. The phrase could refer to “the introduction” of something great. What about “appearances?” No bad connotations there. Good or bad things could make appearances. “Colonies?” Colonies of things might be bad, but you can also have colonies of people; let’s call this one somewhere between neutral and negative. “Infestations?” Bingo! An “infestation” is never a good thing. You wouldn’t refer to an “infestation” of positively-connoted things. (e.g. “infestation of kittens,” “infestation of gifts,” and “infestation of friends” all sound strange.) B is the correct answer; if the author wants to emphasize his or her dislike for the lichen and black mold in the Lascaux caves, referring to their presence as “infestations” will help get that meaning across.

Rephrasing a Phrase or Clause

Onto considering rephrasing questions that present a goal for a phrase or clause. The following question focuses on how a phrase interacts with the tone and style of the passage.

Sample Question #4

The author wants to maintain a formal tone throughout the passage. Which of the following options best accomplishes this?

- A. blew up
- B. grew in renown
- C. became infamous
- D. NO CHANGE (“got really popular”)

This time, we’re looking for the answer choice that most helps give the passage a formal tone. The sentence in question is as follows:

Visitors poured into the site as it got really popular, but such a high rate of tourism has damaged the art, causing chemical changes as well as the introduction of lichen and black mold, as well as other deposits.

“Got really popular” is a very casual phrase, not a formal one. “Got” isn’t the most specific verb, and the use of “really” instead of a stronger adjective again lacks specificity and creates a low-key tone. You would expect to hear someone use the phrase “got really popular” in casual conversation, not in a formal paper. That’s not the best option; let’s consider the other answers. What about “blew up?” This is again a casual expression of something increasing in popularity. If you wouldn’t hear it in an academic paper or write it in an assignment, chances are it’s not a formal expression. “Became infamous” starts to look like a good option until you realize that “infamous” and “famous” are not synonyms—“infamous” is negatively connoted and specifically means *famous for a bad reason*. This varies too much from the original meaning of the phrase “got really popular” to be correct. We don’t want to introduce new shades of meaning into the



passage; we only want to adjust the tone. B is the correct answer; “grew in renown” elevates the formality of the passage’s tone while retaining the same meaning as “got really popular.”

Let’s consider a phrase-based question that focuses on injecting a new idea into the passage instead of creating or maintaining a particular tone and style.

Sample Question #5

The author wants to emphasize that the historical sites being discussed are areas of potential academic interest. Which of the following choices best accomplishes this?

- A. safeguarding them for future studies
- B. shielding them from thoughtless damage and vandalism
- C. defending them against artistic criticism
- D. NO CHANGE (“protecting them for everyone”)

The sentence in question:

Should historical monuments be completely open for the public to examine, or is there greater value in protecting them for everyone?

Again, this is a perfectly correct sentence that doesn’t have any obvious problems. But we’re told that the author wants to emphasize the sites as being of potential interest to academia, so a change might be necessary in order to achieve that goal. After all, “protecting them for everyone” doesn’t suggest an academic focus at all—it’s a general phrase. (Case in point: the use of the word “everyone.”) Another option is likely better for the stated goal. What about “shielding them from thoughtless damage and vandalism?” Again, a perfectly correct phrase that could work in the passage without causing problems, but not an answer choice that works toward creating an academic focus. “Defending them against artistic criticism” begins to approach an academic context in mentioning “artistic criticism,” which could be part of academic debates. The best answer, though, is “safeguarding them for future studies.” This phrase suggests that in the future, people might be interested in studying sites like the Lascaux caves. The focus on the sites being potentially studied in the future portrays them as “areas of academic interest.”

Rephrasing a Sentence

Having worked through rephrasing single words and phrases, let’s take a look at working with entire sentences while trying to focus on the author’s particular goals.

Sample Question #6

The author wants to introduce the Lascaux caves as an example of a historic site that has been damaged. Which of the following sentences best accomplishes this?

- A. You may think this isn’t true, but the Lascaux caves prove it is.
- B. Works of art like those found in the Lascaux caves form the most important type of historic site to protect.
- C. The Lascaux caves show just how selfish some people can be.
- D. NO CHANGE (“The history of the Lascaux caves in France attests to this fact”).



When working with sentences, we need to consider the indicated sentence alongside its “bookend” sentences—the ones that appear immediately before and immediately after.

While many people are willing to travel thousands of miles to view ancient works of art and architecture up close and in person, the more visitors an ancient site has, the more quickly it crumbles. The history of the Lascaux caves in France attests to this fact. Discovered by accident in 1940, the caves are covered in breathtaking Paleolithic artwork portraying a wide variety of animals in motion.

The author’s goal for this sentence is to introduce the caves as an example of a historic site that’s been damaged. That doesn’t sound like it will require a lot of authorial opinion or strong wording, but a lot of the answer choices appear to veer in that direction. Answer choice A introduces a lot of value judgment into the passage, none of which is called for by the prompt. B does something similar thing, but instead of judging people, it makes a comparison between works of art and other types of historic sites, claiming the works of art are the most important ones to protect. That’s not called for in the prompt, either. What about A? While this answer doesn’t introduce any tone issues, its logic is off; nothing in the passage suggests that the author has to work against an audience that doubts the veracity of the previous statement. Thus, the phrasing of “You may think this isn’t true” isn’t called for. This leaves us with the phrasing that we started with, which in this case, accomplishes the stated goal best of all the presented options. It simply connects the general idea of damaged historic sites to the specific example of the Lascaux caves without introducing any elements that aren’t mentioned in the question.

Let’s consider one more sentence-based question—one that focuses on picking out a sentence that serves a specific purpose in the passage.

Sample Question #7

The author wants to end the passage with a call to action. Which of the following sentences best accomplishes this?

- A. Who knows what other sites have yet to be discovered and potentially better preserved?
- B. We should learn from what has happened to the Lascaux caves and be content to appreciate unique historic sites from afar so as not to damage them.
- C. Scientists hope to eventually restore Lascaux caves to the condition in which they were initially discovered, but this will take a great deal of time and effort to fix a problem that could have been avoided.
- D. NO CHANGE (“If you want to experience the beauty of the Lascaux caves, visiting the replica site Lascaux II can help you have the experience of exploring the caves without damaging their fragile art”).

Here’s the relevant portion of the passage. Note that this time, we only have to consider the sentence that precedes the underlined one, because the underlined one is the passage’s concluding sentence:

Overinteraction with the caves has turned them from a fascinating tourist destination into a world wonder desperately in need of help. If you want to experience the beauty of the Lascaux caves, visiting the replica site Lascaux II can help you have the experience of exploring the caves without damaging their fragile art.



The author is looking to end his or her passage with a “call to action,” or in other words, to urge the reader to actually change their behavior or go and do something based on what they’ve read. A, a question, doesn’t do this at all. C is a statement of what scientists intend to do to help restore the caves, but it’s just informative and doesn’t directly ask the reader to do anything. D, the option written in the passage, recommends that readers visit a replica site to avoid damaging the caves, but it begins with a conditional statement, “If you want to experience the beauty of Lascaux caves.” This means that it’s not addressing all readers, which takes its strength down a notch. A call to action is typically a strong statement aimed at all readers, not a weaker one giving some advice to some of them. B is the best answer, as it provides a recommendation as to what everyone should do in lieu of recognizing the damage that people have caused by visiting Lascaux. The “should” is a major clue that a recommendation is being given—in this case, a recommendation about how people should change their behavior to assist in the preservation of historic sites.



To Edit or Not—and Why

Certain ACT English questions present you with a straightforward question: to edit a section in a recommended way, or to not edit it, and leave it the way it's written. These questions, while based around this decision, don't test just your editorial decision-making skills: they're interested in how well you can articulate your reasoning for either changing out the phrasing or leaving it intact. Whether you're being asked about adding, removing, or rephrasing a word, phrase, clause, or sentence, learning to approach this species of question with confidence can help you stay focused during your exam.

To Add Content or Not—and Why

One type of question you might encounter on an ACT English section specifies a sentence and tells you where that sentence might be inserted into the paragraph. The question focuses on whether the writer should make that addition at all, and what reason he or she might have for doing so. These questions test perhaps the broadest understanding of the passage, and they also expect you to imagine yourself in the role of the writer as well as a reader in order to provide the best editing advice.

Practice Passage: The “Discovery” of America

For a long time, Christopher Columbus was widely known as the “discoverer” of America; however, many modern historians disagree with this title. The first European to set foot in North America was likely Leif Erikson, an Icelandic explorer. Erikson is thought to have colonized part of Newfoundland in modern-day Canada long before Columbus set foot in Hispanola in the Caribbean. ^[1] On top of archaeological evidence, Erikson's story is recorded in the centuries-old Icelandic work *Saga of the Icelanders*. ^[2]

It's important to remember, though, that America was inhabited long before any Europeans figured out it was here, perhaps from dates as early as approximately 12,000 BCE. It's important to keep historical discoveries in the correct context! So, the next time you hear about the “discovery” of America, whether it's attributed to 1000 or 1492, remember: cultures had already risen and fallen in America by whichever date is applied.

Sample Question

The author of this passage is thinking about adding in the following line at the point in the passage marked by ^[1]:

“One settlement in particular, L'Anse aux Meadows, was erected around 1000 CE and provides evidence that Vikings, and perhaps Erikson, were in America long before Columbus's famous voyage in 1492.”

Should the writer add this line to the passage?

- A. Yes, because it provides additional evidence about why the Eurocentric view of the discovery of America is limited.
- B. Yes, because it provides details about the archaeological evidence suggesting that Leif Erikson was the first European to reach America.
- C. No, because it argues against the idea that Christopher Columbus was the first European to discover America.
- D. No, because it is outside the scope of the passage.



This type of question might take you a bit by surprise the first time you come across one. The way in which the test makes you outline the reasoning behind your editing decision may remind you more of an ACT Reading question than the grammatical-error-based questions which form most of the ACT English section. The important thing is to be as prepared to answer these questions as you are to answer the grammatical-error-based ones, and to think carefully and critically not only about whether the writer should add the sentence in question, but why he or she should or should not.

After considering the sentence, consider the spot in which the author wants to insert it into the passage:

Erikson is thought to have colonized part of Newfoundland in modern-day Canada long before Columbus set foot in Hispanola in the Caribbean. One settlement in particular, L'Anse aux Meadows, was erected around 1000 CE and provides evidence that Vikings, and perhaps Erikson, were in America long before Columbus's famous voyage in 1492. On top of archaeological evidence, Erikson's story is recorded in the centuries-old Icelandic work *Saga of the Icelanders*.

The nice thing about this type of question is that the answer choices are clearly divided into two camps: the “Yes, Add the Sentence” camp and the “Don’t Let That Sentence Near the Paragraph” camp. This sentence flows nicely in the paragraph and is right on-topic in discussing archaeological evidence of Leif Erikson and the Vikings having been in America. If we decide that it would make a nice addition to the passage, we just have to figure out why: is it “because it provides additional evidence about why the Eurocentric view of the discovery of America is limited,” or “because it provides details about the archaeological evidence suggesting that Leif Erikson was the first European to reach America?” In this case, the answer is clear, as the sentence is considered as an addition to the first paragraph and the topic of the limited nature of the Eurocentric view of the “discovery” of America isn’t broached until the second paragraph. This sentence would make a good addition to the passage because it discusses details of archaeological evidence which the sentence that follows it adds to (“On top of archaeological evidence, ...”).

Sample Question

The author of this passage is thinking about adding in the following line at the point in the passage marked by **2**:

“The *Saga* is perhaps the most well-known example of Icelandic literature and focuses on individuals and their families and descendants.”

Should the writer add this line to the passage?

- A.** Yes, because it provides additional detail about the relationship between the *Saga of the Icelanders* and Leif Erikson’s story.
- B.** Yes, because it provides background information as to why the *Saga of the Icelanders* is a reliable source of historical information.
- C.** No, because it provides details about the *Saga of the Icelanders* that are beyond the scope of the passage.
- D.** No, because it incorrectly suggests that literature is more reliable than archaeological evidence.



Let's consider this sentence inserted into the passage at the specified spot, the end of the first paragraph:

On top of archaeological evidence, Erikson's story is recorded in the centuries-old Icelandic work *Saga of the Icelanders*. The *Saga* is perhaps the most well-known example of Icelandic literature and focuses on individuals and their families and descendants.

It's important to remember, though, that America was inhabited long before any Europeans figured out it was here . . .

Does something seem off about this addition? While details about the *Saga* could very well have a place in this passage, these details don't have anything to do with the rest of the passage. They're far too general. Information about the story that specifically pertains to Leif Erikson might be relevant, but a general description of the *Saga of the Icelanders* is unnecessary, as the author introduces the *Saga* as "the centuries-old Icelandic work" in the preceding sentence. That's enough information for readers to understand the work's connection with the passage's main idea, the discovery of America by Leif Erikson. This sentence's isn't contributing anything that helps that main idea progress, so it should not be added to the passage. But why not? Is it because the line "provides details about the *Saga of the Icelanders* that are beyond the scope of the passage," or because "it incorrectly suggests that literature is more reliable than archaeological evidence?" Reading carefully, you can realize that the archaeological evidence and the *Saga* both support the idea that Leif Erikson's discovery of America preceded Columbus's. They're supporting the same point! So, the correct answer is that it provides details that are beyond the scope of the passage. Paying attention to exactly how you came to the conclusion that the line doesn't belong in the passage can be extremely helpful in picking out the correct answer.



To Remove Content or Not—and Why

Certain ACT English questions may indicate a sentence in the passage and ask you whether it belongs in the passage at all. Make sure you're focusing on the passage as a whole when you analyze whether the sentence contributes to it or should be omitted.

Practice Passage: Recyclable Plastics: What Do Those Symbols Mean?

[1] Most people are aware that a symbol composed of three arrows forming a circle indicates that a container is somehow associated with recycling. [2] Far fewer individuals know what the little letters and numbers in the center of this symbol indicate. [3] These markings designate the particular kind of plastic from which the item is made. [4] The letters are acronyms that designate the item's broad category of plastic and provide detail about the specific thickness, density, and durability of the material. [5] For instance, "PET" and "PETE" both stand for the same thing, "post-consumer polyethylene terephthalate," while "HDPE" stands for "high-density polyethylene." [6] The numbers provide the same information in a different way—a PET bottle can be designated by a numeral 1 inside the recycling arrows, while an HDPE bottle can be identified by a 2. [7] By referring to these symbols, consumers can discern where each item should be recycled. [8] Recycled materials should always be placed in the correct stream. [9] While this system is very accurate, it is not consumer-friendly; people glancing over a product's packaging they wish to recycle might encounter the symbol and not know what it means, resulting in their recycling it incorrectly. [10] So, the next time you purchase something made of plastic, make sure to decode these symbols!

Sample Question

The author is considering removing Sentence 4 from the passage. Should he or she do this?

- A. Yes, because this sentence is beyond the scope of the passage because it provides unnecessary details about only letters when letters and numbers are each being discussed.
- B. Yes, because this sentence disrupts the flow of the passage.
- C. No, because the passage would lose valuable details and its parallel structure of discussing both letters and numbers would be disrupted.
- D. No, because it is functioning as a transition in order to introduce a concept different from the one that has been discussed thus far in the passage.

We first need to consider Sentence 4: "The letters are acronyms that designate the item's broad category of plastic and provide detail about the specific thickness, density, and durability of the material." Ok, so this sentence is telling us exactly what the letters stand for. How does it fit into the paragraph as a whole? The two sentences before it tell us that recycling symbols include letters and numbers that provide information about the specific type of plastic from which something is made. The sentence immediately after it provides examples of such letters and their meanings, and the sentence after that explains how numbers relate.

Should we remove this sentence? No, it fits well in the paragraph's two-pronged discussion of both letters and numbers. If we removed this sentence, the passage wouldn't mention letters in general at all; it would jump to examples and then proceed to discussing the numbers in recycling symbols. Plus, this sentence



lets us know details about what information is conveyed by these markings. It's a pretty useful sentence that doesn't appear to stick out as extraneous, and it connects nicely with the sentence that follows it, which starts with the phrase "For instance" and provides examples specifically of the meanings of various letters, which the sentence in question discusses more generally.

Given that reasoning, we can consider the two answer choices that begin with "No": A and B. A says we should keep the sentence because it's providing useful information and contributing to parallel structure; B says we should keep it because it's a transition to a new idea. Well, it's not a transition to a new idea; the sentence in question is functioning to convey information about the letters in recycling symbols, one of two major points the passage is addressing, along with numbers in those symbols. If we removed the sentence, it would disrupt the paragraph's parallel structure of discussing both letter and numbers, though. That means that the correct answer is C.

Let's consider another sample question:

Sample Question

The author is considering removing Sentence 8 from the passage. Should he or she do this?

- A.** Yes, because the sentence is extraneous and doesn't fit the tone of the rest of the passage.
- B.** Yes, because the sentence provides information that conflicts with other information in the passage.
- C.** No, because the sentence provides information necessary to understanding the passage's main idea.
- D.** No, because the sentence is transitioning to the passage's conclusion.

Sentence 8 states the following: "Recycled materials should always be placed in the correct stream." Before it in the passage, we find, "By referring to these symbols, consumers can discern where each item should be recycled." After it, we find discussion of the pros and cons of this recycling symbol system: "While this system is very accurate, it is not consumer-friendly . . ."

Did Sentence 8 stick out to you as you were initially reading the passage? You may have noticed its slight shift in tone. Whereas the rest of the passage is declarative and aims to provide general information to its reader until the last line, this sentence tells readers what they "should" do, as if providing direct instructions. Sentence 8 might also have seemed a bit extraneous because of the way in which it repeats information conveyed by the sentence that precedes it:

[7] By referring to these symbols, consumers can discern where each item should be recycled. [8]
Recycled materials should always be placed in the correct stream. [9]

Sentence 7, in telling us that the symbols allow consumers to figure out where to recycle each item, conveys the implicit idea that it is important that items be placed in the correct location. If where each type of plastic was recycled wasn't an issue, the symbols wouldn't be necessary. Thus, Sentence 8 isn't necessary to this passage, and we can omit it.

Having made that decision, we can turn our attention to deciding between the two "Yes" answer choices. A says that we should omit Sentence 8 because it's extraneous and differs in tone, while B says that we should omit it because it's info conflicts with info elsewhere in the passage. Well, viewing Sentence 7 side-by-side with Sentence 8 should demonstrate that the problem with Sentence 8 isn't that it contains conflicting information—it's that it says something directly that's already been conveyed in the passage. Answer A, which identifies both the extraneous information issue and the slight shift in tone, is spot-on.



To Rephrase Content or Not—and Why

Rephrasing questions are perhaps the toughest of this question type, as whether a sentence belongs in a passage or not can eventually grow to be a clear-cut decision, at least much more so than weighing different options that often only differ in terms of subtle connotations. When rephrasing, don't limit your focus only to the indicated portion of the passage. Make sure to pay attention to elements of context such as parallel structure and the sequence in which information is introduced in the passage to discern all the relevant ways in which the presented phrasing options differ from one another.

Practice Passage: The Eiffel Tower

[1] The Eiffel Tower in modern times is viewed as a symbol of Paris, France in specific and Europe overall, but it hasn't always been such a popular tourist destination. [2] Originally planned to be the highlight of the 1889 World's Fair, the tower was opposed by numerous individuals. [3] Much of this opposition was composed of artists, and their complaints were primarily visual in nature: the tower, they argued, would be a blot on the Parisian skyline and steal the thunder from the city's prominent landmarks located near where it would be constructed. [4] Eiffel, the architect who built the tower and after whom it is named, wasn't worried. [5] In addressing the artists' concerns, he drew flattering comparisons between the to-be-constructed tower and the Pyramids of Giza. [6] And indeed, after Eiffel completed his tower, the city grew to regard it proudly. [7] Originally intended to be taken apart twenty years after its construction, it was never dismantled, and today it still stands as one of the most famous architectural cultural icons in the world.

To Rephrase Word Choice or Not—and Why

Let's start at the narrowest focus: that of changing out a single word or two in a passage. Connotation is often the name of the game at this zoomed-in level, so make sure to think not just about what each word means in terms of dictionary definitions ("denotation"), but what other feelings and associations it conjures up ("connotations").

Sample Question

The author is considering changing the underlined word "tourist destination" in Sentence 1 to "icon." Should he or she make this change?

- A. Yes, because "icon" works better in parallel with "symbol" than does "tourist destination."
- B. Yes, because "tourist destination" has only tropical connotations that don't apply to the Eiffel Tower.
- C. No, because "tourist destination" is less visually specific than "icon."
- D. No, because the passage discusses the international appeal of the Eiffel Tower more than it discusses the French population's reaction to it.



Let's consider the sentence in which the word in question appears:

[1] The Eiffel Tower in modern times is viewed as a symbol of Paris, France in specific and Europe overall, but it hasn't always been such a popular tourist destination.

Ok, what's going on in this sentence? Simplified, the sentence's structure goes, "The Eiffel Tower is viewed as a symbol, but it hasn't always been such a popular destination. "Destination" is working in parallel with another noun, "symbol." That's something to consider. In addition, notice that the rest of the passage doesn't discuss tourism apart from the 1889 World's Fair, a specific event. If the Eiffel Tower "hasn't always been such a popular tourist destination," that clashes a bit with the information about how it was designed to be the main attraction at the World's Fair. That would make it a pretty popular tourist destination.

Let's consider the answer choices. Should we not swap out "tourist destination" for "icon" because "tourist destination" is less visually specific? The two options are both general phrases that don't indicate any specific imagery, so this isn't the correct answer. Should we not edit the phrasing because the passage discusses the international appeal of the Eiffel Tower more than it discusses the French population's reaction to it? No; while the passage does discuss the Eiffel Tower being built for the 1889 World's Fair, it discusses the specific reaction of French people to the building of it, not the opinion of residents of other countries. So, we should make the edit. But why? Because "tourist destination" has tropical connotations that don't apply to the Eiffel Tower, or because "icon" works better in parallel with "symbol?" "Tourist destination" *can* have tropical connotations, but it can also be used to describe colder locales. For example, Hawaii is a tourist destination that happens to be tropical, and Yellowstone National Park is a tourist destination that happens to not be tropical. The best answer is that the suggested change should be made because "icon" works better in parallel with "symbol" than "tourist destination" does. These two nouns are both referring to the Eiffel Tower in a specific role, that of a famous, representative structure. Choosing two synonyms to describe the tower in modern times as opposed to the tower when it was initially built and not as popular as it is today makes the sentence flow well and helps readers realize that the same thing is being referred to by each noun. A is the correct answer!

Let's consider another question, this time one that includes additional information about the passage's topic to help you decide between the presented phrasing options:

Sample Question

The author is considering changing the underlined phrase "the highlight" in Sentence 2 to "a feature." Given that the tower was designed to attract arguably the most attention out of any exhibit at the 1889 World's Fair, should he or she make this change?

- A.** Yes, because "a feature" puts more emphasis on the tower as the main attraction of the 1889 World's Fair
- B.** Yes, because the visual connotation of "the highlight" clashes with the subject matter.
- C.** No, because "the highlight" conveys a more specific focus on the tower as the most prominent exhibit France planned for the 1889 World's Fair.
- D.** No, because other exhibits at the 1889 World's Fair also wanted to attract a lot of attention.



Here's Sentence 2:

[2] Originally planned to be the highlight of the 1889 World's Fair, the tower was opposed by numerous individuals.

So, should we change “a highlight” to “a feature?” Before we consider whether we should make the change or not, consider what that change would do to the sentence and the passage. How does “the highlight” differ from “a feature?” While considering, we should keep in mind that we're told that “the tower was designed to attract arguably the most attention out of any exhibit at the 1889 World's Fair.” The question wouldn't provide information like this if it wasn't relevant. Well, one answer choice uses the word “the” (a definite article) while the other uses the word “a” (an indefinite article). It turns out that this choice of article makes a pretty big difference to the meaning of each option. “A feature” is grammatically equal with every other attraction at the 1889 World's Fair; the “a” conveys that it is one of many. In contrast, “the highlight” conveys that it is *the* main attraction—that it is more important than the other attractions. This matches the information that we're told about the Eiffel Tower having been “designed to attract arguably the most attention out of any exhibit at the 1889 World's Fair.” Plus, “highlight” conveys some of this focused attention, whereas “feature” doesn't. It looks like changing out “the highlight” for “a feature” wouldn't be a very good editing decision, but let's consider each of the answer choices to make sure.

Should we change “the highlight” to “a feature” because “a feature” emphasizes how the tower is the main attraction of the 1889 World's Fair? Nope—we just determined the opposite to be true: “the highlight” emphasizes this more than “a feature.” Should we change “the highlight” to “a feature” because the visual connotation of “the highlight” clashes with the subject matter? Did you read that answer choice only to wonder, “Wait, *does* “the highlight” visually clash with its subject matter?” Many people probably did. As a rule of thumb, if you didn't notice anything at all wrong with a word, phrase, or sentence that an answer choice claims contains a specific phrasing error it, the answer choice should be considered with some suspicion. In this case, “the highlight” doesn't clash with its subject matter at all, and that answer choice is incorrect. So, our initial idea was correct: we should not change “the highlight” to “a feature.” But why? You can correctly figure out that you need to make a change or not and still get the question wrong if you can't identify why that change should or shouldn't be made. So, should we leave the phrasing as-is because other exhibits at the 1889 World's Fair also wanted to attract a lot of attention, or because “the highlight” conveys a more specific focus on the tower as the most prominent exhibit France planned for the 1889 World's Fair. If the latter sounds familiar, it's because it matches the previous reasoning about the options perfectly. Even though other exhibits at the 1889 World's Fair surely also wanted to attract a lot of attention, this has nothing to do with the purpose for which the Eiffel Tower was designed. The correct answer is C!



To Rephrase a Phrase or Clause or Not—and Why

Let's zoom out a bit and consider questions that ask you to decide whether phrases and clauses should be rephrased.

Sample Question

The author is considering changing the underlined phrase “a blot on the Parisian skyline” in Sentence 3 to “a silhouette in the Paris cityscape.” Should he or she make this change?

- A. Yes, because “a silhouette” has positive visual connotations, whereas “a blot” has negative visual connotations.
- B. Yes, because “cityscape” better suits the subject matter than does “skyline”
- C. No, because “a blot” has connotations that have to do with “ink” that “a silhouette” doesn't have.
- D. No, because “a blot” is interpreted as a bad thing, whereas “a silhouette” is not.

Sentence 3, for your consideration, with this question's indicated phrase underlined:

[3] Much of this opposition was composed of artists, and their complaints were primarily visual in nature: the tower, they argued, would be a blot on the Parisian skyline and steal the thunder from the city's prominent landmarks located near where it would be constructed.

Skimming over the answer choices, did you notice that three of them have to do with [connotations](#), and two specifically with which option has positive connotations and which has negative connotations? Let's keep connotation in mind as we consider the phrase that's in the sentence to begin with. The sentence is discussing artists' complaints about the Eiffel Tower before it was built. They had two main complaints: that it would overshadow other major Parisian landmarks and not look nice in the city from a distance because it would be so much taller than other buildings. This sounds like we're looking for a term to describe the Eiffel Tower that has negative connotations—these artists certainly aren't happy and excited that it is going to be built!

Since many of the answer choices discuss connotation, let's consider each option's connotations before we attempt to match our reasoning to an answer choice. “A blot on the city's skyline”—does that sound like a nice thing? No, not at all. What about “a Silhouette in the Paris cityscape?” That could be a good thing or a bad thing; it's hard to tell. It's pretty neutral. It's certainly not negative like “a blot on the city's skyline” is negative. It makes sense that when the passage explains the artists' complaints, it might use negatively connoted language, so leaving “a blot on the Parisian skyline” in the passage is looking like a pretty smart choice. Ok, time to consider the answer choices.

Answer choice C may stick out a bit because “blot” does have some connotations to do with ink (e.g. “ink blot”). Does that have anything to do with the passage's subject matter, though? Nope! We can ignore answer choice C. Make sure to distinguish between a reason that is true and an entire answer choice that is true! Incorrect answers may still include true statements that just function as bad reasons for making a given editing decision. “Cityscape” doesn't suit the subject matter any better than “skyline” does—again, if you have to really wonder about which option suits the passage better, they might suit the passage equally, as in this case. Answer B is incorrect. This leaves us with A and D. Should we change “a blot . . .” to “a silhouette . . .” in the passage because “a silhouette” has positive connotations and “a blot” has negative connotations, or should we leave “a blot . . .” in the passage because “a blot” is interpreted as a bad thing whereas “a silhouette” is not? Did you notice that both of these answer choices are essentially



using the same reasoning and only differing in which editing decision they support? Read them carefully, and you can figure out that answer choice A doesn't make any sense. Yes, it's true that "a silhouette" is less negatively connoted than "a blot," but we are looking to make use of those negative connotations in the sentence, which is describing why the artists objected to the Eiffel Tower. It makes more sense for people to object to something considered a bad thing ("a blot . . .") than to something that is neutral or good ("a silhouette . . ."). Answer choice D is correct!

Let's consider another phrase-based question, this time one that concerns a verb phrase:

Sample Question

The author is considering changing the underlined phrase "steal the thunder from" in Sentence 3 to "compete for attention with." Should he or she make this change?

- A.** Yes, because "compete for attention with" makes the opposition seem less reliable in their research and opinions.
- B.** Yes, because "compete for attention with" is both more formal and more appropriate in meaning.
- C.** No, because "steal the thunder from" is more appropriate given the meteorological significance of the tower discussed in the passage.
- D.** No, because "steal the thunder from" better conveys the opposition's distaste for the planned Eiffel Tower.

Here's Sentence 3 with the indicated part of the sentence underlined:

[3] Much of this opposition was composed of artists, and their complaints were primarily visual in nature: the tower, they argued, would be a blot on the Parisian skyline and steal the thunder from the city's prominent landmarks located near where it would be constructed.

Let's consider each answer choice individually. Answer choice D claims that the writer should not make this change because "steal the thunder from" conveys the distaste of the artists for the planned Eiffel Tower better than "compete for attention with" does. That's not true; neither phrase conveys distaste notably more than the other one. Consider answer choice C: is the Eiffel Tower's "meteorological significance" ever discussed in the passage? Nope! We can ignore that answer choice, too, thus figuring out that the correct answer involves making the suggested change. Ok, if "steal the thunder from" should be changed out for "compete for attention with," why is that so? Let's take a moment to consider the subtleties of each phrase. "Steal the thunder from" is a casual phrase most often used for people; to "steal someone's thunder" is to draw attention to oneself when it more properly should be directed to someone else for that person's accomplishment. On the other hand, "compete for attention with" is a straightforward, more formal phrase that is often used to describe people or things. Let's now consider the remaining answer choices, A and B: should the phrase be swapped out because "compete for attention with" makes the artists seem "less reliable in their research and opinions?" If you got a little confused when reading that answer choice and wondered if you missed something to that effect in the passage, you're probably not the only one. This is an answer choice that sounds reasonable, until you realize that nothing about "compete for attention with" makes the artists seem unreliable. If anything, the phrase's formality makes the artists seem more reliable than when they are the subject of the more causal "steal the thunder from." This leaves us with one answer choice, B: yes, "steal the thunder from" should be changed out for "compete for attention with" because the latter is both more formal and more appropriate in meaning. We've found the correct answer using process of elimination.



To Rephrase a Sentence or Not—and Why

Let's look at one more example of why-rephrase-or-not questions, this time focusing on one that asks about changing out an entire sentence. Sharpen up your sentence-level context skills!

Sample Question

The author is considering changing Sentence 5, “In addressing the artists’ complaints, he drew flattering comparisons between the planned tower and the famous Pyramids of Giza,” to “In addressing the artists’ concerns, he compared the planned tower with the enormous Pyramids of Giza, which can be seen for miles around.” Should he or she make this change?

- A. Yes, because more detail about the Pyramids of Giza is helpful and relevant.
- B. Yes, because while the reader is expected to have heard of the Pyramids of Giza, he or she might not be aware that they are large.
- C. No, because the alternate sentence emphasizes the negative features that the opposition was worried about and Eiffel was addressing.
- D. No, because the new sentence doesn’t describe how Eiffel intended the comparison to be taken, while the first sentence specifies the comparisons as “flattering.”

Ok, seeing as we’re thinking about changing an entire sentence, let’s look at that sentence in context with the sentences that precede and follow it. Paying close attention to the transitions between these three sentences will help shed light on whether the initial phrasing or the revised phrasing is a better fit for the passage.

[4] Eiffel, the architect who built the tower and after whom it is named, wasn’t worried. [5] In addressing the artists’ concerns, he drew flattering comparisons between the to-be-constructed tower and the Pyramids of Giza. [6] And indeed, after Eiffel completed his tower, the city grew to regard it proudly.

Sentence 4 tells us that the eponymous Eiffel “wasn’t worried.” Sentence 5 provides information about how he responded to the artists’ concerns and complaints. Sentence 6 tells us how the scenario ended, with the city eventually being proud of the tower. So, Sentence 5 functions as a transition describing how Eiffel exactly responded to his critics. With that in mind, let’s review the phrase options independent of the question, considering how they differ.

Our two options for the passage are “In addressing the artists’ complaints, he drew flattering comparisons between the planned tower and the famous Pyramids of Giza” and “In addressing the artists’ concerns, he compared the planned tower with the enormous Pyramids of Giza, which can be seen for miles around.” At first glance, these options may seem very similar—potentially too similar for a test-taker to distinguish confidently between them. But take an extra moment and consider the details: while each sentence conveys the same big-picture idea (“Eiffel told his critics not to worry and compared the tower to the Pyramids of Giza”), they differ in their details: the first refers to the artists’ “complaints,” the second to their “concerns.” There’s a small difference there that might be relevant: a “concern” is not necessarily a “complaint.” “Complaint” is the stronger, more negative word. Furthermore, each sentence ends with a different focus: the first one calls the Pyramids of Giza “famous,” and the rephrased option calls them “the enormous



Pyramids of Giza, which can be seen for miles around.” Ok, so our sentences differ in these two particular ways. On to the answer choices!

First up: answer choice A. Does either option contain “more detail” about the Pyramids of Giza than the other? Not noticeably. One calls them “famous” and the other emphasizes their large size. Changing between them doesn’t result in one that provides detail that’s “helpful and relevant.” Let’s ignore this answer choice for now. Answer choice B: this one hinges on the idea that the reader doesn’t know that the Pyramids of Giza are large, and that the description of the pyramids’ size should be included because (one can infer) the reader might otherwise not realize this. The Pyramids of Giza—indeed, any architectural structures called “pyramids”—are inherently expected to be large. It’s not solid reasoning to say that the size descriptor should be included because audiences wouldn’t realize this otherwise. Since neither of the “yes” answer choices is looking great, let’s assume that we should not make the suggested change and consider the two “no” answer choices.

Ok, remember to correctly differentiate the options as you read! You don’t want to miss the question because you mixed them up. This issue becomes increasingly relevant as we move from considering single words to larger and larger phrases, clauses, and full sentences. So, should we keep the answer choice that calls the pyramids “famous” instead of using the one that emphasizes their large size because the new sentence doesn’t include the word “flattering?” This is what answer choice D comes down to. Take a moment to consider the passage: why would Eiffel make an unflattering comparison? He’s the one who is arguing for the Eiffel Tower to be built and trying to make it look appealing in the face of criticism. One can assume that the comparison would be flattering. Answer choice D isn’t looking that great. Let’s consider answer choice C: that we should not change the indicated sentence to emphasize the Pyramids’ size because the revised sentence would emphasize a negative feature of the Eiffel Tower, the one that the artists were worried about. Bingo! That’s it. Why would Eiffel want to emphasize that the Tower would be large when the artists are specifically concerned about how it will affect—and potentially dominate—the Paris skyline? To pick up on this, we have to go back further than a single sentence before the indicated one in the paragraph, but once we notice this logic, answer choice C makes perfect sense. Don’t forget to scan the passage for ways in which sentences can interact over relatively long distances!



Identifying the Function of Passage Elements

This may come as a surprise, but not every question on the ACT English section asks you either to identify a grammatical error or to make an editing decision. Some questions ask you about the function of certain parts of the passage—what exactly a specific word, phrase, or sentence is doing in the context of the text. These questions test your ability to read a term in context directly, unlike other problems, which ask you to gather this information to weigh in on a writing decision that needs to be made or an error that needs to be corrected. It's worth practicing a few of these problems so as not to get taken by surprise by any you happen to encounter on your test!

Practice Passage: Young's Double-Slit Experiment

[1] Physics has attempted to model the behavior of light for hundreds of years, but sometimes experiments demonstrate that an accepted model of light doesn't correctly approximate the behavior of the real thing. One classic experiment that dramatically altered scientists' understanding of light was performed by Thomas Young around the turn of the nineteenth century. This experiment involved sunlight as a light source, a pinhole camera, a blank wall, and a sheet of paper with multiple slits cut into it. This last feature has given the experiment its common name of "the double-slit experiment." When focused light hit the paper with the slits in it, it behaved in a fascinating way: it formed a pattern involving a gradient of light and dark bands on the wall. If light were behaving as a stream of particles, it would travel straight to the wall, but it did not do this. Instead, it behaved as waves emitted from two point sources (the slits). These waves interfered with each other to cause the patterns that appeared on the walls. [2] During Young's time, many scientists, notably Sir Issac Newton, believed in the "corpuscular" theory of light; they thought that light behaved like a stream of particles. [3] Young's discovery, though, provided undeniable support for a wave theory of light. A combination of experiments supporting facets of the corpuscular and wave theories led people to conclude that light exhibits a "wave-particle duality" that classical mechanics cannot explain, but quantum mechanics can.



Identifying the Function of Punctuation

Certain function-identification questions might not ask about specified text; instead, they might concern punctuation. Certain punctuation marks have a range of potential interpretations; these can form the basis of function-identification questions easily, as test-takers are asked which of a range of potential uses is at work and what the resulting meaning is in the context of the passage.

Sample Question

In the sentence immediately following point [2], what is the function of the quotation marks surrounding the underlined word “corpuscular”?

- A. The quotation marks indicate the author is suspicious of this particular theory and thinks it is unreliable.
- B. The quotation marks indicate that the author is quoting Sir Isaac Newton, who gave the theory its name.
- C. The quotation marks indicate that this is the commonly recognized if unofficial name of the theory.
- D. The word “corpuscular” is explicitly defined in the passage, and the quotation marks indicate to which word the definition relates.

As usual, let’s first consider the indicated quotation marks in the sentence in which they’re used:

[2] During Young’s time, many scientists, notably Sir Isaac Newton, believed in the “corpuscular” theory of light; they thought that light behaved like a stream of particles.

If you haven’t reviewed all the different ways in which quotation marks can be used, it may be helpful to [do so now](#). They certainly don’t always convey a direct quotation! In this case, no one is being quoted. The quotation marks are functioning in a different way. Knowing that they don’t indicate direct quotation allows us to ignore answer choice B, which states that the marks convey a direct quotation from Sir Isaac Newton. Nope! This leaves us with three potential answer choices. But wait—is “corpuscular” explicitly defined in the passage? No—no definition is provided, so D doesn’t make sense either. This means that the quotation marks are either indicating that the author is using the term “corpuscular” with suspicion in order to convey how he or she thinks the associated theory is unreliable (A) or that the quotation marks convey that this is not the official name of the theory, but how it’s commonly referred to (C). Consider the passage: how does the author feel about the corpuscular theory? He or she is simply conveying that many scientists of Young’s time believed this theory; we don’t get a lot of information about whether the author him- or herself adheres to the theory, and it’s not cast in a suspicious light at all. That means that C is the best answer. “Corpuscular” is surrounded by quotation marks to convey that it is the theory’s unofficial but common name.



Identifying the Function of a Word

Identifying the function of a single word requires you to consider it as one puzzle piece amongst many that come together to form the ideas and the “big picture” of the passage. Consider how the indicated word interacts with these multiple levels of meaning when trying to pick the answer choice that best explains what it’s doing in the passage.

Sample Question

In the sentence immediately following point **1**, the underlined word “accepted” serves to indicate which of the following?

- A.** The models in question have been peer-reviewed by other scientists but may or may not be believed by the general public.
- B.** The general public holds the models in question to be true despite scientists’ qualms about them being unreliable.
- C.** Both scientists and the general public think that these models are the best estimations of lights’ behavior we currently have developed.
- D.** These models of lights’ behavior are the only ones that the writer thinks to be true, even though most people disagree with him or her.

Let’s consider the word in context in the sentence in which it appears:

1 Physics has attempted to model the behavior of light for hundreds of years, but sometimes experiments demonstrate that an accepted model of light doesn’t correctly approximate the behavior of the real thing.

Though “accepted” may look like a verb and often be used as a verb, in this particular sentence, it’s functioning as a participle—a verb acting like an adjective. The question isn’t asking us for a synonym, as in certain ACT Reading questions, or a dictionary definition. Instead, the question concerns the effect that this particular word has on the passage—what does it indicate? All of the answer choices share a few features—“accepted” is referring to “an accepted model of light,” and each answer choice reflects some judgment about the veracity of these “accepted” models and specifying a particular group by which these models are “accepted.” With that in mind, let’s look at what makes each answer choice unique. Does the sentence specify anything about scientists accepting the models vs. the general public accepting the models? No, it just says that the models are “accepted,” not specifying scientists in particular. There’s no science-general public conflict going on in the sentence, so we can discard answer choices A and B, since each of those hinge on either scientists or the general public believing the theories while the other group does not. This leaves us with C, that both scientists and the general public think these models are the best we currently have, and D, that these models are the only ones the writer accepts, despite disagreement. Wondering how the writer got involved in all this? It’s a good question, since the passage isn’t written in first-person point of view and the writer never inserts him- or herself into the passage at all. Instead, he or she just relates facts. The writer doesn’t indicate that he or she supports the indicated models despite the disagreement of many other people, so answer choice D doesn’t make sense. C is the best answer—even though the passage doesn’t differentiate between scientists and the general public, its calling the theories “accepted” indicates that both scientists and laypeople think that these models are the best we currently have.



Identifying the Function of a Phrase or Clause

Let's consider questions about the function of phrases and clauses. While not conveying entire thoughts and not limited to the shades of meaning that single words convey, these grammatical units can provide additional details, introductory contextualization for the points they precede, points, counterpoints—the list goes on! Each question is going to be unique, as each phrase or clause will be uniquely related to the context in which it appears. Try to get a good sense of what each clause indicates by itself before considering it in the passage.

This next question considers the meaning of a particular phrase:

Sample Question

In the sentence immediately following point [2], what is the function of the phrase set apart from the rest of the sentence by commas, “notably Sir Isaac Newton?”

- A. The phrase provides a specific example of a famous scientist who thought the theory being discussed was true, lending it credibility.
- B. The phrase provides an example of a scientist who argued against the theory being discussed, casting doubt on its credibility.
- C. The phrase serves to introduce Newton, the scientist the passage discusses in its conclusion.
- D. The phrase serves to compare Newton with Young, suggesting that they held similar beliefs.

For convenience, here is that sentence again:

[2] During Young's time, many scientists, notably Sir Issac Newton, believed in the “corpuscular” theory of light; they thought that light behaved like a stream of particles.

To put the question a different way, why would the author want to mention Sir Isaac Newton in the sentence and in the passage at this specific spot? Newton isn't mentioned anywhere else. Simply recognizing that allows us to discard C, which calls Newton “the scientist the passage discusses in its conclusion.” It does not, so this answer is incorrect. Let's consider how Newton is interacting with the other sentence concepts. How does he relate to the “corpuscular” theory? He believes that theory to be true, so B can't be correct; he's not being mentioned as “an example of a scientist who argued against the theory being discussed.”

At this point, we need to decide between answer choices A and D. Take another look at the specified phrase: “notably” is showcasing Newton in a way, suggesting that he is “notable” (*important*) in some regard. Is he important because he's being compared to Young? No, he's not compared to Young at all, though the two scientists are mentioned in the same sentence. Answer A is correct: Newton is mentioned to provide a specific example (this fits with the meaning of “notably”!) of a famous scientist who thought the theory being discussed (the “corpuscular” theory) was true, lending it credibility. Paying attention to which scientists support which theories is a useful tactic on the ACT, and not just on the test's reading section!



Ok, time to consider a question about the function of a specified clause:

Sample Question

What is the function of the underlined clause “they thought that light behaved like a stream of particles” that follows point **2** in the passage?

- A.** The clause explains that the “corpuscular” theory held that the particles of light travel in a stream, whereas the other theory the passage discusses suggested that the particles behave like a gas, bouncing around randomly.
- B.** The clause explains how Young and Sir Isaac Newton’s beliefs were very similar
- C.** The clause repeats information from earlier in the passage to provide a transition to a completely different topic
- D.** The clause provides helpful explanation about what the “corpuscular” theory of light entails

Sentence 2, one more time, in addition to the sentence that follows it:

2 During Young’s time, many scientists, notably Sir Issac Newton, believed in the “corpuscular” theory of light; they thought that light behaved like a stream of particles. Young’s discovery, though, provided undeniable support for a wave theory of light.

The indicated clause is continuing a discussion of the “corpuscular” theory of light before the passage talks about how the results of Young’s experiment (described earlier) affected the debate about light. The clause isn’t transitioning to a completely different topic, so C is incorrect. The passage also never tells us which side of the debate Young was on—whether he thought that the “corpuscular” theory was correct or not. Furthermore, the clause doesn’t compare Young’s and Newton’s beliefs. This means that B is incorrect. A and D are somewhat similar, though A appears to go into more detail than D. Don’t just pick A because it seems more specific! Sometimes the extra details included in an answer choice can make it incorrect. For instance, consider A: “The clause explains that the ‘corpuscular’ theory held that the particles of light traveled in a stream . . .” is perfectly correct, but “. . . whereas the other theory the passage discusses suggests that the particles behave like a gas, bouncing around randomly” is incorrect! The other theory of light that the passage discusses is the “wave” theory of light, which suggests that light behaves like a wave, not like a gas. Nothing is mentioned in the passage about light behaving “like a gas” with particles “bouncing around randomly.” A is incorrect, and the correct answer is D, the more general statement that the indicated clause “provides helpful explanation about what the ‘corpuscular’ theory of light entails. Positioned immediately after the first mention of the “corpuscular” theory, the clause functions as an implicit definition of what that theory’s adherents believe.



Identifying the Function of a Sentence

Certain function-identification questions may point to an entire sentence and ask you what it's doing in the passage. If you get overwhelmed by having to consider an entire sentence, ask yourself: what would the passage lose if this sentence were deleted? Noting the particular problems that would be caused by a sentence's conspicuous absence can help you pinpoint exactly why it's necessary.

Sample Question

What is the function of the sentence that follows point [3] in the passage?

- A. To draw out the conflict between the “corpuscular” theory and the results of Young’s experiment
- B. To demonstrate how Young’s evidence supported the “corpuscular” theory
- C. To suggest that the wave theory of light needed refinement at this point in history
- D. To convey that Young initially misinterpreted the results of his own experiment

When looking at entire sentences, be sure to consider the sentences that bookend them—those that appear immediately before and immediately after. If those rely on other neighboring sentences and it doesn't make much sense to consider the “bookend” sentences alone, work farther backwards or forwards in the passage as needed until you can get a good idea of which sentences the indicated one relies on and how it connects to them.

During Young's time, many scientists, notably Sir Isaac Newton, believed in the “corpuscular” theory of light; they thought that light behaved like a stream of particles. [3] Young's discovery, though, provided undeniable support for a wave theory of light. A combination of experiments supporting facets of the corpuscular and wave theories led people to conclude that light exhibits a “wave-particle duality” that classical mechanics cannot explain, but quantum mechanics can.

For example, our sentence begins with “Young's discovery.” That phrase refers back to the middle of the passage, where Young's experiment is explained. It may be helpful to consider that portion of the passage, too. This may look like a lot of material for one question to concern, but remember: we're working at the level of sentences, here! You don't need to worry about the details of individual words or phrases, just about the “gist” of the entire sentence.

When focused light hit the paper with the slits in it, it behaved in a fascinating way: it formed a pattern involving a gradient of light and dark bands on the wall. If light were behaving as a stream of particles, it would travel straight to the wall, but it did not do this. Instead, it behaved as waves emitted from two point sources (the slits). These waves interfered with each other to cause the patterns that appeared on the walls. During Young's time, many scientists, notably Sir Isaac Newton, believed in the “corpuscular” theory of light; they thought that light behaved like a stream



of particles. ³ Young's discovery, though, provided undeniable support for a wave theory of light. A combination of experiments supporting facets of the corpuscular and wave theories led people to conclude that light exhibits a “wave-particle duality” that classical mechanics cannot explain, but quantum mechanics can.

Note how the results of Young's experiment are explained without referencing how Young actually interpreted his experiment. This means that D can't be correct—we're not told at all how Young interpreted his experiment's results, so the sentence in question definitely isn't suggesting that he misinterpreted them! Nothing in the passage suggests that the “wave” theory of light “needed refinement at this point in history,” either; it's presented as an opposing theory, but not necessary a less accurate one. In fact, the sentence in question says that Young's experimental results supported the wave theory of light; this makes the theory look pretty reliable. So, C isn't the answer either. So the answer is either B, that our sentence's function is to demonstrate how Young's experimental results supported the “corpuscular” theory, or . . . wait a second! Did you catch how that's completely wrong? Our sentence explicitly tells us that Young's experimental results supported the other theory—the “wave” theory of light. That means that A is the correct answer: the indicated sentence points out the inconsistency between the results of Young's double-slit experiment and the “corpuscular” theory of light.



Identifying the Effects of Editing

Certain questions on the ACT English section concern the effects of writing and editing, asking you to pinpoint the exact repercussions of particular editing decisions. These questions don't ask you which sentence to add or where to add it, for example, or whether the sentence should be added at all. They proceed on the assumption that the passage's author wants to make a given change (e.g. is going to add an indicated sentence); your job isn't to opine about the decision, but to figure out how it will change the way the passage reads.

Practice Passage: New Zealand's Rabbit and Stoat Problems

New Zealand has historically had to deal with a number of threats to its native ecosystem, many of which were caused by *imprudent but purposeful* human decisions to attempt to adjust the environment's delicate balances. For instance, when European colonists began living in New Zealand, they brought rabbits with them [1]. What the rabbit importers did not consider is that New Zealand, unlike Europe, does not have any endemic predatory mammals. Put another way, no animals that live there eat rabbits! The rabbit population *exploded*, creating an imbalance in the native ecosystem and threatening native plants as well as farmers' crops. At this point, to combat the rabbit population, New Zealand began to import stoats, a type of small predatory mammal that eats rabbits, *despite the protestations of* ornithologists (bird scientists). New Zealand's bird populations suffered greatly from this sudden introduction of a predator for which they were not evolutionarily prepared. The beloved national bird of New Zealand, the kiwi, provides the most overt example of just such an unprepared species. Having adapted to an environment with no native mammalian carnivores, it roams around the ground and has very poor flying skills. [2] The stoat population *wreaked havoc on* the kiwi population [3]. Conservation efforts aimed at reducing the stoat population and protecting kiwis are still ongoing. New Zealand's ecological history serves as a poignant lesson of how dangerous it is for people to attempt to change an ecosystem without fully understanding the repercussions that may arise from such changes.



Identifying the Effects of Adding Content

If a question asks you about the effects of adding content, make sure to consider not only the immediately surrounding context of the addition, but also the consequences of adding that information to the paragraph as a whole. An addition might change the way that certain sentences read later in the text.

Sample Question

At point 1, the author is considering changing the period that follows “rabbits” to a comma and adding the following, which is true:

thinking to make the surrounding environment more like their native countries, where rabbits were plentiful

If the writer makes this addition, it would contribute which of the following to the passage?

- A. Explanation about why colonists choose to bring rabbits over other animals common in their countries
- B. Direct and detailed commentary on the specifics of New Zealand’s environment
- C. Insight into the colonists’ reasoning for bringing rabbits to New Zealand
- D. It would provide a positive perspective on the effects of bringing rabbits in New Zealand

For questions such as this one, it’s important that you have a good grasp on exactly how the passage is functioning before the addition is (hypothetically) made. Having to go back and reread too many parts of the passage can easily get confusing as you mix up what’s being added and what you may have missed during the first read-through. Make sure you keep these distinct in your mind when answering the question!

This particular question is asking us about adding a phrase to the sentence shown below. The phrase has been added and is underlined:

For instance, when European colonists began living in New Zealand, they brought rabbits with them, thinking to make the surrounding environment more like their native countries, where rabbits were plentiful.

It may be helpful to consider why the writer would want to make such a writing decision, but remember: that’s not what’s being asked here. What is our concern is the change the underlined phrase makes to the passage when it is added. Well, the phrase in question is drawing out a comparison between the colonists’ native countries and New Zealand. New Zealand didn’t have rabbits; they were familiar with land that had rabbits. So they brought rabbits to make New Zealand more like their old homes.

Time to consider the answer choices. Is the phrase in question providing “direct and detailed commentary on the specifics of New Zealand’s environment?” One could argue that it’s providing some indirect information—New Zealand didn’t originally have any rabbits before colonists brought them. This is indirect, though, and the answer choice specifies “direct” information. There’s nothing in the phrase being added that specifically and directly provides details about New Zealand’s environment. That’s a distinction significant enough to knock this answer choice out of the running. Ignore C, and let’s consider the others.



D may catch your eye as a potentially correct answer choice, but read carefully: while the phrase certainly doesn't put a particularly negative spin on the introduction of rabbits to New Zealand (certainly not in comparison to the rest of the passage!), the answer choice specifies that it provides "a positive perspective on the effects"—the *effects*—of the rabbits' introduction. This phrase isn't talking about the effects of rabbits being introduced at all—it's talking about the cause of their introduction. We can ignore answer choice D as well.

A may look like a good answer choice initially. The sentence does have to do with "why colonists chose to bring rabbits" with them to New Zealand; however, answer choice A adds in information that makes it incorrect when it specifies that the phrase provides info about "why colonists choose to bring rabbits *over other animals common in their countries*." No other animals are mentioned in the phrase, and we're not given any information that helps to answer the question "Why rabbits as opposed to, say, sheep or partridges?"

Answer choice B heads in the same direction as A initially but doesn't include that incorrect specificity. The added phrase does offer "insight into the colonists' reasoning for bringing rabbits to New Zealand." It tells us exactly why the colonists wanted to bring rabbits to New Zealand: they were "thinking to make the surrounding environment more like their native countries." B is the correct answer! Remember to watch out for answer choices that seem similar but differ in how specific they get. It's not always the case that the more specific one is the correct one!

Sample Question

At point **2**, the author is considering beginning the sentence with the phrase "Within just a few years," and changing the "T" in "The" to a lower-case "t."

If the writer makes this addition, it would:

- A.** emphasize the severity of the rabbits' effects on the kiwi population
- B.** urge readers to help protect the New Zealand kiwi population
- C.** provide evidence that the kiwis were not completely defenseless
- D.** specify the time it took for the stoats to have a significant effect on the kiwi population

Let's consider another, similar question. Considered out of context, what is the phrase "Within a few years" talking about? It's talking about time. While one of the answer choices begins with "specifies the time it took," we should consider the others to make sure we find the correct one. Let's consider the phrase in context:

Within just a few years, the stoat population wreaked havoc on the kiwi population.

Not only is it specifying a time, it's specifying the time it took the kiwi population to be devastated by the stoats. Answer choice D is looking pretty accurate, but let's consider the other ones to be sure.

Answer choices B and C don't make sense. As for B, the phrase does nothing in context to urge readers to help the New Zealand kiwi population. The passage doesn't ever do this directly; it's providing information and helping readers learn something, not asking them to do anything. Answer choice C is also off the mark;



the phrase does the opposite of “provid[ing] evidence that the kiwis were not completely defenseless”; it emphasizes how defenseless they were by detailing just how short of a time it took for them to be greatly affected by the stoats.

This leaves us with answer choices A and D. A, that the phrase “emphasize[s] the severity” of the problem. But rabbits have nothing to do with the problem—stoats are the problem! Rabbits aren’t said to cause kiwis any trouble. Answer choice A is incorrect, making D the correct answer. Make sure to orient yourself in the passage so that when multiple things or characters are in play, you don’t get confused about which part of the passage the phrase is affecting.

Sample Question

At point **3**, the author is considering adding a comma after “population” and following it with the following phrase:

eating kiwi eggs and killing adult kiwis

If the writer were to make this addition, it would:

- A.** suggest that only certain members of the kiwi population are at risk from stoat predation
- B.** explain why kiwi chicks need special protection from stoat attacks
- C.** use detail to help convey the severity of the stoats’ effect on the kiwi population
- D.** help the reader empathize with the stoats

This phrase is telling us exactly what the stoats did to the kiwi population to decrease it. In context, it would read like this:

The stoat population wreaked havoc on the kiwi population, eating kiwi eggs and killing adult kiwis.

Answer choice A is incorrect; what readers take from this phrase is not that only certain members of the kiwi population are at risk of being attacked by stoats, but that stoats are not only eating adult kiwis but also their eggs. While, say, kiwi chicks aren’t mentioned, the phrase doesn’t seem to suggest that they are somehow immune from stoat predation. On the subject of kiwi chicks, B specifically mentions them, but the phrase doesn’t mention them, so it’s not correct to make the logical leap and claim that the phrase is explaining why they need special protection from stoats. The phrase certainly isn’t helping the reader identify with the stoats, either. In the rest of the passage, the writer has worked to make the kiwis seem pitiable and the stoats look like bloodthirsty predators. This phrase is only contributing to that image of the stoats, so D isn’t the correct answer. That leaves us with C, that the phrase “use[s] detail to help convey the severity of the stoats’ effect on the kiwi population.” By pointing out that stoats not only eat adult kiwis, but also their eggs, the phrase is emphasizing how severe the stoats’ effect on the kiwi population is. This means that C is the correct answer.



Identifying the Effects of Removing Content

ACT Questions that ask you how removing content could change a passage also require you to understand how each sentence functions within the passage as written, and what potential holes it could leave in the reader's current experience if removed from the text.

Here is the practice passage as in the last section, reproduced for your convenience:

Practice Passage: New Zealand's Rabbit and Stoat Problems

[1] New Zealand has historically had to deal with a number of threats to its native ecosystem, many of which were caused by imprudent but purposeful human decisions to attempt to adjust the environment's delicate balances. [2] For instance, when European colonists began living in New Zealand, they brought rabbits with them. [3] What the rabbit importers did not consider is that New Zealand, unlike Europe, does not have any endemic predatory mammals. [4] Put another way, no animals that live there eat rabbits! [5] The rabbit population exploded, creating an imbalance in the native ecosystem and threatening native plants as well as farmers' crops. [6] At this point, to combat the rabbit population, New Zealand began to import stoats—small, predatory, weasel-like mammals that eat rabbits—despite the protestations of ornithologists (bird scientists). [7] New Zealand's bird populations suffered greatly from this sudden introduction of a predator for which they were not evolutionarily prepared. [8] The beloved national bird of New Zealand, the kiwi, provides the most overt example of just such an unprepared species. [9] Having adapted to an environment with no native mammalian carnivores, it roams around the ground and has very poor flying skills. [10] The stoat population wreaked havoc on the kiwi population. [11] Conservation efforts aimed at reducing the stoat population and protecting kiwis are still ongoing. [12] New Zealand's ecological history serves as a poignant lesson of how dangerous it is for people to attempt to change an ecosystem without fully understanding the repercussions that may arise from such changes.



Sample Question

If the author were to remove Sentence 4 from the passage, the passage would lose:

- A.** a transition linking one idea to another
- B.** information important to understanding the position the author is arguing against
- C.** helpful paraphrasing of scientific terminology
- D.** nothing; the sentence provides extraneous information

Sentence 4 reads, “Put another way, no animals that live there eat rabbits!” The “put another way” phrase is an immediate and overt clue that this sentence is functioning in direct relation to another one, the one before it: “What the rabbit importers did not consider is that New Zealand, unlike Europe, does not have any endemic predatory mammals.” Sentence 4 isn’t providing any information important to understanding a position the author is arguing against, because throughout the essay, the author really doesn’t argue against any positions, only expressing a definite opinion in the last line, much later in the text than when Sentences 3 and 4 appear.

We can ignore answer choice C for now. Due to how linked Sentence 4 is to the material that precedes it, you might assume that it is a transition; however, take a closer look at it, and you’ll see that it’s only connected to the ideas that come before it and restates them. It’s not transitioning *to* anything. Answer choice A can’t be correct either.

This leaves us with C, that the passage would lose “helpful paraphrasing of scientific terminology” were Sentence 4 to be deleted, and D, that the passage would lose “nothing” as “the sentence provides extraneous information.” This might seem tricky; after all, Sentence 4 *is* restating the information that directly precedes it; doesn’t that inherently make it extraneous? No, it doesn’t, because the rephrasing in this case is doing something valuable for readers: it’s translating “endemic predatory mammal,” which some readers might not understand, into “things that eat rabbits,” which readers should be able to grasp much more easily. Answer choice C is correct.

Sample Question

If the author were to remove the underlined portion of Sentence 6 from the passage and change the second dash to a period, the passage would:

- A.** lose vital information describing stoats
- B.** make it seem as if the colonists’ decision to import stoats was unopposed
- C.** make the colonists seem as if they considered potential environmental effects of importing stoats
- D.** a valuable connection to the discussion of rabbits earlier in the passage

Consider the underlined portion of Sentence 6:

At this point, to combat the rabbit population, New Zealand began to import stoats—small, predatory, weasel-like mammals that eat rabbits—despite the protestations of ornithologists (bird scientists).



Our phrase begins with the preposition “despite,” which signals a significant logical shift. As it’s written in the passage, this sentence’s logic tells us that the colonists imported the stoats even though bird scientists told them not to do so. Omitting this phrase is sure to change the meaning of the sentence dramatically. With that in mind, let’s consider the answer choices.

Well, we can discard answer choice A; omitting the phrase wouldn’t result in “losing vital information about stoats,” as the phrase doesn’t mention stoats at all. After consideration, we can also ignore answer choice D, “a valuable connection to the discussion of rabbits earlier in the passage.” The phrase doesn’t directly connect to the discussion of rabbits earlier in the passage. At a stretch, we might be able to say that the ornithologists saw what happened with the rabbits and wanted to warn the colonists not to make a similar mistake in importing stoats, but given that nothing is said to connect the ornithologists with the discussion of rabbits—they’re bird scientists, not rabbit scientists—answer choice D doesn’t look like the best option.

Let’s consider the remaining answer choices: that removing the indicated phrase would (C) “make the colonists seem as if they considered potential environmental effects of importing stoats” or (B) “make it seem as if the colonists’ decision to import stoats was unopposed.” Both of these seem to relate to the sentence well, as they each discuss an effect on how the reader understands the colonists, and the colonists are discussed in the sentence in question. If we take out the phrase that says that the colonists made their decision “despite” what the ornithologists said, we take out the representation of their opposition. “Making a decision” can mean that everyone agreed; making a decision “despite” some complaints can’t mean that. That means that the correct answer is B, that removing the phrase would make it seem as if the colonists’ decision to import stoats was unopposed.

Sample Question

If the author were to remove Sentence 11 from the passage, the passage would lose:

- A.** a summary statement about the importance of considering all potential effects of making environmental changes
- B.** specific details about how New Zealand is dealing with its stoat problem
- C.** a transition statement linking evolutionary principles to the results of New Zealand’s kiwi and stoat situation
- D.** a statement that shows how historical environmental changes have present-day effects

Here is Sentence 11:

Conservation efforts aimed at reducing the stoat population and protecting kiwis are still ongoing.

“Still ongoing”—this is the first time in the passage up until this point that the author has talked about things that are going on right now, not just things that happened in the past. Let’s consider the answer choices.

Answer choice A might sound potentially correct if you’ve confused Sentence 11 with Sentence 12, as it’s a great description of the latter. Be sure that you’re considering the indicated sentence! While Sentence 11 may seem like it’s summarizing the events of the time between just after the stoats were introduced and the present day, it doesn’t specifically discuss the importance of considering all potential effects of making environmental changes. Answer choice A isn’t the correct answer.



Neither is answer choice B. While Sentence 11 does tell us that New Zealand is trying to deal with its stoat problem, it doesn't provide any "specific details" about how the country is going about this. Answer choice C doesn't make much sense either—while evolution is mentioned in reference to how unprepared kiwis were to deal with stoat predation, it's mentioned earlier in the passage and doesn't pop up in the indicated sentence at all. The correct answer is the remaining one, answer choice D: "a statement that shows how historical environmental changes have present-day effects." This answer accurately describes how Sentence 11 brings the focus of the passage from historical events to the present.

Identifying the Effects of Rephrasing Content

Questions that ask about rephrasing content may only ask about changing a single word or short phrase. These questions ask you to focus on a very specific part of the passage with an eye for how even subtle changes in word choice and tone can affect a reader's experience.

Practice Passage: New Zealand's Rabbit and Stoat Problems

[1] New Zealand has historically had to deal with a number of threats to its native ecosystem, many of which were caused by imprudent but purposeful human decisions to attempt to adjust the environment's delicate balances. [2] For instance, when European colonists began living in New Zealand, they brought rabbits with them. [3] What the rabbit importers did not consider is that New Zealand, unlike Europe, does not have any endemic predatory mammals. [4] Put another way, no animals that live there eat rabbits! [5] The rabbit population exploded, creating an imbalance in the native ecosystem and threatening native plants as well as farmers' crops. [6] At this point, to combat the rabbit population, New Zealand began to import stoats—small, predatory, weasel-like mammals that eat rabbits—despite the protestations of ornithologists (bird scientists). [7] New Zealand's bird populations suffered greatly from this sudden introduction of a predator for which they were not evolutionarily prepared. [8] The beloved national bird of New Zealand, the kiwi, provides the most overt example of just such an unprepared species. [9] Having adapted to an environment with no native mammalian carnivores, it roams around the ground and has very poor flying skills. [10] The stoat population wreaked havoc on the kiwi population. [11] Conservation efforts aimed at reducing the stoat population and protecting kiwis are still ongoing. [12] New Zealand's ecological history serves as a poignant lesson of how dangerous it is for people to attempt to change an ecosystem without fully understanding the repercussions that may arise from such changes.



Questions about the Effect of Rephrasing a Single Word

If you spot a question that asks about how rephrasing one word in the passage will affect its meaning, you might think it's going to be particularly easy—after all, how much meaning could be conveyed by a single word amongst so many others in the passage? Alternatively, you might get a bit suspicious and anticipate that your ability to pick up on subtle meanings at the word-by-word level is about to be tested. Don't think these questions are necessarily going to be easier than others, but don't let the zoomed-in focus of the question throw you off, either. These questions function just like those that test phrases and clauses and test the same skills.

Sample Question

If the author were to change the underlined word “exploded” in Sentence 5 to “increased,” it would _____.

- A. deemphasize the degree to which the rabbit population grew
- B. suggest that New Zealand's rabbit problem was worse than anticipated
- C. convey the colonists' shock at the effect of importing rabbits
- D. suggest that New Zealand had bigger environmental problems to deal with than rabbits at this point in history

Consider the indicated sentence:

The rabbit population exploded, creating an imbalance in the native ecosystem and threatening native plants as well as farmers' crops.

The violent and emphatic connotations of “exploded” make it seem like a pretty strong word—especially in comparison to “increased.” “Increased” is something you might expect to read in an official scientific report, “exploded” perhaps not so much. If the author changes the word “exploded” to “increased,” is it going up or down in intensity? Down. With that in mind, let's look over the answer choices describing this change's effect.

Does downgrading the intensity of this word “suggest that New Zealand's rabbit problem was worse than anticipated?” Not in the slightest. Out goes answer choice B. Does it have anything to do with conveying the colonists' shock at the effect of importing rabbits? Changing this word only changes readers' understanding of how the rabbit population changed in size; it doesn't relate to the colonists' reaction to this change. So, answer choice C isn't correct either. What about answer choice D? It might look potentially correct because the word is decreasing in emphasis/strength, which could potentially work to indirectly emphasize another problem by reducing the emphasis on the rabbit problem. No other problem is mentioned at this point in the passage, though, so that doesn't make much sense. In considering answer choice D, we've stumbled across the correct answer: that changing “exploded” to “increased” would de-emphasize the degree to which the rabbit population grew. That's our correct answer!



Consider another example of an effect-of-editing question that deals with a single word:

Sample Question

If the author were to change the underlined word in Sentence 7 to “abrupt,” it would _____.

- A.** emphasize how quickly the decision to introduce stoats was made
- B.** suggest that the colonists deliberated about introducing stoats for a long time
- C.** draw readers’ attention to how quickly the introduction of a new predator can affect a prey population
- D.** have little effect on the sentence overall

The sentence we need to consider is as follows:

New Zealand’s bird populations suffered greatly from this sudden introduction of a predator for which they were not evolutionarily prepared.

Hmm. Changing “sudden” to “abrupt” doesn’t appear to change much about the sentence. After all, the two words are synonyms that don’t differ greatly in terms of severity, emphasis, or tone. Let’s consider the answer choices.

“Sudden” in this sentence is talking about how quickly the stoats were actually introduced to New Zealand—almost from the birds’ perspective, as the birds are the focus of the sentence. The sentence actually uses the phrasing “for which they were not evolutionarily prepared,” so “sudden” is meant to complement that. The term isn’t talking about the colonists’ decision to introduce the stoats at all; that realization can help us knock out both answer choices A and B, leaving us with C, the idea that the change in terms “draws readers’ attention to how quickly the introduction of a new predator can affect a prey population,” or D, that the shift in word choice doesn’t change the sentence much at all. The shift certainly isn’t emphasizing how quickly the stoats affected the bird population. One might argue that the terms on their own each do that to a degree, but we’re not talking about what the terms each do on their own—we’re talking about the shift between terms. Changing between terms doesn’t affect that sort of emphasis, especially as both terms are about equal in terms of emphasis. The correct answer is D; changing “sudden” to “abrupt” doesn’t affect the sentence’s meaning much at all.



Questions about the Effect of Rephrasing a Phrase or Clause

Just like single-word effect identification questions, those based on phrases or clauses expect you to pay attention to a detailed level of meaning in a passage when considering changes the author might make.

Sample Question

If the author were to change the underlined phrase “despite the protestations of” in Sentence 6 to “against the recommendation of,” it would _____.

- A. reduce the degree to which the ornithologists seem to oppose the decision
- B. emphasize the colonists’ stubbornness in proceeding with a decision the ornithologists thought to be unwise
- C. suggest that the ornithologists’ opinions were unreliable
- D. convey a sense of competition between the ornithologists and the colonists

Consider Sentence 6:

At this point, to combat the rabbit population, New Zealand began to import stoats—small, predatory, weasel-like mammals that eat rabbits—despite the protestations of ornithologists (bird scientists).

Changing “despite the protestations of” to “against the recommendation of” does change the meaning of the sentence somewhat. Going against someone’s recommendation when making a decision means they advise against it, but aren’t actively complaining. Going ahead and doing something “despite the protestations of” someone *does* indicate active complaint and opposition against that action. This shift in meaning also inherently suggests a shift in emphasis; there is a shift in emphasis between not actively complaining and actively complaining about something.

On to the answer choices: changing this phrasing has nothing to do with the reliability of the ornithologists, so C isn’t correct. Neither is D; nothing about the change in phrasing suggests a sense of competition between the ornithologists and the colonists. This leaves us with A and B. The first words of these answer choices are telling: A begins with “reduce” and B begins with “emphasize.” Since changing the wording appears to be reducing the intensity of the phrase, A is looking like a good answer choice, but let’s check B to make sure we’re on the right track. Does rephrasing from “despite the protestations of” to “against the recommendation of” “emphasize the colonists’ stubbornness in proceeding with a decision the ornithologists thought to be unwise?” Not at all. The intensity of the phrase is being decreased by the change, so if anything, it looks like it is deemphasizing how stubborn the colonists look. This means that the correct answer is indeed A. The change in phrasing reduces the degree to which the ornithologists seem to oppose the colonists’ decision to import stoats.



Sample Question

If the author were to change the underlined phrase “wreaked havoc on” in Sentence 10 to “impacted,” it would:

- A.** make the kiwis seem weaker and pitiable
- B.** make the author’s tone more formal, but deemphasize the effect of the stoats
- C.** switch the sentence’s focus from the kiwis to the stoats
- D.** evoke phrasing the passage used in the preceding line

Let’s consider Sentence 10 as it appears in the passage. It’s relatively short and to the point:

The stoat population wreaked havoc on the kiwi population.

“Wreaked havoc on” is a relatively informal but very intense and emphatic phrase that conveys one thing having a huge negative effect on another. “Impacted,” by contrast, doesn’t convey as great an effect as “wreaked havoc on” does, and it doesn’t necessarily indicate a positive or negative effect. So, we are decreasing the intensity of the phrase by changing it to “impacted,” and potentially losing the clear negative gist conveyed by “wreaked havoc on.” With that, let’s look over the answer choices.

We can consider D immediately by comparing this line to the previous line, which is talking about the kiwi: “Having adapted to an environment with no native mammalian carnivores, it roams around the ground and has very poor flying skills.” Nothing about “wreaked havoc on” or “impacted” connects to any particular phrasing in the previous line, so D is incorrect. Changing the phrase also doesn’t change the focus from the kiwis to the stoats, so C isn’t correct either. This leaves us with A and B. Which of the two phrasing options makes the kiwis seem “weaker and pitiable?” The initial one, “wreaked havoc on,” as it conveys the greater effect. So, the change isn’t having that effect; it’s having an opposite one. The correct answer is B. Changing “wreaked havoc on” to “impacted” would make the author’s tone more formal, but it would also deemphasize the effect that the stoats had on the kiwi population.



Analyzing Passage Effectiveness

At the broadest level, ACT English questions can ask you to consider an entire passage. Luckily, the test won't ask you to comb through the entire text looking for a grammatical error. Instead, you'll be asked to take a step back from the detail-oriented nature of many of the other questions and consider whether the writer did what he or she set out to do in writing the passage.

These questions distinct from [ACT Reading questions about passage purpose](#), which ask you to identify a writer's likely purpose instead of being told the writer's purpose and ask if they achieved it; however, learning to consider passage purpose for ACT Reading questions can help you feel comfortable when dealing with the same concepts on the English section in a different manner.

Practice Passage: "The Election of 1800" by Alec Slatky (2013)

The presidential election of 1800 was an unusual and unique contest in American history. The opponents were John Adams and Thomas Jefferson former friends who became rivals. Jefferson had actually been Vice President under Adams, but had not supported the latter's policies. Jefferson leapt at the chance to unseat his former boss. Adams, too, would have enjoyed spending time on his farm more than dealing with political acrimony. Accordingly, he was too prideful to willingly relinquish his office. So, the two men who were most qualified for the position but least excited to have such a stressful job became the presidential candidates.

The campaign was a bitter partisan one and included numerous personal attacks. Critics called Jefferson a Democratic-Republican radical, he would lead the country down the bloody path of the French Revolution. Jefferson's allies fired back, painting Adams as an anti-liberty autocrat. Adams was from Massachusetts. Adams even faced opposition from some extremists among his fellow Federalists and thus lacked the party unity required to win the election.

In the end, Jefferson emerged victorious, winning 73 of the 138 electoral votes. Thomas Jefferson was inaugurated as President, and Aaron Burr was inaugurated as Vice President. The election was a milestone in American history: it was the first that shifted control of the White House to a different party, the first with a modern mudslinging campaign, and the first that truly tested whether the new nation would remain united despite all its divisions.

Quickly jotting down a few notes about what each paragraph does in the context of the entire passage can be particularly helpful when considering questions as broad as these. You want to avoid feeling overwhelmed when suddenly switching from considering comma splices and tense errors to looking across multiple paragraphs, and quick notes can form an outline on which you can build your analysis of the entire piece. For instance, when reading the above passage, after you finish the first paragraph, you might jot down something like "Jefferson, Adams, leading up to election." For the second paragraph, you might write down "Election details." For the third, you might say, "Jefferson wins, effects of this." These may seem too obvious to merit writing down, but glancing over that mini-outline can help with the perspective shift that the sample question shown next calls for.



Sample Question

Suppose the author intended to write an essay about the platform of the Federalist Party. Would this essay fulfill the author's goal?

- A.** Yes, because the presidential election of 1800 promoted Federalist ideals.
- B.** Yes, because John Adams was the candidate who explained the Federalist platform.
- C.** No, because the Federalist Party was not part of the presidential election of 1800.
- D.** No, because the essay focuses on a contest involving the Federalist Party rather than its specific policy proposals.

“No, because the essay focuses on a contest involving the Federalist Party rather than its specific policy proposals” is correct because the essay is about the overall election, not the Federalist Party or their proposals. The two “yes” choices are incorrect because the essay never explains what the platform is. The other “no” choice is incorrect because the Federalist Party was mentioned as one of the competitors in the Election of 1800: “Adams even faced opposition from some extremists among his fellow Federalists.”



The ACT Reading Section

The Reading section of the ACT can put a damper on high hopes for a fantastic composite score for students to whom reading does not come easily. While nearly all high school courses include some form of reading in their curricula, many test students' ability to comprehend and analyze passages of works with which they are already familiar due to coursework, not material they have never seen before, like the ACT Reading section does; furthermore, students' reading comprehension abilities are often tested in English courses focused on a relatively narrow range of content: fictional prose, drama, and poetry. The ACT Reading does test students' abilities to understand prose fiction, but this accounts for only a quarter of the questions on the section. Since the ACT Reading section draws its passages from a much later range of topics, one way to begin to prepare for the exam is to simply read widely across many subjects.

Along with the imposing range of topic material, the ACT Reading section is also challenging because of how necessary good timing is for success. While few people use a stopwatch when they are reading a book for fun, the ACT Reading section asks test-takers to read four passages of approximately eight hundred words each and answer forty questions in total about them. Even answering the questions aside, one must be able to read somewhat quickly in order to make it through all of the presented material. Beginning to review early can leave you with ample time to calibrate your reading speed and prevent you from having to rush or leave questions unanswered on test day.

Passage topics and section timing aside, the questions asked on the ACT Reading section are not simple. Many ask you to make complex inferences and pick up on subtle shades of meaning conveyed in tone and authorial style. Others may ask you to identify information presented that supports the author's assertions, or to analyze the reasoning behind the passage's organization. Practicing with test-specific resources can help you to learn which types of questions to expect and to practice any that give you particular difficulty.

At a Glance

Tests: Students' abilities to comprehend what they read, find details, and think critically about the text (e.g. make inferences and predictions)

Timing: $\frac{35 \text{ Minutes}}{40 \text{ Questions}} = \frac{52.5 \text{ Seconds}}{\text{Question}}$

Section Breakdown:

- About ten questions per single passage and up to 10 questions about a pair of passages
- Three single passages and one set of paired passages per ACT Reading section
- Passages are typically about 750-900 words long—that's about 4-5 pages in a typical paperback novel.



Passage Genres

Table of Contents

Prose Fiction: excerpts from novels and fictional stories

Most challenging aspect: orienting yourself as to what's going on in a passage from the middle of a novel you haven't read. Context clues are key!

Humanities: excerpts from works about the arts, literature, language, philosophy, etc.

Most challenging aspect: keeping pace with challenging rhetorical devices and dense language. Rephrase things in your own words and jot down summaries of paragraphs to help keep up!

Natural Sciences: excerpts from works about scientific subjects like biology, chemistry, and physics.

Most challenging aspect: the jargon! Scientific terminology can throw you for a loop if you're not ready to read in context and not worry about what a concept means or entails outside of the passage. Also, not letting outside knowledge inform your answer choices!

Social Sciences: excerpts from works about subjects like history, linguistics, psychology, and sociology

Most challenging aspect: a combination of jargon and subject matter. Many students won't have had much experience with social science subjects, and they have their own terminology to contend with.

Sample Question

Prose Fiction: Adapted from “The McWilliamses and the Burglar Alarm” in *The Mysterious Stranger and Other Stories* by Mark Twain (1898; 1916)

The conversation drifted along from weather to crops, from crops to literature, from literature to scandal, from scandal to religion; then took a random jump, and landed on the subject of burglar alarms. And now for the first time Mr. McWilliams showed feeling. Whenever I perceive this sign on this man's dial, I comprehend it, and lapse into silence, and give him opportunity to unload his heart. Said he, with but ill-controlled emotion:

“I do not go one single cent on burglar alarms, Mr. Twain—not a single cent—and I will tell you why . . .”

Which of the following does the first paragraph allow us to infer?

- A. The conversation that Mr. McWilliams had with Mr. Twain focused on only a couple of topics.
- B. Mr. McWilliams feels neutral about burglar alarms.
- C. Mr. McWilliams didn't have particularly strong opinions about the topics of conversation that preceded that of burglar alarms.
- D. The discussion of topics in the conversation described led logically to that of burglar alarms.

[See Answer and Explanation](#)

How to Use This Book

Each section in this chapter investigates one ACT Reading topic in-depth and walks you step-by-step through relevant practice questions. Each section concludes with a practice quiz so that you can test your skills before trying some of the full ACT tests at the end of the book.

Chapter Outline

- I. [Review: Close-Reading](#)
- II. [What is Said](#)
 - [Main Idea](#)
 - [Analyzing Argument](#)
 - [Identifying Details](#)
- III. [How It's Said](#)
 - [Determining Context-Dependent Meaning](#)
 - [Analyzing Tone](#)
 - [Point of View](#)
 - [Sequence](#)
- IV. [Why It's Said](#)
 - [Purpose of Dialogue](#)
 - [Purpose of Lines and Passage](#)
- V. [Engaging with What's Said](#)
 - [Summarizing and Paraphrasing](#)
 - [Comparing and Contrasting](#)
 - [Identifying Cause and Effect](#)
 - [Generalizations, Inferences, and Predictions](#)
- VI. [Comparing What's Said: Paired Passages](#)
 - [Comparing and Contrasting Main Ideas](#)
 - [Comparing and Contrasting Arguments](#)
 - [Comparing and Contrasting Language and Rhetoric](#)
 - [Multi-Passage Critical Thinking: Inferences, Conclusions, and Predictions](#)
 - [Drawing Inferences and Conclusions from Paired Passages](#)
 - [Making Predictions Based on Paired Passages](#)



Review: Close Reading

Close reading is the most useful tool in any attempt to analyze a piece of writing. Basically, when you are asked to analyze a piece of writing, you are asked to formulate a thesis (your theory or argument about what is happening in the passage, and why it's important) and to argue in support of that thesis. Close reading is how you gather evidence to support your argument. Close reading focuses on specific details within a text, and uses those details to support a position about the text's overall meaning. Like proofs in Math, or experiments in Science, close reading is the technical skill of English as an academic discipline. The small details you pick up on in your close reading will allow you to construct a sturdy argument.

In close reading a passage there are many things to look for, but, since it's your argument, the most important things to look for, and to look at in more detail, are just what sticks out to you. If you're reading a passage and you notice a word choice, or a repetition, or a sentence that strikes you as odd or interesting, simply underline it or make a small note, then come back and read it more carefully. By noting your impressions as you go, you'll be giving yourself a basis upon which to build an argument based on real, textual evidence, not just your suppositions about the text.

Close reading helps you gather information to explain why a text created a certain effect, if you liked it you can close read the text to find the specific, small details that added up to your enjoyment, and if you didn't enjoy the text, you can close read the text to explain exactly what about it you thought wasn't enjoyable or good.

Analyzing and noticing diction is perhaps the most basic and rewarding area to mine for close reading content. Diction refers to the words a writer chooses to use. That might sound obvious, but there are ways to look deeper into the language of a text to better understand and make claims about the text overall.

For instance, if I say "I hate dogs," that's a lot different than saying that "I am not fond of dogs." The meaning is roughly the same, I don't like dogs (that's not really true, but let's press on with this example), but the intensity of my word choice changes the meaning of the statement. If you're reading a passage and you notice a number of words that jump out at you as either expressing strong, or notably mild emotions, this is a reasonable observation of which to take note. A writer might repeat a specific word several times through a passage, and observing and thinking about this repetition might give you some insight into what they are trying to convey with their writing.

Diction can also help determine the tone of a work. If an author chooses to say: "I took a mid-afternoon perambulation," it creates a radically different effect than saying, "I took a walk at 2:30." Again, the literal meaning is very close, but the diction of one is elevated, and that's worth observing.

You can also take note of how the author chooses to arrange his or her words, in addition to noting the words themselves. A writer's syntax (or sentence structure) can tell you a lot about what they think is important about what they're saying. For instance, if I say: "The briefcase was thrown off the roof by that man." I am using the passive voice, making the briefcase, which received the action of the man, the grammatical subject, and thus making it seem as though the briefcase is the most important thing in the situation. If I frame this sentence in the active voice: "That man threw the briefcase off the roof." The focus of the sentence is now on the agent, the man. If you notice a writer using an unconventional sentence structure, maybe the passive voice, maybe connecting two independent clauses with a semicolon, take a second and think about why the author might have done that, it might give you some clues as to what they think is important about what they're saying or how the different things they're talking about relate to each other. Every sentence communicates not just with the words it uses, but with its fundamental structure.



In analyzing texts (and writing your own) it is vital to keep in mind that every time a writer puts a word, punctuation mark, or even a letter, on the page that writer is making a choice. Close reading allows you to examine these choices in greater detail, and to see how these choices add up (or fail to add up) into a coherent whole.

The main point of close reading is that it underscores the key thing to know when analyzing any text: you need to support every claim you make with evidence. Just as a prosecutor can't walk into a court of law and tell the jury that they should send someone to jail because he "just feels like" the defendant is guilty, a reader cannot make claims about a text without justifying those claims using evidence gathered from the text itself. To go further with this metaphor (Were you close reading this paragraph you might note my use of an extended metaphor, or that I used this annoying parenthetical!), in the case of close reading you get to be both the prosecutor and the detective. First you study the scene of the crime (the text) and form a theory about it, then you gather evidence to support that theory (or maybe you form the theory by gathering evidence), then you organize and argue for that case by citing the evidence you gathered. Fortunately, nobody goes to jail after you solve and prosecute your textual case.

What is really useful about close reading is that it can help you at every step of the analytical process. If you're stuck, and you really don't have an argument or a theory about what is happening in the text, the best way to come up with something is to close read the text, paying careful attention to the text's language, syntax, and tone and seeing if you notice a pattern. Any patterns or quirks you notice can lead you to a theory about what is happening.

On the other hand, if you read a text and you form an immediate, strong opinion about it, the best way to test, and prove, that theory is to go right back to the text, and read it carefully, analyzing the details of the writing to see if, or how, they add up to what you think is happening overall.

In any case, close reading will always be the most useful tool at your disposal to form and argue a thesis, especially in a timed test situation.



What is Said

Certain ACT Reading questions are relatively to the point in the way that they ask about the passage at hand. Some of them might ask you to identify the main idea of a paragraph or of the entire passage, or tell one side from another in a major argument being laid out in the text. Other such questions might ask you to pick out details from the passage or figure out which information supports a given position. Learning to figure out what is said in an ACT Reading passage without skimming over vital information or getting bogged down in unimportant minutiae is a skill vital to good preparation and success on the section.

Section Outline

[Main Idea](#)

[Finding the Main Idea of a Selection from a Passage](#)

[Finding the Main Idea of an Entire Passage](#)

[Practice Passage: Main Idea](#)

[Analyzing Argument](#)

[Questions About the Author's Argument](#)

[Questions About Other People's Arguments](#)

[Questions About the Structure of an Argument](#)

[Practice Passage: Analyzing Argument](#)

[Identifying Details](#)

[Identifying Details in a Specific Paragraph](#)

[Identifying Details in a Full Passage](#)

[Practice Passage: Identifying Details](#)



Finding the main idea of a passage is an integral component in developing reading comprehension skills necessary to succeed at the ACT. The main idea of the passage can differ in each of its constituent paragraphs. Being able to recognize the nuances between these ideas and how they build upon one another to create a coherent narrative is crucial for succeeding on the ACT Reading section.

Practice Passage

Social Science: Adapted from *Logic: Inductive and Deductive* by William Minto (1915)

We cannot inquire far into the meaning of proverbs or traditional sayings without discovering that the common understanding of general and abstract names is loose and uncertain. Common speech is a quicksand.

Consider how we acquire our vocabulary, how we pick up the words that we use from our neighbors and from books, and why this is so soon becomes apparent. Theoretically, we know the full meaning of a name when we know all the attributes that it connotes, and we are not justified in extending it except to objects that possess all the attributes. This is the logical ideal, but between the ought to be of Logic and the is of practical life, there is a vast difference. How seldom do we conceive words in their full meaning! And who is to instruct us in the full meaning? It is not as in the exact sciences, where we start with knowledge of the full meaning. In Geometry, for example, we learn the definitions of the words used, “point,” “line,” “parallel,” etc., before we proceed to use them. But in common speech, we hear the words applied to individual objects; we utter them in the same connection; we extend them to other objects that strike us as like without knowing the precise points of likeness that the convention of common speech includes. The more exact meaning we learn by gradual induction from individual cases. The individual’s extension of the name proceeds upon what in the objects has most impressed him when he caught the word: this may differ in different individuals; the usage of neighbors corrects individual eccentricities. The child in arms shouts “Da” at the passing stranger who reminds him of his father; for him at first it is a general name applicable to every man; by degrees he learns that for him it is a singular name.

It is obvious that to avoid error and confusion, the meaning or connotation of names, the concepts, should somehow be fixed; names cannot otherwise have an identical reference in human intercourse. We may call this ideal fixed concept the Logical Concept. But in actual speech we have also the Personal Concept, which varies more or less with the individual user, and the Popular or Vernacular Concept, which, though roughly fixed, varies from social sect to social sect and from generation to generation.

When we come to words of which the logical concept is a complex relation, an obscure or intangible attribute, the defects of the popular conception and its tendencies to change and confusion are of the greatest practical importance. Take such words as “monarchy,” “civil freedom,” “landlord,” “culture.” Not merely should we find it difficult to give an analytic definition of such words; we might be unable to do so, and yet flatter ourselves that we had a clear understanding of their meaning.

It was with reference to this state of things that Hegel formulated his paradox that the true abstract thinker is the plain man who laughs at philosophy as what he calls abstract and impractical. He holds decided opinions for or against this or the other abstraction, “freedom,” “tyranny,” “revolution,” “reform,” “socialism,” but what these words mean and within what limits the things signified are desirable or undesirable, he is in too great a hurry to pause and consider.

The disadvantages of this kind of “abstract” thinking are obvious. The accumulated wisdom of mankind is stored in language. Until we have cleared our conceptions, and penetrated to the full meaning of words, that wisdom is a sealed book to us. Wise maxims are interpreted by us hastily in accordance with our own narrow conceptions. All the vocabulary of a language may be more or less familiar to us, and yet we may not have learnt it as an instrument of thought.



Main Idea

When tackling reading passages on the ACT Reading section, the best and most natural place to start is to interrogate and figure out the main idea of the passage. The term “main idea” refers, basically to what the passage is about. In order to answer any questions about a passage with confidence, it is a massive help to have a general understanding of the main point or intent of the passage you are reading.

While it's important to know the main idea of a passage for your own understanding, some questions will specifically query the main point of excerpts of entire passages. The nice thing about these questions is that they are the most natural questions to answer, in that these questions most closely mirror the questions anyone asks themselves while reading a passage: What is happening in this passage, and what is the point of this passage?

While main idea questions are quite general in nature, you should always be basing your notion of the main idea of a passage on specific evidence drawn from the passage. The answer to a main idea question on the ACT will always be based directly on the text, you are not being asked to make an [inference](#) in order to answer a question interrogating the main point of a passage or excerpt.

Finding the Main Idea of a Selection from a Passage

The first kind of Main Idea question you will be asked to answer will interrogate the main idea of a small part (usually a paragraph) of a passage. Note that if the question asks you the meaning or main idea of a specific part of a passage, in specifying that part the question is asking you **ONLY** about that part. One paragraph may be conveying a different idea than the one conveyed by the passage as a whole. Be careful to read the question carefully, and answer for only the part of the passage selected if a particular part of the passage is specified.

Sample Question

Which of the following options best captures the main idea of the second paragraph?

- A.** We learn the meaning of most words organically and as a result the exact definition and application of those words will differ from person to person.
- B.** People learn the meaning of words best when those words are specifically and rigidly defined.
- C.** It is impossible to define the vast majority of words because people have their own personal ideas about how each thing could best be described.
- D.** Babies learn words without being intentionally taught them and seem to develop incorrect assumptions about the meanings of certain words.

The second paragraph begins with the author stating, “Consider how we acquire our vocabulary, how we pick up the words that we use from our neighbors and from books.” We thus know that the second paragraph is going to be a consideration of how we acquire words and definitions for things. The author goes on to state, “We hear the words applied to individual objects; we utter them in the same connection; we extend them to other objects that strike us as like without knowing the precise points of likeness that the convention of common speech includes.” This tells us that the author believes we learn words “organically,” or naturally. Finally, the author says, “The individual’s extension of the name proceeds upon what in the objects has most impressed him when he caught the word: this may differ in



different individuals.” This tells us that the author believes the result of our organic learning is that the exact definition and application of words differs from person to person. The other answer choices either summarize a small part of the paragraph, or else they draw incorrect conclusions from the author’s writing. You will not be able to find any evidence to support these summaries in the paragraph specified by the question.

Sample Question

Which of the following options best summarizes the main point of the sixth paragraph?

- A.** In order to fix the problems associated with “abstract” thinking we must collectively focus on deep thinking and a scientific classification of words and names.
- B.** Hegel’s “abstract” thinking is not applicable to the author’s argument because it ignores the preconceptions that blind each individual to the true meaning of words and names.
- C.** Our individual preconceptions blind us to the true meaning of words and prevent us from accurately penetrating the combined knowledge of humanity.
- D.** None of these answers accurately summarizes the point of the sixth paragraph.

The main point of the sixth paragraph is best captured in the following few sentences: “The accumulated wisdom of mankind is stored in language. Until we have cleared our conceptions, and penetrated to the full meaning of words, that wisdom is a sealed book to us. Wise maxims are interpreted by us hastily in accordance with our own narrow conceptions.” Here, the author discusses how people’s individual preconceptions blind them to the true meaning of words, and draws his conclusions on how this individual approach to definition and meaning prevents people from accessing to combined wisdom of humanity. The other answer choices either summarize the wrong paragraph, or else do not properly penetrate to the heart of the author’s argument in this paragraph. Note particularly that while the paragraph in question is concerned with “abstract thinking,” it makes no mention “scientific classification,” so answer A is clearly not correct. Answer B is similarly incorrect in that Hegel is mentioned in the fifth, not the sixth paragraph.

Finding the Main Idea of an Entire Passage

Unless the question specifies a particular selection from the passage, Main Idea questions will be interrogating the general, most important notion or idea from the entire passage. Solving these questions requires pretty much the same process as do questions asking you for the main idea of a particular section. Even though these questions are more general in nature, it is still vital to root your interpretation of the text’s main idea in evidence drawn directly from the passage.

It is also important to note that just as questions specifying a particular section of a passage will be concerned only with that section, questions interrogating the entire passage are focused on the passage as a whole. Some answers will try to trick you by summarizing the main point of a particular paragraph, not the question as a whole.



Sample Question

The main argument made in this passage is that _____.

- A.** a sophisticated understanding of the language people use will facilitate understanding of culture and thought
- B.** it is impossible to learn the roots of words and expressions
- C.** geometry is a more defined science than is abstract philosophy
- D.** None of these answers accurately sums up the main point of the passage.

The key here is taking a holistic, appropriately open view of the entire passage. Answer option B makes a radical oversimplification of the passage's point. Options like this one appear often in main idea questions, and it's important to remember that passages chosen for the ACT Reading are chosen because they are sufficiently complex so as to be mined for a number questions, they will rarely boil down to main ideas that are blunt, simple, or aggressively overarching in their claims. While geometry is mentioned in the passage, a direct comparison of geometry to another discipline is clearly not the main idea of this passage. Now, having eliminated B and C, the only thing to decide is if answer option A is accurate, which in this case it is, both the first and last paragraph (usually good places to look for hints at the main idea) make this goal clear.



Practice Passage: Main Idea

Social Science: Adapted from “Goffman’s Theory of Institutions” by Joseph Ritchie (2015)

Sociological inquiry often investigates members of society considered to be on its outer edges. These individuals often live in precarious and vulnerable situations. In 1961, Erving Goffman published the book *Asylums: Essays on the Social Situation of Mental Patients and Other Inmates*. This book outlined the theory of a total institution as seen in prisons and asylums for the clinically insane. Goffman’s interests and theory helped to reveal the inner mechanics of asylums and the process of institutionalization that takes place within a total institution.

Sociologists—such as Goffman—revolutionized the field by introducing ethnographic methodologies that sought to understand social phenomena through direct observations and interactions. Prior to this, sociologists conducted what has been satirically referred to as armchair sociology. This methodology characterized early sociological inquiries. Pioneers of the field engaged with society in a philosophical manner that left them disengaged from their targets of investigation. Sociologists of the ethnographic school, however, actively engage in activities of participant observation with the subcultures under investigation. Instead of theorizing from a distance, Goffman carried out fieldwork within an actual asylum. Immersion into the institution and social embeddedness with its actors enabled Goffman to accurately document and theorize the characteristics of and methods of socialization used by total institutions.

According to Goffman’s observations and subsequent theories, a total institution seeks to erode the relationships of an individual with the outside world and consume their personal identities and daily activities. A total institution can be described as a physical space of work or residence where like-situated individuals are cut off from the greater society and lead a confined life regulated by a formal administration. The end goal of a total institution is to break down and deconstruct

the barriers that separate the spheres of sleep, play, and work in an individual’s life by conducting all of these aspects of life in the same location under the same authority. In these institutions, Goffman stated that there is an intentional divide between a large, managed group and a supervisor, which often results in feelings of submissiveness and reluctance to leave the institutionalized setting on the part of the “inmates.” This suggests that these restrictive environments lead to the institutionalization of an individual into the group and away from his or her previous, independent life.

In these structures, an individual’s admission procedures shape and engineer the new member in what may be described as a process of programming. This programming of an individual is characterized by a “leaving off” of one’s identity and a “taking on” of one supplied by the establishment. Members of these establishments are alienated from their previous lives and encircled by the ideals and principles of the new institution. Inmates in asylums were subjected to mortifying experiences of initiation. Goffman observed that they were stripped of their street clothes, shaved bald, deloused, and given an identification number. As a result, inmates became cogs of the total institution: human representations of the ideals and characteristics of the asylum. Goffman noted that prolonged exposure to similar institutions results in a phenomenon known as “disculturation,” an un-training that renders an individual temporarily incapable of managing certain features of daily life outside the structures of the institutions.

Sociologists often study groups forgotten or ignored by society. In this manner, sociological investigations seek to understand the fringes of the social world. As a result, sociologists can critically observe the society and advocate for change. Goffman’s work illuminated issues associated with vulnerable populations at asylums and other institutions. Ethnographic field studies have continued this tradition and in doing so have theorized the causes of many of society’s ills. Goffman’s work is just one example of sociology’s ability to delve into an understudied region of society, propose explanations of issues, and theorize possible avenues of reform.



1. Which of the following choices best describes the main idea of the entire passage?
 - A. The main point of the passage is to describe how sociology can reform some of society's ills
 - B. The main point of the passage is to describe Goffman's life and career in sociology
 - C. The main point of the passage is to outline Goffman's research, methods, and subsequent interventions to American mental institutions
 - D. The main point of the passage is to describe Goffman's motivation for social inquiry: his wife was institutionalized in an asylum
2. Which of the following choices best describes the main idea of the first paragraph?
 - A. An introduction of Goffman's fieldwork, methods, and book *Asylums*
 - B. The importance of studying the "forgotten victims of society"
 - C. The mechanics of a total institution
 - D. None of these
3. Which of the following choices best describes the main idea of the second paragraph?
 - A. The methods of the pioneers of sociology
 - B. The methods of ethnographers
 - C. Philosophical methods used in sociology
 - D. The revolution from armchair sociology to ethnographic participant observation
4. Which of the following choices best describes the main point of the third and fourth paragraphs?
 - A. Sociology and its effect on the reformation of mental institutions
 - B. The characteristics of a total institution
 - C. The process of "disculturation"
 - D. Theorizations on the characteristics of total institutions and the processes of socialization used by asylums
5. Which of the following choices best describes the main idea of the third paragraph?
 - A. An explanation of the methods of socialization used by a total institution to "program" inmates
 - B. An explanation of the characteristics of a total institution
 - C. An explanation of the barriers that separate the spheres of sleep, play, and work in an individual's life
 - D. An explanation of the primary end goal of a total institution
6. Which of the following choices best describes the main idea of the fourth paragraph?
 - A. An explanation of the characteristics of a total institution
 - B. An explanation of the methods of socialization used by an institution to "program" inmates
 - C. An explanation of Goffman's idea of "disculturation"
 - D. All of the other answers are correct.
7. Which of the following choices best describes the main idea of the last paragraph?
 - A. A discussion of Goffman's reformation of American insane asylums
 - B. A discussion of sociology's ability to reform some of society's ills
 - C. An explanation of ethnographic methods
 - D. An explanation of how sociologists develop theories



Practice Passage: Main Idea: Answer Key

1. Which of the following choices best describes the main idea of the entire passage?

- A. The main point of the passage is to describe how sociology can reform some of society's ills
- B. The main point of the passage is to describe Goffman's life and career in sociology
- C. The main point of the passage is to outline Goffman's research, methods, and subsequent interventions to American mental institutions
- D. The main point of the passage is to describe Goffman's motivation for social inquiry: his wife was institutionalized in an asylum

2. Which of the following choices best describes the main idea of the first paragraph?

- A. An introduction of Goffman's fieldwork, methods, and book *Asylums*
- B. The importance of studying the "forgotten victims of society"
- C. The mechanics of a total institution
- D. None of these

3. Which of the following choices best describes the main idea of the second paragraph?

- A. The methods of the pioneers of sociology
- B. The methods of ethnographers
- C. Philosophical methods used in sociology
- D. The revolution from armchair sociology to ethnographic participant observation

The main point of the entire passage was to describe Goffman's research, methodology, and subsequent interventions on American mental care. The passage includes information on sociology's ability to initiate reform for observed ills in society; however, this is not the primary focus of the passage. The passage does not contain information about Goffman's early life and career in sociology; therefore, this choice is incorrect. Last, it is true that Goffman was motivated to study asylums because of his wife's condition; however, this passage did not reveal any information on this topic and it is not the main point.

The main idea of the first paragraph is to introduce Goffman's, fieldwork, methods, and book. This paragraph contains ideas related to the importance of studying the victims of society and the mechanics of a total institution; however, it does not explain these in detail. They are components of and not directly related to the main idea which was to provide an abstract and frame for Goffman's work and the body of the passage.

The main idea of the second paragraph is to detail sociology's methodological progression from theorizations bred from armchair philosophy to conclusions informed by ethnographic participant observation. The methods of sociology's pioneers as well as ethnographers were described but only represent components of the main idea and not its entirety. Likewise, the choice pertaining to philosophical methods used in sociology was used to explain armchair sociology; therefore, it is incorrect and not representative of the whole main idea of the paragraph.



4. Which of the following choices best describes the main point of the third and fourth paragraphs?

- A. Sociology and its effect on the reformation of mental institutions
- B. The characteristics of a total institution
- C. The process of “disculturation”
- D. Theorizations on the characteristics of total institutions and the processes of socialization used by asylums

5. Which of the following choices best describes the main idea of the third paragraph?

- A. An explanation of the methods of socialization used by a total institution to “program” inmates
- B. An explanation of the characteristics of a total institution
- C. An explanation of the barriers that separate the spheres of sleep, play, and work in an individual’s life
- D. An explanation of the primary end goal of a total institution

6. Which of the following choices best describes the main idea of the fourth paragraph?

- A. An explanation of the characteristics of a total institution
- B. An explanation of the methods of socialization used by an institution to “program” inmates
- C. An explanation of Goffman’s idea of “disculturation”
- D. All of the other answers are correct.

7. Which of the following choices best describes the main idea of the last paragraph?

- A. A discussion of Goffman’s reformation of American insane asylums
- B. A discussion of sociology’s ability to reform some of society’s ills
- C. An explanation of ethnographic methods
- D. An explanation of how sociologists develop theories

The main point of the third and fourth paragraphs of this passage is to outline Goffman’s theorization on the characteristics of a total institution and the process of socialization of inmates into social spaces such as asylums. The characteristics of a total institution as well as the process of “disculturation” were part of the third and fourth paragraphs, respectively; however, neither is sufficient to explain the main point of the two paragraphs. Last, the effects of sociology upon reformation of societal ills was part of the first and last paragraphs and not associated with these two.

The main idea of this paragraph is an explanation of the primary characteristics of a total institution. The paragraph did explain the goals of the institution as well as the barriers that separate the spheres of life; however, they were merely components of the main idea and not the essence of the paragraph. The paragraph outlined the traditional characteristics used by Goffman to characterize spaces as total institutions. Last, the paragraph did not explain the methods of socialization used by institutions to program inmates.

The main idea of this paragraph is to provide an explanation of the methods of socialization used by an institution to “program” inmates. The paragraph is clearly centered on Goffman’s observed process of socialization from initiation into the asylum to the effects of prolonged exposure to institution life (i.e. “disculturation”). “Disculturation” is an important part of this story but not the focus of the paragraph. Likewise, the characteristics of the institution were described in the previous paragraph. They play a role in the methods of socialization but are not the focus of this section of the passage.

The main idea of the passage is associated with a discussion of sociology to reform some of society’s ills. The paragraph mentions Goffman’s contribution to reforming mental health asylums; however, this was not the focus of this section of the passage. The passage did not focus on methodology or theorization in any way; therefore, answers associated with these concepts are incorrect.



Analyzing Argument

Certain authors aim to convince readers of their opinions, other passages attempt to map out academic debates, and some passages do both, with the author describing the academic “lay of the land” surrounding a debate while also making a pitch for his or her own particular take on it. No matter the form of an argumentative ACT Reading passage or what is being argued about, you need to be able not only how different perspectives interact, but keep track of who agrees with whom.

Practice Passage

Natural Science: Adapted from “Recent Views as to Direct Action of Light on the Colors of Flowers and Fruits” in *Tropical Nature, and Other Essays* by Alfred Russel Wallace (1878)

The theory that the brilliant colors of flowers and fruits is due to the direct action of light has been supported by a recent writer by examples taken from the arctic instead of from the tropical flora. In the arctic regions, vegetation is excessively rapid during the short summer, and this is held to be due to the continuous action of light throughout the long summer days. “The further we advance towards the north, the more the leaves of plants increase in size as if to absorb a greater proportion of the solar rays. M. Grisebach says that during a journey in Norway he observed that the majority of deciduous trees had already, at the 60th degree of latitude, larger leaves than in Germany, while M. Ch. Martins has made a similar observation as regards the leguminous plants cultivated in Lapland.” The same writer goes on to say that all the seeds of cultivated plants acquire a deeper color the further north they are grown, white haricots becoming brown or black, and white wheat becoming brown, while the green color of all vegetation becomes more intense. The flowers also are similarly changed: those which are white or yellow in central Europe becoming red or orange in Norway. This is what occurs in the Alpine flora, and the cause is said to be the same in both—the greater intensity of the sunlight. In the one the light is more persistent, in the other more intense because it traverses a less thickness of atmosphere.

Admitting the facts as above stated to be in themselves correct, they do not by any means establish the theory founded on them; and it is curious that Grisebach, who has been quoted by this writer for the fact of the increased size of the foliage, gives a totally different explanation of the more vivid colors of Arctic flowers. He says, “We see flowers become larger and more richly colored in proportion as, by the increasing length of winter, insects become rarer, and their cooperation in the act of fecundation is exposed to more uncertain chances.” (*Vegetation du Globe*, col. i. p. 61—French translation.) This is the theory here adopted to explain the colors of Alpine plants, and we believe there are many facts that will show it to be the preferable one. The statement that the white and yellow flowers of temperate Europe become red or golden in the Arctic regions must we think be incorrect. By roughly tabulating the colors of the plants given by Sir Joseph Hooker as permanently Arctic, we find among fifty species with more or less conspicuous flowers, twenty-five white, twelve yellow, eight purple or blue, three lilac, and two red or pink; showing a very similar proportion of white and yellow flowers to what obtains further south.



Analyzing Argument

Argument questions can focus on the author, on other individuals, or ask you what roles certain parts of the passage in relation to the argument at hand. Let's consider sample questions from each of these categories.

Questions About the Author's Argument

When asked about the argument the author is making, you won't necessarily be asked directly about his or her argument. You might be asked to identify the particular points of another individual's argument with which they disagree, or about why they present certain information—is it to support their own ideas? Dismantle a counter-argument? Maybe make the other side look unreliable? Authorial argument questions can take many forms, so let's consider a few of them now.

Sample Question

In this passage, the author _____.

- A.** disagrees with Martins but agrees with Grisebach
- B.** disagrees with Hooker but agrees with Martins
- C.** disagrees with the “recent writer” quoted in the first paragraph, but agrees with Grisebach
- D.** disagrees with all of the writers and scientists mentioned in the passage

Answering this question requires you to read closely, as many theories are mentioned throughout the passage and keeping track of them can be quite challenging. In the first paragraph, the writer quotes a “recent writer,” who then quotes evidence in the form of observations by M. Grisebach and M. Ch. Martins. In the second paragraph, the writer says that he agrees with the evidence of the “recent writer” (in other words, Grisebach and Martins), but not with the theory the “recent writer” has come up with to explain that evidence. So, the author disagrees with the “recent writer,” but agrees with Grisebach, because the author goes on to quote Grisebach's own theory, with which the author agrees.

Sample Question

The author's critique of the theory presented in the first paragraph is that _____.

- A.** they are true, but do not support the theory established based on them
- B.** the facts supporting the theory are false, so the theory is also false
- C.** only some of the facts are true, casting doubt on the reliability of the theory as a whole
- D.** the facts were gathered in an unscientific manner and are thus not reliable, making the theory doubtful

At the start of the second paragraph, the author says, “Admitting the facts as above stated to be in themselves correct, they do not by any means establish the theory founded on them.” So, the correct answer is that “[the facts] are true, but do not support the theory established based on them.”



Sample Question

The author brings up Joseph Hooker's research in order to _____.

- A. disprove the theory of the "recent writer" quoted in the first paragraph
- B. support Martins' theory
- C. demonstrate that the colors of flowers change at varying latitudes
- D. suggest that a follow-up experiment be performed to check his results

The author brings up Joseph Hooker's research near the end of the second paragraph, stating, "By roughly tabulating the colors of the plants given by Sir Joseph Hooker as permanently Arctic, we find among fifty species with more or less conspicuous flowers, twenty-five white, twelve yellow, eight purple or blue, three lilac, and two red or pink; showing a very similar proportion of white and yellow flowers to what obtains further south." This immediately follows the sentence, "The statement that the white and yellow flowers of temperate Europe become red or golden in the Arctic regions must we think be incorrect." In this sentence, the author is doubting the veracity of the "recent writer" quoted in the first paragraph. The author then uses Hooker's evidence to disprove the theory of the "recent writer," because if the theory of the "recent writer" were correct, there would be very few white or yellow flowers in the Arctic and many red or golden ones, and Hooker's evidence shows that this is not the case, as most of the Arctic flowers he observed were white. So, the correct answer is that the author uses Joseph Hooker's evidence to "disprove the theory of the 'recent writer' quoted in the first paragraph." "Provide evidence in favor of the author's theory, which disagrees with all of the previously mentioned scientists' statements" cannot be the correct answer because the author is in agreement with M. Grisebach.

Questions About Other People's Arguments

You can also be asked about the opinions and arguments of other people mentioned in ACT Reading passages. Be careful to realize that you are hearing about these positions indirectly, from an author who may or may not be biased in favor of certain ideas and thus purposely presenting others' opinions in a favorable or unfavorable light, depending on how they interact with the author's own views.

Sample Question

The "recent writer" quoted in the first paragraph believes that _____.

- A. cultivated flowers have lighter colors in the south and darker colors in the north
- B. M. Ch. Martins' theory is incorrect
- C. light is less persistent in the north than in the south
- D. the green color of plants becomes more intense in the south

Answering this question requires you to read the first paragraph very closely and to go back and figure out what exactly the "recent writer" is asserting, whether or not the author of the passage agrees with those assertions. Let's consider each of the answer choices one by one:

B cannot be the correct answer, as the "recent writer" is quoted as mentioning M. Ch. Martins to bolster his own assertion about leaf size and latitude. C cannot be the correct answer because the author, in referring to the "recent writer," says that "the same writer goes on to say that all the seeds of cultivated plants acquire a deeper color the further north they are grown . . . This is what occurs in the Alpine flora,



and the cause is said to be the same in both—the greater intensity of the sunlight.” Answer choice D cannot be correct because the author, in discussing the “recent writer,” says, “The same writer goes on to say that all the seeds of cultivated plants acquire a deeper color the further north they are grown, white haricots becoming brown or black, and white wheat becoming brown, while the green color of all vegetation becomes more intense.” A is the correct answer! We can find evidence supporting it in that the author says (discussing the “recent writer”) “The same writer goes on to say that all the seeds of cultivated plants acquire a deeper color the further north they are grown, white haricots becoming brown or black, and white wheat becoming brown, while the green color of all vegetation becomes more intense.”

Questions About the Structure of an Argument

Arguments unfold with a certain structure, often beginning with a thesis statement or proposition and then following that statement with evidence to back it up and addressing any counterarguments critics may put forward. Reading argumentative texts can help you feel at ease when dealing with such pieces on the ACT Reading section and to quickly identify parts of an argument as they turn up. Such familiarity can come in very handy if certain questions focus not on what argument is being made in the passage, but how it is being made.

Sample Question

What role does the underlined sentence play in the passage as a whole?

- A. It provides evidence that the phenomenon being discussed exists, but does not support one theory more than the other.
- B. It provides evidence that supports the theory of the writer quoted in the first paragraph, but casts doubt on other theories.
- C. It offers an opinion as to the validity of the theory of the “recent writer” quoted in the first paragraph.
- D. It provides a counterargument opposing the theory of the “recent writer” quoted in the first paragraph.

The sentence underlined is “The further we advance towards the north, the more the leaves of plants increase in size as if to absorb a greater proportion of the solar rays.” To answer this question correctly, you have to pay a great deal of attention to the way in which it is presented in the passage. It is quoted as evidence that the “recent writer” uses to support his or her theory that leaf size differs in this way due to a change in the intensity of the sunlight. So, neither “It provides a counterargument opposing the theory of the ‘recent writer’ quoted in the first paragraph” nor “It demonstrates that the ‘recent writer’ quoted in the first paragraph is unreliable” can be the correct answer. Since the statement in question is just presenting evidence, and not an opinion, “It offers an opinion as to the validity of the theory of the ‘recent writer’ quoted in the first paragraph” cannot be the correct answer either.

This leaves us with two possible answer choices: “It provides evidence that supports the theory of the writer quoted in the first paragraph, but casts doubt on other theories,” and “It provides evidence that the phenomenon being discussed exists, but does not support one theory more than the other.” The author of the passage, in the second paragraph, says that “the facts as above stated” are “in themselves correct, they do not by any means establish the theory founded on them.” Given this, along with the fact that the underlined sentence’s evidence never casts doubt on any theories in the passage, the correct answer is “It provides evidence that the phenomenon being discussed exists, but does not support one theory more than the other.”



Practice Passage: Analyzing Argument

Prose Fiction: Adapted from “Errors in Our Food Economy” in Scientific American Supplement No. 1082 Vol. XLII (September 26th, 1896)

Scientific research, interpreting the observations of practical life, implies that several errors are common in the use of food.

First, many people purchase needlessly expensive kinds of food, doing this under the false impression that there is some peculiar virtue in the costlier materials, and that economy in our diet is somehow detrimental to our dignity or our welfare. And, unfortunately, those who are most extravagant in this respect are often the ones who can least afford it.

Secondly, the food which we eat does not always contain the proper proportions of the different kinds of nutritive ingredients. We consume relatively too much of the fuel ingredients of food, such as the fats of meat and butter, and the starch which makes up the larger part of the nutritive material of flour, potatoes, sugar, and sweetmeats. Conversely, we have relatively too little of the protein of flesh-forming substances, like the lean of meat and fish and the gluten of wheat, which make muscle and sinew and which are the basis of blood, bone and brain.

Thirdly, many people, not only the well-to-do, but those in moderate circumstances, use needless quantities of food. Part of the excess, however, is simply thrown away with the wastes of the table and the kitchen; so that the injury to health, great as it may be, is doubtless much less than if all were eaten. Probably the worst sufferers from this evil are well-to-do people of sedentary occupations.

Finally, we are guilty of serious errors in our cooking. We waste a great deal of fuel in the preparation of our food, and even then a great deal of the food is very badly cooked. A reform in these methods of cooking is one of the economic demands of our time.

1. It is the author’s opinion that we should consume less/fewer _____ and more _____.
 - A. wheat gluten . . . potatoes
 - B. fish . . . butter
 - C. sweetmeats . . . lean meat
 - D. sugar . . . fat
2. The primary argument of the second paragraph is:
 - A. People who can least afford the expense are most likely to be convinced that there is some benefit accrued by spending more money on food than is necessary.
 - B. Poorer people are just as inclined as wealthy people to be overly extravagant in their food purchases.
 - C. Wealthy people have created an injustice in society by attaching a sense of dignity to the purchase of needlessly expensive food, which has the effect of reducing the buying power of poorer people.
 - D. Society has betrayed the poorer people in society by requiring them to spend more money on food than is necessary in order to maintain a sense of dignity.
3. The primary argument of this essay is that _____.
 - A. humans misuse their food resources heavily to the detriment of individual health and social equality
 - B. poorer people are particularly vulnerable to changes in the food market
 - C. the economic divide between wealthy and poor people is contributing negatively to the supply of food in the world
 - D. if humanity is to survive the massive growth in population, people need to get smarter about the types and quantities of food they consume
4. Which of these factors does the author believe is most relevant to why the excessive preparation of food is less injurious to our health than the other mistakes and fallacies he discusses?
 - A. Much of the excess, instead of being eaten, is simply thrown away.
 - B. The preparation of excess food encourages social gathering and bonding.
 - C. The excess food can be saved and eaten later.
 - D. The consumption of excess food on one occasion allows us to consume less food at a later date.



Practice Passage: Argument: Answer Key

1. It is the author's opinion that we should consume less/fewer _____ and more _____.

- A. wheat gluten . . . potatoes
- B. fish . . . butter
- C. sweetmeats . . . lean meat
- D. sugar . . . fat

2. The primary argument of the second paragraph is:

- A. People who can least afford the expense are most likely to be convinced that there is some benefit accrued by spending more money on food than is necessary.
- B. Poorer people are just as inclined as wealthy people to be overly extravagant in their food purchases.
- C. Wealthy people have created an injustice in society by attaching a sense of dignity to the purchase of needlessly expensive food, which has the effect of reducing the buying power of poorer people.
- D. Society has betrayed the poorer people in society by requiring them to spend more money on food than is necessary in order to maintain a sense of dignity.

In the third paragraph, the author states,

We consume relatively too much of the fuel ingredients of food, such as the fats of meat and butter, and the starch which makes up the larger part of the nutritive material of flour, potatoes, sugar, and sweetmeats. Conversely, we have relatively too little of the protein of flesh-forming substances, like the lean of meat and fish and the gluten of wheat, which make muscle and sinew and which are the basis of blood, bone and brain.

So, he or she thinks we should consume less fatty meat, butter, and starch in the form of flour, potatoes, sugar, and sweetmeats. This allows us to narrow down our answer choices to “sweetmeats . . . lean meat” and “sugar . . . fat.” So, does the author think we should eat more lean meat, or more fat? The author says in the second sentence that we should eat more lean meat, fish, and wheat gluten, so “sweetmeats . . . lean meat” is the correct answer. The author mentions fat when discussing things of which we consume too much in the first sentence.

The primary argument of the second paragraph is actually two ideas combined. First, the idea that “many people purchase needlessly expensive kinds of food” is argued. This idea is expressed in many of the answer choices, so we need to combine it with the second idea, which is that “unfortunately, those who are most extravagant in this respect are often the ones who can least afford it.” So, those people who can least afford the expense are most likely to needlessly spend money. Only one of these answer choices reflects this argument. The rest are primarily concerned with attaching blame, either to “society” or to “wealthy people,” which the author does not do.



3. The primary argument of this essay is that _____.
- A. Humans misuse their food resources heavily to the detriment of individual health and social equality.
- B. Poorer people are particularly vulnerable to changes in the food market.
- C. The economic divide between wealthy and poor people is contributing negatively to the supply of food in the world.
- D. If humanity is to survive the massive growth in population, people need to get smarter about the types and quantities of food they consume.
4. Which of these factors does the author believe is most relevant to why the excessive preparation of food is less injurious to our health than the other mistakes and fallacies he discusses?
- A. The consumption of excess food on one occasion allows us to consume less food at a later date.
- B. The preparation of excess food encourages social gathering and bonding.
- C. The excess food can be saved and eaten later.
- D. Much of the excess, instead of being eaten, is simply thrown away.

While it is true that the differences between wealthy people and poor people in the purchase, preparation, and consumption of food is an important theme in this text, it is more accurate to say that the primary theme is that all “humans misuse their food resources heavily.” The author employs examples of the ways in which poor and wealthy people both do this to demonstrate that this mistake is not unique to one or the other, but rather is a universal tendency of humanity.

When discussing the excessive preparation of food, the author says, “Part of the excess, however, is simply thrown away with the wastes of the table and the kitchen, so that the injury to health, great as it may be, is doubtless much less than if all were eaten.” This is the only example he gives for which he provides a mitigating factor, so we know that he believes this particular mistake is less injurious to our health than others which he discusses. The reason he gives is that, rather than consuming this excess food, we generally just throw it away. No doubt he still believes this to be a problem, but the harm it does to individuals is reduced by the fact that so much of it is not eaten.



Identifying Details

Not all questions on the ACT Reading exam ask you to interpret a passage. Some questions on the exam seek to test your understanding of the passage through your ability to recognize and contextualize specific details contained within the passage. These questions will not simply ask you to regurgitate content straight from the passage, they will test your ability to synthesize the information with which these details provide you. Even if you are not a particularly detail oriented person, these questions still contain the encouraging feature that all information needed to answer detail questions will be contained directly in the passage. If you have read the passage carefully and fully understood its content, you should never be guessing on a details question, because the answer is contained somewhere in the passage.

Sample Passage: Identifying Details

Humanities: Adapted from *Laughter: An Essay on the Meaning of the Comic* by Henri Bergson (1914 ed.)

What does laughter mean? What is the basal element in the laughable? What common ground can we find between the grimace of a merry- andrew, a play upon words, an equivocal situation in a burlesque and a scene of high comedy? What method of distillation will yield us invariably the same essence from which so many different products borrow either their obtrusive odor or their delicate perfume? The greatest of thinkers, from Aristotle downwards, have tackled this little problem, which has a knack of baffling every effort, of slipping away and escaping only to bob up again, a pert challenge flung at philosophic speculation. Our excuse for attacking the problem in our turn must lie in the fact that we shall not aim at imprisoning the comic spirit within a definition. We regard it, above all, as a living thing. However trivial it may be, we shall treat it with the respect due to life. We shall confine ourselves to watching it grow and expand. Passing by imperceptible gradations from one form to another, it will be seen to achieve the strangest metamorphoses. We shall disdain nothing we have seen. Maybe we may gain from this prolonged contact, for the matter of that, something more flexible than an abstract definition,--a practical, intimate acquaintance, such as springs from a long companionship. And maybe we may also find that, unintentionally, we have made an acquaintance that is useful. For the comic spirit has a logic of its own, even in its wildest eccentricities. It has a method in its madness. It dreams, I admit, but it conjures up, in its

dreams, visions that are at once accepted and understood by the whole of a social group. Can it then fail to throw light for us on the way that human imagination works, and more particularly social, collective, and popular imagination? Begotten of real life and akin to art, should it not also have something of its own to tell us about art and life?

At the outset we shall put forward three observations which we look upon as fundamental. They have less bearing on the actually comic than on the field within which it must be sought.

The first point to which attention should be called is that the comic does not exist outside the pale of what is strictly HUMAN. A landscape may be beautiful, charming and sublime, or insignificant and ugly; it will never be laughable. You may laugh at an animal, but only because you have detected in it some human attitude or expression. You may laugh at a hat, but what you are making fun of, in this case, is not the piece of felt or straw, but the shape that men have given it,--the human caprice whose mould it has assumed. It is strange that so important a fact, and such a simple one too, has not attracted to a greater degree the attention of philosophers. Several have defined man as "an animal which laughs." They might equally well have defined him as an animal which is laughed at; for if any other animal, or some lifeless object, produces the same effect, it is always because of some resemblance to man, of the stamp he gives it or the use he puts it to.



Identifying Details

There are basically two species of detail questions on the ACT Reading exam, those that ask you to account for details from a specific paragraph or section of the reading, and those that ask you to account for details from the passage as a whole, with no specific section isolated in which to look for the answer.

We will start by looking at the more direct of these two types: the questions about specific paragraphs. While these questions are obviously a little bit easier than their more expansive counterparts, the fundamental process by which you will be finding and understanding these details is the same for both types.

Identifying Details in a Specific Paragraph

A very important thing to keep in mind when approaching detail questions is that the questions will, most often, not ask for a detail using the exact language used in the text, they will re-phrase or paraphrase the particulars of the detail as it is laid out in the passage. This means that the most important aspect of these detail questions is to read the question carefully BEFORE looking at the questions. It is vital to form your own understanding of the passage overall, and the individual sections and paragraphs contained within the paragraph.

The test is written and constructed in such a way that you must be secure in your own understanding of a text before attempting to answer for the specific details contained within that text. You may even find it helpful to make a brief note alongside each paragraph quickly summarizing (in your own words) its contents.

Sample Question

Which of the following statements is not established in the first paragraph?

- A. Scholars have tried and failed to define the foundation of humor.
- B. We should try to define the comic spirit.
- C. The comic spirit, and our relationship to it, changes and grows over time.
- D. The comic spirit may offer insight into the human imagination.

In order to raise the difficulty of detail questions, the test will often ask you to identify which of a group of options was NOT stated, identified, supported, or argued for in the passage, as is the case here.

So, let us embark on our spirited journey through this question! First, we should return to the first paragraph and read it again, but this time with the statements named in the question firmly in mind.

The passage opens with a series of rhetorical questions, in which the author establishes his focus for the passage that follows. Now, having thus established the focus of the passage the author begins to make some direct statements. First he states that “the greatest of thinkers,” a group in which he includes the Greek philosopher Aristotle, have “tackle[d] this little problem.” The author then makes clear that this “little problem” is not little in difficulty, as it has “baffled” these great minds, and been an extreme and difficult subject of “philosophic speculation.” Now, we can swiftly eliminate option A. Philosophers, including Aristotle, are scholars, and the author’s characterization of the relationship of scholars to this problem clearly support option A.



Remember that, for questions seeking the answer that does NOT appear in the text, as soon as you find a section you believe supports one of the options you can eliminate that option. One option for answering these types of questions involves going through the passage and finding the answer options that are supported within the text. This way, you will end up, ultimately, with your answer after having eliminated all the options that DID appear, in some form, in the passage.

In this case, however, the very next sentence (beginning with “our excuse”) we find a direct contradiction of one of our answer options! The author makes clear that he “does not aim at imprisoning the comic spirit within a definition.” From the author’s tone and word choice, he makes clear that he views attempts to “define the comic spirit” as inherently negative and foolhardy (since the passage obviously emphasizes the goodness of humor, it would not make sense to “imprison” it). It does not require much interpretation to stretch the author’s statement that his entire justification for “attacking the [intellectual] problem” of the comic spirit to the statement that “we” should NOT “try to define the comic spirit.” Therefore, the answer is B.

If we want to double check this answer, we can continue through the passage and find that the statements “The comic spirit, and our relationship to it, changes and grows over time,” (“watching [the comic spirit] grow and expand. Passing by imperceptible gradations from one form to another”) and “The comic spirit may offer insight into the human imagination.” (“Can it then fail to throw light for us on the way that human imagination works, and more particularly social, collective, and popular imagination?”) well-supported in the first paragraph.

Had you, on your first reading of the passage, noted the passage’s treatment of the concept of humor as a fluid, “living,” being you might have instinctively known that B was the least likely, since it was the least conceptually expansive in its treatment of the subject. So, you can see how a general understanding of the content of a passage will help you answer detail questions.

Identifying Details in a Full Passage

Not all detail questions will do you the favor of providing a specific paragraph in which to look. Some questions on the test will instead ask you to find whether or not statements, details, or arguments were made anywhere in the passage. In these types of questions, since you may not have time to parse the entire passage, line-by-line, your overall understanding of the passage’s subject, tone, and overall argument is even more crucial to answering for these details.

Sample Question

Which of the following is a claim supported by the passage?

- A. There is no method to the comic spirit; what is funny is completely random.
- B. Non-human animals, like dogs or cats, can be funny in their own right.
- C. If you laugh at a hat, you are laughing at the shape given to that hat by humans.
- D. A joke is funnier when told by a professional comedian.

This question, rather than asking us which claim is NOT made in the passage, is asking us to find the claim that IS made somewhere in the passage. There are two reasonable approaches to doing so. The first is to find the one claim that is supported by the passage, and the second is to find and eliminate the three claims that are NOT supported by the passage.



Say we're pressed for time, we can scan the passage for the word "hat." After a couple of tense moments, we find it in the third paragraph! Reading the full sentence in which hats appear ("You may laugh at a hat, but what you are making fun of, in this case, is not the piece of felt or straw, but the shape that men have given it,--the human caprice whose mould it has assumed."), we can easily see that option C is pretty much directly supported by the passage.

A fuller reading of the passage reveals that options A and C are directly refuted in the first and third paragraphs of the passage respectively. Option D simply does not appear in the passage; there is no mention of professional comedians anywhere.

Once again, an overall understanding of passage's content, and the author's attitude toward that content is helpful. The third paragraph's discussion of the uniquely human aspects of laughter is notable, and makes a number of interesting claims. If you had noted that this paragraph was about the role of the level of human involvement in objects of laughter, you may have simply gone right back to it and answered this question quite quickly and decisively.



Natural Science: Adapted from *Ice-Caves of France and Switzerland* by George Forrest Browne (1865)

This account states that the cave is in the county of Thorn, among the lowest spurs of the Carpathians. The entrance, which faces the north, and is exposed to the cold winds from the snowy part of the Carpathian range, is eighteen fathoms high and nine broad; and the cave spreads out laterally, and descends to a point fifty fathoms below the entrance, where it is twenty-six fathoms in breadth, and of irregular height. Beyond this no one had at that time penetrated, on account of the unsafe footing, although many distant echoes were returned by the farther recesses of the cave; indeed, to get even so far as this, much step-cutting was necessary.

When the external frost of winter comes on, the account proceeds, the effect in the cave is the same as if fires had been lighted there: the ice melts, and swarms of flies and bats and hares take refuge in the interior from the severity of the winter. As soon as spring arrives, the warmth of winter disappears from the interior, water exudes from the roof and is converted into ice. In the dog-days, the frost is so intense that a small icicle becomes in one day a huge mass of ice; but a cool day promptly brings a thaw, and the cave is looked upon as a barometer, not merely feeling, but also presaging, the changes of weather. The people of the neighborhood, when employed in field-work, arrange their labor so that the mid-day meal may be taken near the cave, when they either ice the water they have brought with them, or drink the melted ice, which they consider very good for the stomach. It had been calculated that six hundred weekly carts would not be sufficient to keep the cavern free from ice. The ground above the cave is peculiarly rich in grass.

In explanation of these phenomena, Bell threw out the following suggestions, which need no comment. The earth being of itself cold and damp, the external heat of the atmosphere, by partially penetrating into the ground, drives in this native cold to the inner parts of the earth, and makes the cold there more dense. On the other hand, when the external air is cold, it draws forth towards the surface the heat there may be in the inner part of the earth, and thus makes caverns warm. In support and illustration of this view, he states that in the hotter parts of Hungary, when the people wish to cool their wine, they dig a hole two feet deep, and place in it the flagon of wine, and, after filling up the hole again, light a blazing fire upon the surface, which cools the wine as if the flagon had been laid in ice.

1. Which of the following is NOT true of the residents who live near the cave?
 - A. They use the cave as a predictor of the weather.
 - B. They use the resources of the cave, but make no claims about the potential health benefits of doing so.
 - C. They often eat and drink near the cave.
 - D. All of these statements are true of the residents who live near the cave.
2. Which of the following is not true of the cave?
 - A. The cave is north-facing.
 - B. The interior of the cave has a regular height of six fathoms.
 - C. No one has explored the depths of the cave.
 - D. The cave contains, at times, a huge amount of ice.
3. In the third paragraph the author presents _____.
 - A. another researcher's theories about the cave
 - B. his own theories about the working of the cave
 - C. a summary of the dimensions of the cave
 - D. a summary of the way locals use and understand the cave
4. What effect does the arrival of external frost have on the cave?
 - A. The ice in the cave melts, and animals seek shelter inside.
 - B. Water falls from the top of the cave and freezes, creating huge amounts of ice.
 - C. The footing at the entrance of the cave becomes particularly unsteady.
 - D. The frost becomes so intense that a tiny icicle can become a huge mound of ice.
5. In the warmer parts of Hungary, _____.
 - A. people are known for making the finest local wines
 - B. people burn wine for heat and light
 - C. there exist numerous caves like the one described earlier in the passage
 - D. people cool wine by burying it and lighting the soil on fire
6. The exact amount of ice taken out of the cave is _____.
 - A. six hundred carts per week
 - B. six hundred carts per month
 - C. not specified in the passage
 - D. No ice is taken out of the cave.



Practice Passage: Identifying Details: Answer Key

1. Which of the following is NOT true of the residents who live near the cave?

- A. They use the cave as a predictor of the weather.
- B. They use the resources of the cave, but make no claims about the potential health benefits of doing so.
- C. They often eat and drink near the cave.
- D. All of these statements are true of the residents who live near the cave.

2. Which of the following is not true of the cave?

- A. The cave is north-facing.
- B. The interior of the cave has a regular height of six fathoms.
- C. No one has explored the depths of the cave.
- D. The cave contains, at times, a huge amount of ice.

3. In the third paragraph the author presents _____.

- A. another researcher's theories about the cave
- B. his own theories about the working of the cave
- C. a summary of the dimensions of the cave
- D. a summary of the way locals use and understand the cave

In the second paragraph we are told that the “people of the neighborhood” (residents who live near the cave) specifically “arrange their labour so that the mid-day meal may be taken near the cave, where they either ice the water they have brought with them, or drink the melted ice.” So, here we see that answer C is true, and thus cannot be the answer, nor can D. We are left with two possible answers, we can either eliminate answer A by looking earlier in the second paragraph (“the cave is looked upon as a barometer, not merely feeling, but also presaging, the changes of weather”) or by carefully reading about the locals’ reason for drinking the water, which does, in fact, assert specific health benefits (“which they consider very good for the stomach”).

The answer to this passage can be found, mostly, in the first paragraph. Early on we are told that the cave “faces north,” and we can swiftly eliminate answer choice A from contention. After the entrance of the cave is described, the author directly states that: “beyond this no one [has] penetrated on account of unsafe footing.” Thus, we can eliminate answer choice C. Answer D is implicitly true (since the cave is explicitly defined as an ice cave), and specifically in the second paragraph (“a small icicle becomes in one day a huge mass of ice”). We are left, then, with B, which gives an inaccurate measurement of the caves’ height, which we are specifically told is “of irregular height,” and for which we are given no specific measurement. The six-fathom measurement is the “breadth” or width of the interior of the cave, not its height.

Here, we have a detail question that explicitly directs us to the third paragraph. Answer choice C describes the first paragraph, and answer choice D more directly addresses the second paragraph. So, the real key to answering this question is determining whether the theories about the cave are the author’s own, or summaries of another researcher. The first sentence situates the theories (“explanation of these phenomena”) as belonging to “Bell,” which he presents straightforwardly as they “need no comment” from the author.



4. What effect does the arrival of external frost have on the cave?
- A. The ice in the cave melts, and animals seek shelter inside.
- B. Water falls from the top of the cave and freezes, creating huge amounts of ice.
- C. The footing at the entrance of the cave becomes particularly unsteady.
- D. The frost becomes so intense that a tiny icicle can become a huge mound of ice.
5. In the warmer parts of Hungary, _____.
- A. people are known for making the finest local wines
- B. people burn wine for heat and light
- C. there exist numerous caves like the one described earlier in the passage
- D. people cool wine by burying it and lighting the soil on fire
6. The exact amount of ice taken out of the cave is _____.
- A. six hundred carts per week
- B. six hundred carts per month
- C. not specified in the passage
- D. No ice is taken out of the cave.

The reaction of the cave to the various seasons is described at length in the second paragraph. While “the arrival of external frost” is not an actual season, it is associated with winter, and it is the specific phenomenon of the frost, rather than the time of year, that creates the effect in the cave. From an overall reading of the second paragraph, we know that the main thing of note about this specific cave is that it has a relationship to weather that seems to be the inverse of what one would expect: when the external frost arrives, the ice in the cave melts and “flies and bats and hares take refuge in the interior,” in the “spring” the cave grows cold, and in the “dog days” of summer the cave grows so cold that a “small icicle becomes in one day a huge mass of ice.” The answer, then, is clearly A.

The “hotter” parts of Hungary are described as a part of Bell’s explanation for the strange reactions of the cave to various weather conditions. In seeking to explain how the cave could grow colder in proportion to the weather outside getting hotter, he apparently called upon the example of the practice in “the hotter parts of Hungary” of “dig[ging] a hole two feet deep, and plac[ing] in it a flagon of wine, and, after filling up the hole again, light[ing] a blazing fire upon the surface, which cools the wine.” So, we can see from a reading of the third paragraph that the locals of warm areas of Hungary bury wine and burn the surface, specifically to “cool the wine.” There is no mention of ice caves in Hungary, nor of the quality of Hungarian wine.

First, we can eliminate option D, as we know that locals “ice the water they have brought with them,” by, necessarily, removing ice from the cave. Then, we can scan the passage for the mention of “six hundred carts” of ice, and we find that the mention of this quantity of ice is used in a hypothetical calculation of how much ice would need to be removed for the cave to be free of ice. Thus, we can see that the actual amount of ice taken out of the is not specified in the passage.



How It's Said

There are many different ways an author can say essentially the same thing, and certain ACT Reading questions concern the aspects of prose that affect how an author specifically conveys a particular meaning. Certain questions may ask you to use context clues to figure out the meanings of unfamiliar words or phrases, while other questions may tackle tone or point of view as their topics. Still others may look at the order in which information is presented in a passage, or the order of the events conveyed in the passage. While these questions may initially seem rather difficult because of the subtle nature of their topics, practice can help you learn to tackle them with ease.

Section Outline

[Determining Context-Based Meaning](#)

[Context-Based Meaning of Unfamiliar Words](#)

[Words with Multiple Meanings](#)

[Context-Dependent Meaning of Phrases and Clauses](#)

[Analyzing Tone](#)

[Tone of a Sentence or Paragraph](#)

[Tone of an Entire Passage](#)

[Words that Describe Tone](#)

[Practice Passage: Analyzing Tone](#)

[Point of View](#)

[Identifying Author and Audience](#)

[Identifying the Author's Directly Stated Views](#)

[Inferring the Author's Views on a Topic](#)

[Practice Passage: Point of View](#)

[Sequence](#)

[Sequence of Passage Text](#)

[Sequence of Events](#)

[When Passage Text Sequence Does Not Equal Passage Event Sequence](#)

[Identifying the Reasoning Behind Sequence](#)

[Inferring the Previous Topic](#)

[Inferring the Next Topic](#)

[Practice Passages: Sequence](#)



Determining Context-Based Meaning

The ACT Reading section can include questions that test your ability to identify the meanings of various words and phrases; however, this doesn't mean that you need to start drilling vocabulary words! The skill being tested in these questions isn't rote memorization, or even breadth of vocabulary—the test is interested in how well you can pay attention to the elements of the text surrounding an indicated word or phrase and use it to ascertain its meaning.

Getting a good handle on context and the clues it can provide is fundamental for success when facing these questions. For a brief discussion of context and its importance, [click here](#).

Example Passage

Prose Fiction: Adapted from *The Life and Adventures of Robinson Crusoe* by Daniel Defoe (1719)

I was now, in the months of November and December, expecting my crop of barley and rice. The ground I had manured and dug up for them was not great; for, as I observed, my seed of each was not above the quantity of half a peck, for I had lost one whole crop by sowing in the dry season. But now my crop promised very well, when on a sudden I found I was in danger of losing it all again by enemies of several sorts, which it was scarcely possible to keep from it; as, first, the goats, and wild creatures which I called hares, who, tasting the sweetness of the blade, lay in it night and day, as soon as it came up, and eat it so close, that it could get no time to shoot up into stalk.

This I saw no remedy for but by making an enclosure about it with a hedge; which I did with a great deal of toil, and the more, because it required speed. However, as my arable land was but small, suited to my crop, I got it totally well fenced in about three weeks' time; and shooting some of the creatures in the daytime, I set my dog to guard it in the night, tying him up to a stake at the gate, where he would stand and bark all night long; so in a little time the enemies forsook the place, and the corn grew very strong and well, and began to ripen apace.

But as the beasts ruined me before, while my corn was in the blade, so the birds were as likely to ruin me now, when it was in the ear; for, going along by the place to see how it throve, I saw my little crop surrounded with fowls, of I know not how many sorts, who stood, as it were, watching till I should be gone. I immediately let fly among them, for I always had my gun with me. I had no sooner shot, but there rose up a little cloud of fowls, which I had not seen at all, from among the corn itself.

This touched me sensibly, for I foresaw that in a few days they would devour all my hopes; that I should be starved, and never be able to raise a crop at all; and what to do I could not tell; however, I resolved not to lose my corn, if possible, though I should watch it night and day. In the first place, I went among it to see what damage was already done, and found they had spoiled a good deal of it; but that as it was yet too green for them, the loss was not so great but that the remainder was likely to be a good crop if it could be saved.

I stayed by it to load my gun, and then coming away, I could easily see the thieves sitting upon all the trees about me, as if they only waited till I was gone away, and the event proved it to be so; for as I walked off, as if I was gone, I was no sooner out of their sight than they dropped down one by one into the corn again. I was so provoked, that I could not have patience to stay till more came on, knowing that every grain that they ate now was, as it might be said, a peck-loaf to me in the consequence; but coming up to the hedge, I fired again, and killed three of them. This was what I wished for; so I took them up, and served them as we serve notorious thieves in England—hanged them in chains, for a terror to others. It is impossible to imagine that this should have such an effect as it had, for the fowls would not only not come at the corn, but, in short, they forsook all that part of the island, and I could never see a bird near the place as long as my scarecrows hung there. This I was very glad of, you may be sure, and about the latter end of December, which was our second harvest of the year, I reaped my corn.



Determining Context-Based Meaning

Did you notice that the previous passage contained some words and phrasing that aren't common parlance any more? That's because it's from a work of fiction published in 1719. It's reasonable to assume that the English language will have changed a bit over the course of nearly three hundred years! This archaic text makes it clear how important context clues can be in figuring out what exactly a passage means by them. Let's consider a few questions that fall into two general types: those that ask about the meaning of a single word, and those that ask about the meaning of a phrase or clause. Questions that concern a single word can be further divided into those that ask about words with which you are unfamiliar, and those that ask about words you've seen before. These latter questions may seem simpler, but can be extra tricky if the word in question has more than one accepted meaning!

Context-Based Meaning of Unfamiliar Words

Let's consider a sample question about a word that is most likely unfamiliar to you.

Sample Question

In the second paragraph, the underlined word “apace” most closely means _____.

- A. nearby
- B. with difficulty
- C. quickly
- D. without warning

It's unlikely that you know which answer is correct right off the bat, so getting this question right will be a matter of discerning what the word's meaning has to be from the way it is used in the context of the passage. Consider part of the sentence that precedes it:

... I set my dog to guard [my arable land growing crops] in the night, tying him up to a stake at the gate, where he would stand and bark all night long; so in a little time the enemies forsook the place, and the corn grew very strong and well, and began to ripen apace.

In the first half of the sentence, the narrator has his dog guard his crops at night. The sentence concludes with the narrator reporting that “in a little time” “the enemies” (birds, in this case) stopped trying to eat his crops, and his corn grew well and began to ripen ... “apace.” What could “apace” mean? There seems to be an overall sense of cause-and-effect going on in this sentence, so “apace” might mean something like “therefore,” only appearing after the event being caused is described. Or, it might mean “well,” since the corn is growing well, but that means that “apace” would be a bit redundant, since the earlier part of the sentence says that “the corn grew very strong and well.”

We know that one of the answer choices is correct, so instead of trying to pinpoint the exact meaning of “apace” from its context, let's use the context to narrow down the possible answer choices and identify the correct one. The sentence doesn't convey anything about the distance between the corn and anything else, and nothing in the surrounding context of other sentences suggests that “apace” could logically mean “nearby,” so we can ignore answer choice A. Since we're told that the corn “grew very strong and well,” it seems that it ripening would have surprised anyone, so for “apace” to mean “without warning” doesn't make much sense either. Similarly, the fact that we're told that the corn “grew very strong and well” and



told nothing to the contrary that would suggest it had a hard time growing means that it wouldn't logically follow from the earlier part of the sentence for "apace" to mean "with difficulty." That means that there's only one answer choice that makes logical sense: "quickly." The sentence uses the phrase "in a little time" a bit before it uses "apace," so it's reasonable to assume that the corn might grow quickly, especially if it is, as we are told, "[growing] strong and well." A is the correct answer.

You can approach any context-based meaning question in this manner: come up with your own idea of what the word could mean based on its context, and/or knock out answer choices until you've identified the one that makes the most sense given the specific usage of the word in the passage.

Words with Multiple Meanings

At first glance, context-based meaning questions that ask you about the definitions of familiar words may look entirely too easy in comparison to the rest of the questions you encounter on the ACT Reading section. You might suspect that there is some sort of "catch" to the question, and you'd be correct in your suspicions: these questions are rarely as straightforward as they initially seem to be. Like context-based meaning questions testing unfamiliar words, these are testing your ability to read the passage for context clues. They often contains answers that look like they are obviously correct, but upon further examination, are not! Take, for instance, the following sample question:

Sample Question

In the last paragraph, the underlined word "scarecrows" refers to _____.

- A. corn dollies
- B. straw men
- C. birds' remainst
- D. dead rabbits

"I know what 'scarecrows' means," you might be thinking to yourself as you read this question, ready to pick B, "straw men" as your answer and save time for more challenging problems. But do you really know what it means in context? Consider the passage's usage of the term:

... but coming up to the hedge, I fired again, and killed three of [the pests]. This was what I wished for; so I took them up, and served them as we serve notorious thieves in England—hanged them in chains, for a terror to others. It is impossible to imagine that this should have such an effect as it had, for the fowls would not only not come at the corn, but, in short, they forsook all that part of the island, and I could never see a bird near the place as long as my scarecrows hung there. This I was very glad of, you may be sure . . .

There no mention of anything resembling a traditional "straw men" scarecrows in this passage, so B cannot be correct, no matter how common a definition it is for the term when used in other contexts. Context-based meaning questions that use familiar words will often use terms that have multiple meanings, and expect to catch you off-guard by presenting you with correct meanings that don't work in the passage's context. Don't let them fool you!



Now that we've figured out that B isn't the correct answer, let's figure out which answer choice *is* correct. The passage does talk about corn quite a bit, but it's not because the narrator is constructing "corn dollies"—he's growing a crop of corn. No mention is ever made of him making anything out of corn stalks, so A can't be correct. This leaves us with C and D—dead birds or dead rabbits. What type of animal did the narrator actually shoot in the passage? To determine this, we have to look even further back in the passage for extended context, and things get a little tricky. Considering this question and skimming the passage starting with its first paragraph, we find that the narrator realized that the narrator, in guarding his crop, has "enemies of several sorts, which it was scarcely possible to keep from it; as, first, the goats, and wild creatures which I called hares, who, tasting the sweetness of the blade, lay in it night and day, as soon as it came up, and eat it so close, that it could get no time to shoot up into stalk." This suggests that the correct answer might be D, "dead rabbits." Keep reading to make sure, though, and you'll find the last detail we need to solve this question in the third paragraph:

... going along by the place to see how it throve, I saw my little crop surrounded with fowls, of I know not how many sorts, who stood, as it were, watching till I should be gone. I immediately let fly among them, for I always had my gun with me. I had no sooner shot, but there rose up a little cloud of fowls, which I had not seen at all, from among the corn itself.

The part of the passage that specifically talks about the narrator shooting at anything doesn't involve him shooting at any rabbits—it involves him shooting at "fowls"—birds. Thus, the correct answer is C, "birds' remains." After taking so many steps to determine this answer to what may have first seemed like an obvious question, it's clear why you should approach any context-based meaning question involving a familiar word with caution and suspicion! It probably involves some form of catch.

Let's consider another sample question demonstrating the same thing in a slightly different way:

Sample Question

As it is used in the passage, the underlined word "blade" in the first paragraph most nearly means _____.

- A. leaf
- B. spade
- C. knife
- D. hedge

You likely recognize "blade" and immediately jump to answer choice C, "knife," as the best option. But let's check how the passage uses the term:

But now my crop promised very well, when on a sudden I found I was in danger of losing it all again by enemies of several sorts, which it was scarcely possible to keep from it; as, first, the goats, and wild creatures which I called hares, who, tasting the sweetness of the blade, lay in it night and day, as soon as it came up, and eat it so close, that it could get no time to shoot up into stalk.

"Blade" does not mean "knife" in this passage; it means something that the goats and hares are eating; what's more, they are eating it "so close" that it then can't "shoot up into stalk." So, this tells us that "blades" become "stalks." They grow. "Blade" has to indicate something that can grow and be eaten, then.



We can ignore answer choices B and C and concentrate on A, “leaf,” and D, “hedge.” In deciding between these two, consider what plant the author is growing in the passage: it’s not a plant that makes up hedges—it’s corn. D isn’t the correct answer as hedges aren’t mentioned anywhere in the passage. A, “leaf,” is the best answer. The author is using “blade” like we use the term when we talk about “blades of grass.”

Let’s look at one more example, this time with a new twist:

Sample Question

As it is used in the passage, the underlined word “peck” in the first paragraph most nearly means _____.

- A. a unit of agricultural measurement
- B. jab with a pointed beak
- C. a slight sensation of hunger
- D. annoy with repeated questions

Once again, you may think that you’ve found an obvious question with an obvious answer, B, “jab with a pointed beak.” After all, the passage even talks about the narrator fighting off birds from his crops, right? As usual, with context-based meaning questions, it is imperative that you identify the word in the passage and consider how it is used *there*, not in common parlance, as the test often picks words in situations where these differ greatly:

I was now, in the months of November and December, expecting my crop of barley and rice. The ground I had manured and dug up for them was not great; for, as I observed, my seed of each was not above the quantity of half a peck, for I had lost one whole crop by sowing in the dry season.

While the sentence structure may sound strange due to the text being hundreds of years old, “My seed of each was not above the quantity of half a peck” gives you all of the information you need to answer this question correctly. Skimming the answer choices, did you notice something that makes this question different from the others we’ve considered? In this case, two of the answer choices (A and C) provide definitions of nouns, and the other two (B and D) provide definitions of verbs. This means that the first thing we need to do is figure out if “peck” is being used as a noun or a verb in the passage. Applying a little grammatical knowledge can be extremely helpful in scenarios like these: “peck” is contained in the prepositional phrase “of half a peck,” and in order to have half of something, it has to be, well, something—in grammatical terms, a noun. You can’t have half a verb! This simple observation allows us to knock out answer choices B and D. So, is “peck” being used to mean “a unit of agricultural measurement” or “a slight sensation of hunger?” Hunger could very well be discussed in the passage, as the narrator is worried about the amount of crops he will be able to grow, but there’s a big neon sign of a context clue hanging out right before the term in the passage that makes choosing between these answer choices very easy: “peck” appears in the phrase “the quantity of half a peck.” “Half a peck,” and by extension, a “peck,” are each designations of some agricultural quantity. Don’t quite know the exact details of how much seed is in a “peck?” That’s ok—all you need to do is say that’s how the word is being used. You don’t need to understand all of the subtleties of the term. The correct answer is A!



Context-Dependent Meaning of Phrases and Clauses

Certain context-based meaning questions ask about phrases and clauses instead of single words, but you can approach them in the same way—and with the same suspicions about any familiar-sounding phrases.

Sample Question

As it is used in the passage, the phrase “in the ear” underlined in the third paragraph most nearly means _____.

- A. failed
- B. finished growing
- C. rotten
- D. ripened

You might initially try to attribute a literal meaning to “in the ear,” but the author doesn’t talk about ears as a body part in the passage; he talks about ears of corn. Many of our answer choices have to do with meanings that could easily be applied to corn, so let’s consider the passage’s usage of “in the ear” to figure out what it means by it:

But as the beasts ruined me before, while my corn was in the blade, so the birds were as likely to ruin me now, when it was in the ear; for, going along by the place to see how it throve, I saw my little crop surrounded with fowls, of I know not how many sorts, who stood, as it were, watching till I should be gone.

Ok, this sentence is using some interesting parallel structure. Just as the beasts ruined the corn while it was “in the blade,” the birds were likely to ruin it “in the ear.” This phrase definitely has something to do with corn growing, but what? We can ignore answer choice A, “failed,” because if the crop failed, it doesn’t seem logical that the birds could threaten it at that point. Similar reasoning holds for answer choice C, “rotten.” How could birds threaten a rotten crop, and furthermore, why would the narrator care? We’re left to decide between answer choices B and D. Logic can help us here, too: if the crop were finished growing completely, the narrator would likely harvest it as soon as possible, especially if it were threatened by birds. He doesn’t do this in the passage; instead, he guards it, so it can continue growing. So, the crop must not be done growing yet when it is “in the ear,” making the best answer choice D, “ripened.”

Let’s consider one more phrase-based sample question:



Sample Question

As it is used in the passage, the phrase “let fly among” underlined in the third paragraph most nearly means _____.

- A. defended oneself from
- B. released something into the wild in the direction of
- C. shot at
- D. yelled at

Once again, you may initially think that since the phrase in question involves the word “fly,” and we’re talking about birds in this passage, that the answer must be B, “released something into the wild in the direction of,” which could apply to birds as the subject. Consider the passage, though: the birds are the object of “let fly among,” not the subject.

I saw my little crop surrounded with fowls, of I know not how many sorts, who stood, as it were, watching till I should be gone. I immediately let fly among them, for I always had my gun with me. I had no sooner shot, but there rose up a little cloud of fowls, which I had not seen at all, from among the corn itself.

The most telling part of this context is the prepositional phrase that follows our phrase, “for I always had my gun with me.” This clue on its own can help us knock out answer choices B and D; we know that the phrase “let fly at” has to have something to do with the narrator having a gun. Is the narrator defending himself from the birds, or shooting at them as the attacker? The narrator is the attacker in the passage; he’s not defending himself from his birds—he’s defending his crops from them. This means that the correct answer is C.



Practice Passage: Context-Dependent Meaning

History: Adapted from An Account of the Remarkable Occurrences in the Life and Travels of Col. James Smith . . . (1799; ed. Glugg & Elliott, 1834)

After the departure of these warriors we had hard times; and though we were not altogether out of provisions, we were brought to short allowance. At length Tontileaugo had considerable success, and we had meat brought into camp sufficient to last ten days. Tontileaugo then took me with him in order to encamp some distance from this winter-cabin, to try his luck there. We carried no provisions with us; he said he would leave what was there for the women and children, and that we could shift for ourselves. We steered about a south course up the waters of this creek, and encamped about ten or twelve miles from the winter-cabin. As it was still cold weather, and a crust upon the snow, which made a noise as we walked, and alarmed the deer, we could kill nothing, and consequently went to sleep without supper. The only chance we had, under these circumstances, was to hunt bear-holes; as the bears, about Christmas, search out a winter lodging-place, where they lie about three or four months without eating or drinking. This may appear to some incredible, but it is well known to be the case by those who live in the remote western parts of North America.

The next morning early we proceeded on, and when we found a tree scratched by the bears climbing up, and the hole in the tree sufficiently large for the reception of the bear, we then felled a sapling or small tree against or near the hole, and it was my business to climb up and drive out the bear, while Tontileaugo stood ready with his gun and bow. We went on in this manner until evening without success. At length we found a large elm scratched, and a hole in it about forty feet up, but no tree high suitable to lodge against the hole. Tontileaugo got a long pole and some dry rotten wood, which he tied in bunches with bark; he climbed up a nearby tree and carried with him his rotten wood, fire, and pole. The rotten wood he tied to his belt, and to one end of the pole he tied a hook and a piece of rotten wood, which he set fire to, as it would retain fire almost like punk, and reached this hook from limb to limb as he went up. When he got up with his pole he put dry wood on fire into the hole; after he put in the fire he heard the bear snuff, and he came speedily down, took his gun in his hand, and waited until the bear would come out; when it did appear he attempted taking sight with his rifle; but it being then too dark to see the sights, he set it down by a tree, and instantly bent his bow, took hold of an arrow, and shot the bear a little behind the shoulder. I was preparing also to shoot an arrow, but he called to me to stop, there was no occasion; and with that the bear fell to the ground.

1. Based on context, what is the meaning of the underlined clause in line 3, “brought to short allowance?”
 - A. saved money by doing something a certain way
 - B. given less money by a group due to one’s actions
 - C. forced to ration supplies out of necessity
 - D. forced to feed more people with the same amount of supplies
2. Based on context, what is the meaning of the underlined word in line 10, “shift?”
 - A. move slightly
 - B. fend
 - C. time worked at a job on a given workday
 - D. a change in position or opinion
3. Based on context, what is the meaning of the underlined word in line 40, “punk?”
 - A. Wood so decayed that it is good for tinder
 - B. Wood that burns quickly when set alight
 - C. Bark used to tie firewood together
 - D. Fresh green wood that holds a flame well
4. Based on context, what is the meaning of the underlined phrase in line 47, “to see the sights?”
 - A. To scout an area with military strategy in mind
 - B. To peruse an area to find materials and supplies
 - C. To see one’s target through the scope of a gun
 - D. To see areas of local interest while traveling
5. Based on context, what is the meaning of the underlined word in line 51, “occasion?”
 - A. need
 - B. reason motivating a celebration
 - C. prompt
 - D. hurry



Practice Passage: Context-Dependent Meaning: Answer Key

1. Based on context, what is the meaning of the underlined clause in line 3, “brought to short allowance?”

- A. saved money by doing something a certain way
- B. given less money by a group due to one's actions
- C. forced to ration supplies out of necessity
- D. forced to feed more people with the same amount of supplies

Let's look at the passage's first sentence, where the indicated phrase appears:

After the departure of these warriors we had hard times; and though we were not altogether out of provisions, we were brought to short allowance.

“Hard times” clues us in that the phrase likely has a negative meaning, as the two appear to be causally related. Money isn't what is causing the narrator and his group problems, though—it's specifically the low amount of “provisions.” Recognizing that nowhere in the passage does the narrator mention money allows us to knock out answer choices A and B. Even though “allowance” can refer to *an amount of money given regularly*, this isn't how the term is being used in this passage.

Considering C and D, the answer choices that have to do with the amount of supplies at hand, you can note that D doesn't make sense. The sentence quoted above refers to “the departure of these warriors,” so the narrator's group presumably has to feed fewer people, not more people. The troubles with the amount of available supplies don't appear to be related to an increase in the number of people that rely on them. C is the best answer, conveying “allowance” as the providing of an allowance of food, or rationing.

2. Based on context, what is the meaning of the underlined word in line 10, “shift?”

- A. move slightly
- B. fend
- C. time worked at a job on a given workday
- D. a change in position or opinion

Part of the following clause appears in line 10:

... he said he would leave what was there for the women and children, and that we could shift for ourselves.

Again, food and supplies (“provisions”) form are topic of this conversation. Tontileaugo says that he will leave the provisions with other people and that he and the narrator can “shift” for themselves. “Shift” is being used as a verb, so neither C nor D can be correct. While each answer choice could refer to a potential meaning of “shift,” both of these meanings are nouns. A, “Move slightly,” doesn't make sense in the context of the passage, but B, “fend,” does. The narrator and Tontileaugo can “fend for themselves” by going hunting, which is what happens in the rest of the text.



3. Based on context, what is the meaning of the underlined word in line 40, “punk?”

- ➔ A. Something that it is good for tinder as it burns slowly
B. Wood that burns quickly when set alight
C. Bark used to tie firewood together
D. Fresh green wood that holds a flame well

4. Based on context, what is the meaning of the underlined phrase in line 47, “to see the sights?”

- A. To scout an area with military strategy in mind
B. To peruse an area to find materials and supplies
➔ C. To see one’s target through the scope of a gun
D. To see areas of local interest while traveling

5. Based on context, what is the meaning of the underlined word in line 51, “occasion?”

- ➔ A. need
B. reason motivating a celebration
C. evoke
D. hurry

Here is the line that appears in the indicated sentence:

The rotten wood he tied to his belt, and to one end of the pole he tied a hook and a piece of rotten wood, which he set fire to, as it would retain fire almost like punk, and reached this hook from limb to limb as he went up.

“Punk” is compared to the “piece of rotten wood, which he set fire to,” because the rotten wood is like “punk” in that “it would retain fire almost like punk.” Note that we are not specifically told that “punk” is a type of wood, merely that the discussed rotten wood is *like* punk. The best answer choice is A, as it is the only one that specifies that punk is something that “burns slowly.”

Here is the relevant portion of the passage:

... [Tontileaugo] heard the bear snuff, and he came speedily down, took his gun in his hand, and waited until the bear would come out; when it did appear he attempted taking sight with his rifle; but it being then too dark to see the sights, he set it down by a tree, and instantly bent his bow, took hold of an arrow, and shot the bear . . .

“To see the sights” could be used to describe seeing areas of local interest while traveling, but that’s not what’s going on here, so D is incorrect. The situation isn’t military, either, so A is incorrect as well. While provisions are a topic of the passage, Tontileaugo and the narrator are not looking around the area for materials and supplies in the scene in which the phrase is used. Instead, Tontileaugo is aiming his rifle at the bear, but it is “too dark” to “see the sights.” This means that “seeing the sights” in this passage has to refer to something to do with vision, as you can’t do it if it’s “too dark.” C is the correct answer.

“Occasion” can be used as a noun or a verb, so let’s look at how it’s used in the passage:

I was preparing also to shoot an arrow, but [Tontileaugo] called to me to stop, there was no occasion; and with that the bear fell to the ground.

The clause “There was no occasion” indicates that “occasion” is used as a noun. This means that C, “evoke,” can’t be correct. D is incorrect because “hurry” is not a meaning of “occasion” and doesn’t make sense in context because it would suggest that the narrator would still need to shoot at the bear, just that he could do it at a slower pace, when he doesn’t have to shoot at the bear at all. The word’s context has nothing to do with celebrations (and there arguably *would* be reason for a celebration after they successfully killed the bear!) so B is incorrect. This leaves us with A, the correct answer: there was no need for the narrator to shoot at the bear because Tontileaugo already shot and killed it.



Analyzing Tone

“It’s not just how what you say, it’s how you say it,” is just one of a litany of wisdoms told to young children whose fundamental truth, and utility, stretches from the school yard to the grave. One of the many instances in which this advice will help you is when you are taking the ACT Reading section. ACT Reading questions will ask you to account for the content, context, and meaning of the reading passages with which they present you, and they will also ask you to answer questions based on the tone of the passage.

In reading comprehension and analysis, we use “tone” to refer to the author’s attitude towards what s/he is speaking about, as conveyed by the author’s choice of words and literary techniques. The more common meaning of tone, which refers to the quality of a person’s voice. For instance, one can radically alter the meaning of the phrase “thank you” using only the tone in which one says it. “Thank you,” if said sincerely is polite, but if it is said in a sneering or sarcastic manner, it can be deeply rude.

Although we cannot hear an author’s voice as we read, tone is an equally vital aspect in written discourse. An author’s diction (word choice), use of comparisons, punctuation, and, in particular, their characterization of people or subjects can all help you to determine the author’s tone or attitude towards the subject matter.

Tone of a Sentence or Paragraph

As with content questions on the ACT Reading exam, questions about a passage’s tone will sometimes specify a particular section, paragraph, or sentence for analysis. Just as the subject and content of a passage may shift from one paragraph to another, so too may the tone.

The methods for analyzing the tone a sentence or section within a passage are fundamentally similar to the methods for analyzing the overall tone of the passage; you will be looking for the same clues and signals of tone, you will simply be doing it on a more focused, or more global scale depending on the question.

The first thing to determine when approaching questions about an author’s tone is to decide whether you think the author is putting forward a positive, negative, or neutral attitude towards the subject they are discussing (or the subject the question specifies). Then, once you’ve decided that, you need to move on to deciding how strongly the author is putting forward that attitude. The questions will ask you to determine not just the basic kind of tone, but the strength of that tone. For instance, if I say that my friend Roger is “a nice man,” I’m speaking of him in a positive manner. If I say that Roger is “an outstanding gentleman,” I am still speaking of him positively, but I am doing so in a stronger, more effusive manner. The best clues to figuring out the tone of a sentence or paragraph are the words, and kinds of words, the author chooses. Words like “egregious,” “amazing,” “exemplary,” and “tragic” are all strong words expressing a strong, non-objective feeling towards the subject. As you go through a paragraph or passage, make note of any words that stand out to you as expressing a strong, or a measured, attitude towards the subject matter.



Practice Passage:

Adapted from “Concerning Tobacco” in *What is Man? And Other Essays* by Mark Twain (1906)

As concerns tobacco, there are many superstitions. And the chiefest is this—that there is a standard governing the matter, whereas there is nothing of the kind. Each man’s own preference is the only standard for him, the only one which he can accept, the only one which can command him. A congress of all the tobacco-lovers in the world could not elect a standard which would be binding upon you or me, or would even much influence us.

The next superstition is that a man has a standard of his own. He hasn’t. He thinks he has, but he hasn’t. He thinks he can tell what he regards as a good cigar from what he regards as a bad one—but he can’t. He goes by the brand, yet imagines he goes by the flavor. One may palm off the worst counterfeit upon him; if it bears his brand he will smoke it contentedly and never suspect.

Children of twenty-five, who have seven years of experience, try to tell me what is a good cigar and what isn’t. Me, who never learned to smoke; me, who came into the world asking for a light.

Sample Question

The author’s tone in the first paragraph is best described as _____.

- A. strident
- B. nostalgic
- C. irreverent
- D. objective

The first step to determining the tone of this paragraph is to ascertain what the author is characterizing, and how he is characterizing them. The tone, initially, seems somewhat academic or objective (“As concerns tobacco, there are many superstitions”), but we quickly realize that “the matter” the author is talking about is not a scientific, nor academic one, but rather a matter strictly of preference. Thus, the author’s use of a faux-academic tone is not intended to mock those who would attempt to apply such standards to a matter of preference. The belief he so quickly and easily classifies as a “superstition” is, in fact, the presence of any objective standard of quality for tobacco. The author’s invocation of an imagined “congress of all tobacco-lovers” is intended to mock the very idea of imposition of standards onto the practice of choosing tobacco by showing it to be ridiculous. The tone here is not strident (expressing doubt or misgiving in a desperate or extremely urgent manner), nor is it nostalgic (there is no reference to an idealized past), it is irreverent. The attitude is irreverent because it is treating the idea of “standards governing” the matter at hand, not as important standards at all (as a reverent attitude towards those standards most certainly would), but rather as mere superstitions that can easily be debunked.



Sample Question

In the first sentence of the third paragraph, the author's use of the phrase "children of twenty-five" signals a(n) _____ tone.

- A. angry
- B. effusive
- C. ambivalent
- D. mocking

Here, the question is providing us with the exact phrase that creates the tone they wish us to identify. This is somewhat of a reverse-engineered question, we begin with the clue telling us about the author's tone, and then must identify the tone from the clue with which the question provides us. A good way to identify the tone here is to think about other words the author could have used instead of 'children.' The most neutral such term would have been "persons," which would have taken no stance on the maturity of those discussed. Instead the author specifies an age that is clearly within the bounds of adulthood (or at least young-adulthood), and asserts that the twenty-five year olds he is talking about are not even adolescents, but "children." The specific choice of such non-conventional language rules out "ambivalent," as an option, as the author would not choose such a specific and notable phrase to describe persons about whom he had mixed and undecided feelings. Nor, does this choice signal an effusive attitude, calling a twenty-five year old a child is hardly praise, especially not exuberant, bountiful praise. This leaves us with "angry" and "mocking" as options, and both are possible, the key here is to select the most direct and accurate option. More than anything, the use of this expression is intended to mock the twenty-five year olds, anger is certainly a possible motivation for that, but mockery is more certain, and more directly applicable. The overall tone of the passage is light, so it would take a stronger, more pointed phrase to signal anger over mockery.



Tone of an Entire Passage

While questions interrogating specific passages are designed to test your ability to identify tone based on the specific tools an author uses to convey their attitude towards a subject, questions interrogating the tone of the entire passage ask you to make a judgment about the prevailing tone of the entire passage overall. A passage may shift in tone depending on the subject being discussed. For instance, say I am writing an essay on baseball, I may have one paragraph about pitching (which is a subject I am passionate and effusive about), one paragraph about hitting (which is a subject that I find irritating, and enjoy mocking), and one paragraph about fielding (which is a subject that I find fascinating). If a question asked you about the tone of the entire essay, you would have to look at the introduction of the topic, the transition between subjects, and the overall balance of techniques and language surrounding the overriding main topic of baseball in order to answer accurately.

Practice Passage

Adapted from “Feathers of Sea Birds and Wild Fowl for Bedding”
from *The Utility of Birds* by Edward Forbush (ed. 1922)

In the colder countries of the world, the feathers and down of waterfowl have been in great demand for centuries as filling for beds and pillows. Such feathers are perfect non-conductors of heat, and beds, pillows, or coverlets filled with them represent the acme of comfort and durability. The early settlers of New England saved for such purposes the feathers and down from the thousands of wild-fowl which they killed, but as the population increased in numbers, the quantity thus furnished was insufficient, and the people sought a larger supply in the vast colonies of ducks and geese along the Labrador coast.

The manner in which the feathers and down were obtained, unlike the method practiced in Iceland, did not tend to conserve and protect the source of supply. In Iceland, the people have continued to receive for many years a considerable income by collecting eider down, but there they do not “kill the goose that lays the golden eggs.” Ducks line their nests with down plucked from their own breasts and that of the eider is particularly valuable for bedding. In Iceland, these birds are so carefully protected that they have become as tame and unsuspicious as domestic fowls. In North America, where they are constantly hunted they often conceal their nests in the midst of weeds or bushes, but in Iceland, they make their nests and deposit their eggs in holes dug for them in the sod. A supply of the ducks is maintained so that the people derive from them an annual income.

In North America, quite a different policy was pursued. The demand for feathers became so great in the New England colonies about the middle of the eighteenth century that vessels were fitted out there for the coast of Labrador for the express purpose of securing the feathers and down of wild fowl. Eider down having become valuable and these ducks being in the habit of congregating by thousands on barren islands of the Labrador coast, the birds became the victims of the ships' crews. As the ducks molt all their primary feathers at once in July or August and are then quite incapable of flight and the young birds are unable to fly until well grown, the hunters were able to surround the helpless birds, drive them together, and kill them with clubs. Otis says that millions of wildfowl were thus destroyed and that in a few years their haunts were so broken up by this wholesale slaughter and their numbers were so diminished that feather voyages became unprofitable and were given up.

This practice, followed by the almost continual eggging, clubbing, shooting, etc. by Labrador fishermen, may have been a chief factor in the extinction of the Labrador duck, that species of supposed restricted breeding range. No doubt had the eider duck been restricted in its breeding range to the islands of Labrador, it also would have been exterminated long ago.



Sample Question

Based on the author's tone, we can tell that he thinks that _____.

- A. the Labrador feather voyages shouldn't have given up so easily
- B. the Icelandic way of collecting eider down is preferable to the practices employed in North America
- C. Icelandic eider ducks have become too tame
- D. the use of eider down in bedding is a barbaric practice

This is a species of tone question you will, on occasion, encounter on the ACT Reading section. The question asks you to use your understanding of the author's overriding tone to evaluate a hypothetical statement, and to determine what the author's attitude towards the hypothetical statement. Throughout the passage the author describes the methods of collecting down in North America in a way that makes them seem barbaric and casts the ducks as the helpless victims of the hunters. Because of this, we can eliminate the answer choice A that suggests the author would have encouraged the continuation of the Labrador feather voyages. While the author does suggest that eider ducks have become tame, he does not make any value judgment on the extent of this trait, so the option suggesting that they are "too tame" (answer choice C) is not supported. Similarly, while he describes the use of eider down in bedding materials and suggests that the North American methods of collecting down are barbaric, he doesn't suggest that the use of eider down as a whole is a barbaric practice; he seems to support the collection of down using the Icelandic methods that don't hurt the ducks. If you were looking for one particular sentence with which to justify these assertions about the author's attitude towards the two methods of obtaining down, "The manner in which the feathers and down were obtained, unlike the method practiced in Iceland, did not tend to conserve and protect the source of supply," from the beginning of the second paragraph would be the most acute example of this overall tone and attitude.

Sample Question

The author's tone, overall, through the passage is best described as _____.

- A. academic
- B. flippant
- C. indifferent
- D. reverent

Here, we are asked straightforwardly to describe the overall tone of the passage. We can eliminate "flippant" pretty easily as the author's treatment of the subject matter is studied, and appears educated and serious. As we saw in the previous question the author is not "indifferent," as he shows a clear preference for the Icelandic practices over the North American ones, and his reference to the extinction of the Labrador duck shows that he is aware of the gravity of the situation he describes. While the author does favor the Icelandic practice, the tone is not strictly reverent, it is analytical, or you might say "academic." Other clues for the academic tone are the elevated diction, and the citation of evidence in support of all the author's claims.



Words That Describe Tone

The ACT Reading section uses a number of adjectives to describe an author's tone with which students may not be familiar. Here, we'll provide you with a list of some of the more commonly used ACT Reading tone adjectives that may not be so commonly used elsewhere. Familiarizing yourself with these words will help prevent the unfortunate situation of not knowing what some of the answer options mean.

Didactic describes an author who is giving instructions. Didactic can describe a teacher's tone as they explain a lesson. Watch out for a lot of imperatives (e.g. "One will see...") in didactic passages. If an author is didactic you might describe them as "bossy."

Strident describes passages that express their message in an urgent, almost desperate manner. There is a difference between a critical tone (one which analyzes and finds fault with something, but discusses it in a clear, relatively objective manner) and a strident tone. You might describe an author as "self-righteous" if you found their tone to be strident.

Sentimental, like strident, describes an author who is emotional about the subject, rather than expressing an objective, analytical attitude. The difference is that a sentimental tone is not negative, rather it tends to indulge in overly positive portrayals of events or people discussed. A related term to this is "**effusive**," which refers to a tone which "gushes" or extravagantly expresses (usually positive) emotion about the subject.

Contemplative describes a thoughtful, relaxed tone. A contemplative tone coolly analyzes an interesting issue. This can be seen as similar to an objective tone (a tone which seeks to remove bias and emotion from the discussion), the difference here is that a contemplative tone can be personal, and talk about the author's feeling, it just does so in a calm, thoughtful manner.

Ambivalent describes the tone of an author who is of, at least, two minds about their subject. If, say, an author foregrounds doubts about one side of the issue they're talking about, and then proceed to express doubts about the other side of that issue, their tone is ambivalent. Ambivalence is characterized by uncertainty.

Nostalgic describes the tone of a passage that treats the past in a romanticized manner. Often passages describing a prior time from the author's life will be considered nostalgic (as long as that time is characterized in a positive, wistful manner)

Somber describes a serious and perhaps depressing, gloomy tone.

Bewildered describes the tone of an author who is puzzled by the subject matter. The authors in these cases will not be ambivalent, rather they will not understand the basic workings of what they are talking about. Often authors are bewildered by the behavior of a person they are talking about.

Contemptuous describes the tone of an author who disdains, looks down on, and dislikes a subject they are talking about. It is similar in meaning to another common adjective in the ACT Reading: "condescending." A condescending tone is one that clearly situates the author as "above" the subject matter and the reader.

Flippant describes the tone of an author who is not taking the subject matter seriously.

Macabre describes the tone of a passage that is concerned with dark things, usually death. Edgar Allan Poe passages are often described as "macabre."

Ribald describes the tone of an author who uses crude or offensive expressions, usually for humorous effect.



Passage A: Humanities:

Adapted from *The Prince* by Nicholas Machiavelli (1513; trans. Marriott, 1908)

A prince ought to have no other aim or thought nor select anything else for his study than war and its rules and discipline, for this is the sole art that belongs to him who rules. The first cause of your losing a state is to neglect this art; and what enables you to acquire a state is to be master of the art. Francesco Sforza, through being martial, from a private person became Duke of Milan, and the sons, through avoiding the hardships and troubles of arms, from dukes became private persons. For among other evils that being unarmed brings you, it causes you to be despised, and this is one of those ignominies against which a prince ought to guard himself, as is shown later on.

1. The tone of Passage A is best described as _____.

A. sentimental
B. somber
C. didactic
D. macabre

Passage B: Humanities:

Adapted from "The Decay of Friendship" in Issue 23 of *The Idler* by Samuel Johnson (September 23rd, 1758)

Life has no pleasure higher or nobler than that of friendship. It is painful to consider that this sublime enjoyment may be impaired or destroyed by innumerable causes, and that there is no human possession of which the duration is less certain.

Many have talked in very exalted language, of the perpetuity of friendship, of invincible constancy, and unalienable kindness; and some examples have been seen of men who have continued faithful to their earliest choice, and whose affection has predominated over changes of fortune, and contrariety of opinion.

But these instances are memorable, because they are rare. The friendship which is to be practiced or expected by common mortals, must take its rise from mutual pleasure, and must end when the power ceases of delighting each other.

Many accidents therefore may happen by which the ardor of kindness will be abated, without criminal baseness or contemptible inconstancy on either part. To give pleasure is not always in our power; and little does he know himself who believes that he can be always able to receive it.

Those who would gladly pass their days together may be separated by the different course of their affairs; and friendship, like love, is destroyed by long absence, though it may be increased by short intermissions. What we have missed long enough to want it, we value more when it is regained; but that which has been lost till it is forgotten, will be found at last with little gladness, and with still less if a substitute has supplied the place. A man deprived of the companion to whom he used to open his bosom, and with whom he shared the hours of leisure and merriment, feels the day at first hanging heavy on him; his difficulties oppress, and his doubts distract him; he sees time come and go without his wonted gratification, and all is sadness within, and solitude about him. But this uneasiness never lasts long; necessity produces expedients, new amusements are discovered, and new conversation is admitted.

2. The tone of Passage B is best described as _____.

A. serious and solemn
B. laudatory and playful
C. pedantic and harsh
D. poetic and quixotic

3. The author's tone in the third paragraph of Passage B is best described as _____.

A. contemptuous
B. definitive
C. flippant
D. bewildered



Practice Passages: Analyzing Tone: Answer Key

1. The tone of Passage A is best described as _____.

A. sentimental
B. somber
→ C. didactic
D. macabre

The best clue in this passage is the frequent use of “ought.” Anyone who is repeatedly saying what a type of person “ought” do, and this construction appears twice in the passage, can probably be described as “didactic.” The author is here laying down maxims for what princes “ought” do in order to maintain power, and the potential “causes” of “losing a state.” We can also arrive at the answer. The tone here, discussing the “avoiding the hardships and trouble of arms” as just obstacles that must be overcome is hardly “sentimental.” The tone is never sad, so “somber” is inappropriate. While revolutions and “losing a state” are mentioned, and these are potentially violent, the violence nature is not specified, and the passage would have to be specifically focused on “macabre” aspects of the subject matter in order for that term to be an appropriate description of the tone.

2. The tone of Passage B is best described as _____.

→ A. serious and solemn
B. laudatory and playful
C. pedantic and harsh
D. poetic and quixotic

The first step to answering this question is to accurately define each of the eight terms used in the answer options. A “serious” tone is one that treats the subject matter with gravity; a “solemn” tone is one that treats the matter in a somber or grave manner. A “laudatory” tone is one that expresses praise; a “playful” tone is one that treats the subject in a light and lively manner. A “pedantic” tone treats the manner in a “nit-picky” overly detail-oriented manner; a “harsh” tone treats the subject matter in an unsparing fashion. A “poetic” tone emphasizes poetic imagery or modes of thinking; a “quixotic” tone is extremely idealistic and optimistic.

So, given these definitions, we can see that “serious and solemn” is the best answer here. The author treats friendship in a serious and positive manner, emphasizing that there “is no higher pleasure” than the one derived from friendship. He is also, however, deeply conscious of the “accidents” and “criminal baseness or contemptible inconstancy” that endanger all friendships, and end many. While the tone is laudatory (of friendship as a concept) it is hardly playful. Solemn is a much better description of the tone than is harsh, and the passage is not pedantic. While the passage is written in a somewhat poetic (or at least beautifully written), the tone is too melancholic to be considered quixotic.

3. The author's tone in the third paragraph of Passage B is best described as _____.

→ A. contemptuous
B. definitive
C. flippant
D. bewildered

The key to answering this question is noticing the abundance of the word “must” in the paragraph specified. Anytime one repeats what “must” happen or be done, one must consider the adjective “definitive” to describe the tone (as they are making very definite statements). A “bewildered” author will not be making such strong, definitive claims. A “contemptuous” will almost always feature pointed, direct condemnation, and that is not the case here.



Point of View

Every author is sure to have opinions about the topics he or she writes about, and those opinions form important parts of certain passages. Beyond being able to recognize the use of first-person or third-person point of view, being able to pick up on cues littered throughout the text and discern how the author feels about various concepts is a skill that the ACT Reading section can also quiz you on, so it's best to be as prepared as possible!

Practice Passage

Natural Science: Adapted from “Introduced Species That Have Become Pests” in *Our Vanishing Wild Life, Its Extermination and Protection* by William Temple Hornaday (1913)

The man who successfully transplants or “introduces” into a new habitat any persistent species of living thing assumes a very grave responsibility. Every introduced species is doubtful gravel until panned out. The enormous losses that have been inflicted upon the world through the perpetuation of follies with wild vertebrates and insects would, if added together, be enough to purchase a principality. The most aggravating feature of these follies in transplantation is that never yet have they been made severely punishable. We are just as careless and easygoing on this point as we were about the government of the Yellowstone Park in the days when Howell and other poachers destroyed our first national bison herd, and when caught red-handed—as Howell was, skinning seven Park bison cows—could not be punished for it, because there was no penalty prescribed by any law. Today, there is a way in which any revengeful person could inflict enormous damage on the entire South, at no cost to himself, involve those states in enormous losses and the expenditure of vast sums of money, yet go absolutely unpunished!

gust of wind blew out of his study, through an open window, his living and breeding specimens of the gypsy moth. The moth itself is not bad to look at, but its larvae is a great, overgrown brute with an appetite like a hog. Immediately Mr. Trouvelot sought to recover his specimens, and when he failed to find them all, like a man of real honor, he notified the State authorities of the accident. Every effort was made to recover all the specimens, but enough escaped to produce progeny that soon became a scourge to the trees of Massachusetts. The method of the big, nasty-looking mottled-brown caterpillar was very simple. It devoured the entire foliage of every tree that grew in its sphere of influence.

The gypsy moth spread with alarming rapidity and persistence. In course of time, the state authorities of Massachusetts were forced to begin a relentless war upon it, by poisonous sprays and by fire. It was awful! Up to this date (1912) the New England states and the United States Government service have expended in fighting this pest about \$7,680,000!

The spread of this pest has been retarded, but the gypsy moth never will be wholly stamped out. Today it exists in Rhode Island, Connecticut, and New Hampshire, and it is due to reach New York at an early date. It is steadily spreading in three directions from Boston, its original point of departure, and when it strikes the State of New York, we, too, will begin to pay dearly for the Trouvelot experiment.

30 The gypsy moth is a case in point. This winged calamity was imported at Maiden, Massachusetts, near Boston, by a French entomologist, Mr. Leopold Trouvelot, in 1868 or 69. History records the fact that the man of science did not purposely set free the pest. He was endeavoring with live specimens to find a moth that would produce a cocoon of commercial value to America, and a sudden

35



Point of View

Questions that concern point of view often focus on the author's opinions, but they can also ask you to make generalizations about the author's background based on what he or she knows, or what type of audience for which the passage is intended. Picking out explicit statements of authorial opinion on a subject can also form the basis of a question, and you can also be asked to take a few more critical-thinking steps and infer how he or she likely feels about a topic relevant to the passage or predict how he or she would react to a new development. For these questions, try to imagine yourself in the author's shoes, maybe like a character in a play. How does he or she feel about the topics he or she is writing about? What topics would you bring up or avoid in conversation if you were chatting with him or her? Briefly considering the author's opinions from a broad perspective can help you realize when an answer choice sounds reasonable or is completely inaccurate.

Identifying Author and Audience

Let's consider two sample questions: one that asks you about the author, and another that asks you about the audience of the passage.

Sample Question

The author of this passage is most likely _____.

- A.** a scientist who studies insects
- B.** an environmentalist
- C.** a hunter
- D.** a cattle farmer

We can ignore answer choice D; while the author mentions the national bison herd once found in Yellowstone National Park, he gives no indication that he himself is a cattle farmer. Furthermore, given his negative reaction to Howell's poaching of the national bison herd, it is unlikely that the author is a hunter. While he wants to rid the U.S. of the gypsy moth, he wants to do so for reasons motivated by environmental preservation, not hunting for sport. (Also, one doesn't typically "hunt" moths.)

At this point, we're left with two science-themed answers: A and B. The question becomes one of specificity and detail. M. Trouvelot, the person who originally accidentally released the gypsy moth, is an entomologist, or a scientist who studies insects. Don't conflate him with the author of the passage, though: just because the author is talking about bugs doesn't mean that he is an expert. The author is relaying a story about the introduction of the gypsy moth to the U.S.; nothing in the passage suggests that he has expert-level entomology knowledge. The author introduces the passage by talking about the introduction of non-native species in general, and describes the situation with Howell and the national bison herd (not bugs!) before moving on to discussing the gypsy moth as another example. Because the passage is not entirely focused on bugs and the author doesn't provide any specific information about gypsy moths that would suggest he is a scientist who studies insects, B is the correct answer. The author's focus on the severity of the environmental problems caused by the introduction of non-native species makes "an environmentalist" a good descriptor for him.



Sample Question

This passage's intended audience is most likely which of the following?

- A. Farmers
- B. Scientists
- C. Lawmakers
- D. The general public

Note again that the author doesn't go into scientific detail when describing the gypsy moth; he uses casual language to describe it in the third paragraph: "The moth itself is not bad to look at, but its larvae is a great, overgrown brute with an appetite like a hog." By comparing the larvae's appetite to "a hog," the author is comparing it something the audience will find more familiar. Based on the lack of jargon or specific details, we can presume that the author isn't writing for an audience of scientists, for whom such a general introduction of the moth would be unnecessary. Similar arguments can be constructed to discount answers A and C—while the author mentions things relevant to these groups (the moth might potentially damage farmers' crops and the lack of laws surrounding the introduction of non-native species would be relevant to lawmakers), none of the information seems directed at these groups specifically. The passage is written for a general audience, to introduce them to the problems non-native species of plants and animals can cause. D is thus the best answer.

Identifying the Author's Directly Stated Views

Let's start with questions that provide a lot of direction about the author's views as stated in the passage and gradually move to those that require more inference, prediction, and general extrapolation. For the first question, we'll look at a question that directs you to a specific part of the passage and asks about a specific subject.

Sample Question

The description of the gypsy moth caterpillar found in the passage's second paragraph suggests that the author _____ it.

- A. adores
- B. detests
- C. misunderstands
- D. underestimates

How does the author describe the gypsy moth caterpillar in the second paragraph? Well, we can tell he's not very fond of it at all because he says, "The moth itself is not bad to look at, but its larvae is a great, overgrown brute with an appetite like a hog." Similarly, at the end of the paragraph, he writes, "Every effort was made to recover all the specimens, but enough escaped to produce progeny that soon became a scourge to the trees of Massachusetts. The method of the big, nasty-looking mottled-brown caterpillar was very simple. It devoured the entire foliage of every tree that grew in its sphere of influence." Based on the strong



negative language the author uses when discussing the gypsy moth caterpillars and the damage they cause, we can pick out B, “detests” as the correct answer.

Next, let's take a look at a question that asks you to figure out the author's opinions that are stated explicitly somewhere in the passage, but doesn't tell you where you should start looking for them.

Sample Question

How does the author feel about Howell?

- A.** The author agrees with Howell that invasive species are often problematic.
- B.** The author greatly dislikes Howell for his audacious disrespect for nature.
- C.** The author thinks that Howell made a great mistake in releasing Gypsy moths into the United States.
- D.** The author likes Howell because he helped identify a problem with the consequences available for environmental disruptors.

Let's look at the part of the first paragraph in which the author brings up Howell, paying attention to why he does so:

The most aggravating feature of these follies in transplantation is that never yet have they been made severely punishable. We are just as careless and easygoing on this point as we were about the government of the Yellowstone Park in the days when Howell and other poachers destroyed our first national bison herd, and when caught red-handed—as Howell was, skinning seven Park bison cows—could not be punished for it, because there was no penalty prescribed by any law.

In mentioning Howell, the author is providing an example supporting his argument that harsher legal penalties are necessary for those who harm the environment. The author describes Howell as a “poacher” who “destroyed our first national bison herd” and was “caught red-handed.” From this, we can tell that the best answer choice is B, “the author greatly dislikes Howell for his audacious disrespect for nature.”

One of the other answer choices attempts to get you to confuse Howell with Mr. Trouvelot, who released the gypsy moths—don't fall for that! Check the passage if you are worried at all about confusing the two so you can avoid pitfall answers like that one.

Finally, we'll look at one more question of this same type, but one that doesn't ask about a particular subject, instead directing you to consider the passage as a whole.



Sample Question

Which of the following best describes an opinion held by the author?

- A. Despite spending a great deal of money, the United States will never be rid of the gypsy moth.
- B. Efforts to contain the gypsy moth will improve as technology improves, until all of the moths in the United States have been eradicated.
- C. We should introduce a new species of animal that eats gypsy moths to combat the problems they cause.
- D. It is difficult to say what the future holds for the fate of the gypsy moth in the United States.

The first sentence of the passage's last paragraph provides the information we need to answer this question correctly: the author writes, "The spread of this pest has been retarded, but the gypsy moth never will be wholly stamped out." We can thus definitively say that he thinks that "despite spending a great deal of money, the United States will never be rid of the gypsy moth." This makes A the correct answer.

Inferring the Author's Views on a Topic

Continuing the trend of moving away specific questions that direct you to explicitly stated material to those that provide less information, let's now make the leap to considering those that require inferences. These questions will ask you to draw conclusions about how the author would most likely react to a specific situation given what he or she has expressed in the passage. Let's take a look at a conditional hypothetical, "if-then"-style question:

Sample Question

If the author were to learn that the gypsy moth could be efficiently repelled from trees by coating them with a cheap, natural substance, he would likely feel _____.

- A. exuberant
- B. horrified
- C. unsurprised
- D. anxious

Throughout the passage, the author makes it apparent that he feels that the gypsy moth is a very damaging invasive species that causes a lot of problems in the United States. He calls it a "winged calamity" and, in the third paragraph, describes how it spread:

The gypsy moth spread with alarming rapidity and persistence. In course of time, the state authorities of Massachusetts were forced to begin a relentless war upon it, by poisonous sprays and by fire. It was awful! Up to this date (1912) the New England states and the United States Government service have expended in fighting this pest about \$7,680,000!

From this paragraph, we can tell that if the author were to learn that the gypsy moth could be efficiently stopped from damaging trees, he would be most likely to feel "exuberant," or *excited and happy*. Nothing in the passage supports any of the other answers.



Let's look at one more question, this one asking about which action of the author would be most likely to recommend.

Sample Question

Based on the first paragraph, the author would be most likely to support _____.

- A.** a law severely punishing those who introduce invasive species that damage the environment
- B.** keeping bison out of Yellowstone National Park
- C.** introducing damaging invasive species to the South
- D.** granting Howell clemency for his actions

One of the author's main points in the first paragraph is that harsher legal repercussions are needed for those who release damaging invasive species into the United States. This is clear when the author writes, "The most aggravating feature of these follies in transplantation is that never yet have they been made severely punishable." Thus, we can infer that the author would be most likely to support "a law severely punishing those who introduce invasive species that damage the environment." Though the author does discuss the potential for someone to introduce invasive species to the South, he is not in favor of this, and he clearly doesn't want to grant Howell clemency for his actions. (Furthermore, "clemency" somewhat implies that Howell has been charged with a crime, and the author explains that this isn't the case.)

The author does state, "The enormous losses that have been inflicted upon the world through the perpetuation of follies with wild vertebrates and insects would, if added together, be enough to purchase a principality," and we can therefore assume that he might support cataloging the amount of money invasive species have cost the United States. However, this inference requires a much larger logical leap than does the one that the author would support harsher legal punishments for those who introduce damaging invasive species, making "a law severely punishing those who introduce invasive species that damage the environment" the best answer. If you're unsure when picking between answers to an inference question, it's usually a good idea to see which one is more relevant to the passage's topic and has the most evidence supporting it.



Practice Passages: Point of View**Passage A**

Humanities: Adapted from “Aristotle at Afternoon Tea” by Oscar Wilde (1887)

In society, says Mr. Mahaffy, every civilized man and woman ought to feel it their duty to say something, even when there is hardly anything to be said, and, in order to encourage this delightful art of brilliant chatter, he has published a social guide without which no debutante or dandy should ever dream of going out to dine. Not that Mr. Mahaffy's book can be said to be, in any sense of the word, popular. In discussing this important subject of conversation, he has not merely followed the scientific method of Aristotle which is, perhaps, excusable, but he has adopted the literary style of Aristotle for which no excuse is possible. There is, also, hardly a single anecdote, hardly a single illustration, and the reader is left to put the professor's abstract rules into practice, without either the examples or the warnings of history to encourage or to dissuade him in his reckless career. Still, the book can be warmly recommended to all who propose to substitute the vice of verbosity for the stupidity of silence. It fascinates in spite of its form and pleases in spite of its pedantry, and is the nearest approach, that we know of, in modern literature to meeting Aristotle at an afternoon tea.

1. How does the author of Passage 1 feel about Aristotle's writing?
 - A. The author thinks that the literary style of Aristotle is not desirable or fit for imitation.
 - B. The author cannot understand most of what Aristotle says.
 - C. The author finds Aristotle's work informative and entertaining.
 - D. The author laments that so much of Aristotle's work has been copied and butchered.
2. Which of the following best describes the author's point of view in this passage?
 - A. That of a frustrated socialite
 - B. That of a candid gossip
 - C. That of a playful comedian
 - D. That of a sarcastic literary critic
3. What is one thing the author thinks could be added to Mr. Mahaffy's book to improve it?
 - A. Anecdotes
 - B. An index
 - C. Quotations from other authors
 - D. A discussion of proper dining etiquette



Passage B

Adapted from a letter by T. Thatcher
published in *The Publishers Circular* on
September 27th, 1902

Sir—In these days of increasing rapid artificial locomotion, may I be permitted to say a word in favor of a very worthy and valuable old friend of mine, Mr. Long-Walk? I am afraid that this good gentleman is in danger of getting neglected, if not forgotten. We live in days of bicycles and tricycles, tram cars and motor cars, hansom cabs and ugly cabs; but in my humble opinion good honest walking exercise for health beats all other kinds of locomotion into a cocked hat. In rapid traveling all the finer nerves, senses, and vessels are “rush” and unduly excited, but in walking every particle of the human frame, and even the moral faculties, are evenly and naturally brought into exercise. It is the best discipline and physical mental tonic in the world. The body is healthily brought into normal operation, while, especially in the long distance walk, all the senses and moral faculties are elevated and cultivated healthfully and naturally. Many never know the beauty of it because they never go far enough: exercise and hard work should never be relinquished at any age or by either sex. Walking exercises, together with a well-educated palate, are the greatest physicians in the world: no disease can withstand them. I cannot too strongly recommend for a day’s companionship the society of my old and well-tried friend, Mr. Long-Walk.

Faithfully yours,

T. Thatcher

44 College Green, Bristol.

4. The author is most likely _____.
 - A. an exercise enthusiast
 - B. a respected physician
 - C. a politician
 - D. an automobile manufacturer
5. Which of the following would the author view as the most healthy activity?
 - A. Taking many short-distance walks in one afternoon while running errands
 - B. Taking one extended walk to a location and back
 - C. Running a short distance in order to catch a cab
 - D. Slowly meandering around a park, making frequent stops
6. How does the author feel about automated transportation?
 - A. He dislikes it because it overexcites the senses.
 - B. He dislikes it because one cannot enjoy the details of the scenery.
 - C. He likes it because it is the fastest way to get from point A to Point B.
 - D. He likes it because it lets him walk to a location and then be transported home



Practice Passages: Point of View: Answer Key

1. How does the author of Passage 1 feel about Aristotle's writing?

- A. The author thinks that the literary style of Aristotle is not desirable or fit for imitation.
- B. The author cannot understand most of what Aristotle says.
- C. The author finds Aristotle's work informative and entertaining.
- D. The author laments that so much of Aristotle's work has been copied and butchered.

2. Which of the following best describes the author's point of view in this passage?

- A. That of a frustrated socialite
- B. That of a candid gossip
- C. That of a playful comedian
- D. That of a sarcastic literary critic

3. What is one thing the author thinks could be added to Mr. Mahaffy's book to improve it?

- A. Anecdotes
- B. An index
- C. Quotations from other authors
- D. A discussion of proper dining etiquette

The author describes how Mr. Mahaffy employs the "literary style of Aristotle for which no excuse is possible." From this description it can be inferred that the author finds the writing of Aristotle to be inaccessible, and, additionally, that the author does not think the literary style of Aristotle is fit for imitation. The phrase "no excuse is possible" should clue you in to the author's meaning.

The author's point of view in Passage 1 is concerned with a book—specifically, the author is providing his opinion about a new book, stating what he likes and dislikes about it and ultimately making a recommendation as to whether the person reading his opinion should read the book or not. The content of the book that he is reviewing may evoke socialites and gossips, since it is about the art of conversation, but this doesn't describe the author. The author himself is notably sarcastic in the passage, using a biting tone with a sharp edge to it in his descriptions. The fact that he is acting as a literary critic by offering his criticism of a book combined with his notable sarcasm makes D a better answer than C. We're not given any suggestion that the author is a comedian, and even if his wit is funny at moments, he is overtly acting as a literary critic in the passage.

The author of Passage 1 writes about the missing elements of Mr. Mahaffy's book in the following part of the passage:

In discussing this important subject of conversation, he has not merely followed the scientific method of Aristotle which is, perhaps, excusable, but he has adopted the literary style of Aristotle for which no excuse is possible. There is, also, hardly a single anecdote, hardly a single illustration, and the reader is left to put the professor's abstract rules into practice, without either the examples or the warnings of history to encourage or to dissuade him in his reckless career.

Immediately after criticizing the book for making use of Aristotle's writing style, the author notes in the same negatively connoted breath that the book also lacks anecdotes and illustrations. This helps readers understand



4. The author is most likely _____.
- A. an exercise enthusiast
B. a respected physician
C. a politician
D. an automobile manufacturer

5. Which of the following would the author view as the most healthy activity?
- A. Taking many short-distance walks in one afternoon while running errands
→ B. Taking one extended walk to a location and back
C. Running a short distance in order to catch a cab
D. Slowly meandering around a park, making frequent stops

that the lack of these features is another negative element of the book, in the author's opinion. Therefore, thinking about this conversely, if the author thinks that Mr. Mahaffy's book suffers from a lack of anecdotes and illustrations, he thinks that anecdotes and illustrations could be added to the book to improve it. Since "anecdotes" is the only one of these two features listed as an answer choice, A is correct. Don't let D distract you—the subject of Mr. Mahaffy's book being conversation, it may initially seem like it could most logically benefit from a discussion of proper dining etiquette, which is arguably the most closely related topic of those listed. After consulting the passage itself, though, you'll find that the author only faults the book for its lack of anecdotes and illustrations—he mentions nothing about the lack of a discussion of proper dining etiquette and whether or not he thinks this would improve Mr. Mahaffy's book.

The author is passionate about his subject but not well-informed with research or science; therefore he is likely an amateur enthusiast. He says that walking is better than physicians, so he is not likely one himself. While his letter is persuasive, it has no political bent. Additionally, due to the author's preference for walking instead of "tram cars and motor cars, hansom cabs and ugly cabs" and the fact that he does not mention cars anywhere else in the passage, we cannot assume that he is associated with the automotive industry.

Towards the end of the passage, the author conveys that he thinks long walks to be the most beneficial form of walking:

The body is healthily brought into normal operation, while, especially in the long distance walk, all the senses and moral faculties are elevated and cultivated healthfully and naturally. Many never know the beauty of it because they never go far enough . . .

The author never mentions running, so we can ignore C. A and D both involve forms of short walks, and B is the only answer choice that specifically mentions "one extended walk," so B is the best answer.



6. What opinion does the author give about automated transportation?

- ➔ **A.** He dislikes it because it overexcites the senses.
- B.** He dislikes it because one cannot enjoy the details of the scenery.
- C.** He likes it because it is the fastest way to get from point A to Point B.
- D.** He likes it because it lets him walk to a location and then be transported home

On the subject of automated transportation, the author states the following:

We live in days of bicycles and tricycles, tram cars and motor cars, hansom cabs and ugly cabs; but in my humble opinion good honest walking exercise for health beats all other kinds of locomotion into a cocked hat. In rapid traveling all the finer nerves, senses, and vessels are “rush” and unduly excited, but in walking every particle of the human frame, and even the moral faculties, are evenly and naturally brought into exercise.

Here, the author directly states that he doesn't like automated transportation because “all the finer nerves, senses, and vessels are ‘rush’ and unduly excited.” Put a different way, these modes of transportation overexcite the senses, which matches up with answer choice A.



Sequence

Sequence questions run the gamut of potential difficulties. Some of them are incredibly straightforward, asking you to fetch overtly-stated details directly from the passage. Others are more challenging, dealing with passages in which the text itself and the events described in the text do not follow the same timeline. Still others ask you to consider what is discussed in a passage excerpted from a larger work and use that information to determine what is discussed immediately before and after the passage in the larger work. We'll go over each of these specific question types in this lesson so that you can feel confident in your ability to approach each of them.

Sequence of Passage Text

Let's start with the most direct type of sequence question: those that ask about the passage itself, not the events described. These questions may ask about words, phrases, or even paragraphs, and ask you to consider what they are doing in a particular order or where they are relative to one another in that order.

Practice Passage: Adapted from *Common Sense* by Thomas Paine (1776)

Society in every state is a blessing, but government even in its best state is but a necessary evil, in its worst state an intolerable one; for when we suffer, or are exposed to the same miseries heightened by reflecting that we furnish the means by which we suffer. Government, like dress, is the badge of lost innocence; the palaces of kings are built on the ruins of the bowers of paradise. For were the impulses of conscience clear, uniform, and irresistibly obeyed, man would need no other lawgiver; but that not being the case, he finds it necessary to surrender up a part of his property to furnish means for the protection of the rest; and this he is induced to do by the same prudence that in every other case advises him out of two evils to choose the least. WHEREFORE, security being the true design and end of government, it unanswerably follows that whatever FORM thereof appears most likely to ensure it to us, with the least expense and greatest benefit, is preferable to all others.

In order to gain a clear and just idea of the design and end of government, let us suppose a small number of persons settled in some sequestered part of the Earth, unconnected with the rest; they will then represent the first peopling of any country, or of the world. In this state of natural liberty, society will be their first thought. A thousand motives will excite them thereto, the strength of one man is so unequal to his wants, and his mind so unfitted for perpetual solitude, that he is soon obliged to seek assistance and relief of another, who in his turn requires the same. Four or five united would be able to raise a tolerable dwelling in the midst of a wilderness, but ONE man might labor out the common period of life without accomplishing any thing; when he had felled his timber he could not remove it, nor erect it after it was removed; hunger in the mean time would urge him from his work, and every different want call him a different way.



Sample Question

In the first paragraph, the author _____, whereas in the second paragraph, he _____.

- A.** makes an argument . . . provides an extended example supporting his argument
- B.** takes one position about an issue . . . changes his opinion and takes up the opposing position
- C.** provides a counterargument to a commonly held position . . . argues there is no solution to the problem at hand
- D.** presents his thesis . . . addresses his critics

While topic sentences often provide a good indication of a paragraph's function, the transition here is key in understanding what the author is doing. In the last sentence of the first paragraph, the author claims, "WHEREFORE, security being the true design and end of government, it unanswerably follows that whatever FORM thereof appears most likely to ensure it to us, with the least expense and greatest benefit, is preferable to all others." So, he is saying that the best government is the one that is most likely to keep its citizens safe, and in doing so, he is making an argument. In the second paragraph, he begins, "In order to gain a clear and just idea of the design and end of government, let us suppose . . ." Now, at the end of the first paragraph, he just declared "security being the true design and end of government," and now he is trying to "demonstrate" it in the second paragraph. The best answer is thus that the author makes an argument in the first paragraph and provides an extended example supporting his argument in the second paragraph.

Sequence of Events

Other sequence questions may ask about the order of events described in a passage, not the sequence of its text. Some of these questions may be rather obvious, while others may require you to take your time in order to put events in the correct order.

Practice Passage: Adapted from *The War of the Worlds* by H. G. Wells (1898)

The planet Mars revolves about the sun at a mean distance of 140,000,000 miles, and the light and heat it receives from the sun is barely half of that received by this world. It must be, if the nebular hypothesis has any truth, older than our world; and long before this earth ceased to be molten, life upon its surface must have begun its course. The fact that it is scarcely one seventh of the volume of the earth must have accelerated its cooling to the temperature at which life could begin. It has air and water and all that is necessary for the support of animated existence.



Sample Question

Which of the following accurately represents the order of events as the author believes they happened?

- A. Mars solidified, Earth solidified, life developed on Earth, life developed on Mars
- B. Mars solidified, life developed on Mars, Earth solidified, life developed on Earth
- C. Earth solidified, Mars solidified, life developed on Mars, life developed on Earth
- D. Mars solidified, Earth solidified, life developed on Mars, life developed on Earth

The author discusses the solidification of and the development of life on Mars and Earth in the provided paragraph. Let's narrow down what he says. First, he claims that Mars is older than Earth, but that doesn't tell us anything about when each planet solidified or when life began on each planet. He then states that life on Mars began before the Earth solidified. This allows us to eliminate any answer choice in which "Earth solidified" comes before "life developed on Mars," leaving us with only one answer choice, B: "Mars solidified, life developed on Mars, Earth solidified, life developed on Earth."

When Passage Text Sequence $\neq$ Passage Event Sequence

Some sequence questions concern the sequence of passage text—the order in which paragraphs mention certain topics. Other sequence questions concern the timeline of the events described. The trickiest questions occur when the order in which a passage describes events doesn't match up with the timeline of those events—in other words, when the first event isn't described first. Consider the following sample passage and question for an example of just such a "mismatched" sequence question, and be on your toes!



Practice Passage: Adapted from *The Harvard Classics Shelf of Fiction, Volume 16: Anna Karenina* (1877; 1917 ed., trans. Garnett)

Happy families are all alike; every unhappy family is unhappy in its own way.

Everything was in confusion in the Oblonskys' house. The wife had discovered that the husband was carrying on an intrigue with a French girl, who had been a governess in their family, and she had announced to her husband that she could not go on living in the same house with him.

This position of affairs had now lasted three days, and not only the husband and wife themselves, but all the members of their family and household were painfully conscious of it. The wife did not leave her own room, the husband had not been at home for three days. The children ran wild all over the house; the English governess quarreled with the housekeeper; the kitchen maid and the coachman had given warning.

Three days after the quarrel, Prince Stepan Arkadyevitch Oblonsky—Stiva, as he was called in the fashionable world—woke up at his usual hour, that is, at eight o'clock in the morning, not in his wife's bedroom, but on the leather-covered sofa in his study.

Noticing a gleam of light peeping in beside one of the serge curtains, he cheerfully dropped his feet over the edge of the sofa, and felt about with them for his slippers, a present on his last birthday, worked for him by his wife on gold-colored morocco. And, as he had done every day for the last nine years, he stretched out his hand, without getting up, towards the place where his dressing-gown always hung in his bedroom. And thereupon he suddenly remembered that he was not sleeping in his wife's room, but in his study, and why: the smile vanished from his face, he knitted his brows.

Most unpleasant of all was the first minute when, on coming, happy and good-humored, from the theatre, with a huge pear in his hand for his wife, he had not found his wife in the drawing-room, to his surprise had not found her in the study either, and saw her at last in her bedroom with the unlucky letter that revealed everything in her hand. She, his Dolly, forever fussing and worrying over household details, and limited in her ideas, as he considered, was sitting perfectly still with the letter in her hand, looking at him with an expression of horror, despair, and indignation.

"What's this? This?" she asked, pointing to the letter.

Sample Question

Which of the following occurred first in the story's timeline?

- A. Stiva returned to his house from the theatre with a pear.
- B. Stiva's wife confronted him about a letter.
- C. Stiva woke up on the sofa.
- D. The governess and the housekeeper argued.

Answering this question requires you to realize that the timeline of the story is very different from the order in which events are described in it. While the first answer choice's option to be described in the passage is "The governess and the housekeeper argued," this happened after "Stiva's wife confronted him about a letter," which happened after he "returned to his house from the theatre with a pear."



Identifying the Reasoning Behind Sequence

Other sequence questions may not directly ask you about the order of passage text or events, but instead prompt you to determine why the author has chosen to introduce text and ideas in a certain sequence. In the following sample question, you are asked to discern why the author employs a certain device toward the beginning of the passage. When reading the sample passage, see if you can figure out why the author would want to use this device earlier than later in the text.

Practice Passage: Adapted from *Cowboy Songs and Other Frontier Ballads* by John A. Lomax (1910)

The big ranches of the West are now being cut up into small farms. The nester has come, and come to stay. Gone is the buffalo, the free grass of the open plain—even the stinging lizard, the horned frog, the centipede, the prairie dog, the rattlesnake, are fast disappearing. Save in some of the secluded valleys of southern New Mexico, the old-time round-up is no more; the trails to Kansas and to Montana have become grass-grown or lost in fields of waving grain; the maverick steer, the regal longhorn, has been supplanted by his unpoetic but more beefy and profitable Polled Angus, Durham, and Hereford cousins from across the seas. The changing and romantic West of the early days lives mainly in story and in song. The last figure to vanish is the cowboy, the animating spirit of the vanishing era. He sits his horse easily as he rides through a wide valley, enclosed by mountains, clad in the hazy purple of coming night—with his face turned steadily down the long, long road, “the road that the sun goes down.”

Sample Question

Why does the author start the passage by listing disappearing species of the plains?

- A. To draw attention to the problem of endangered species
- B. To describe the sparse economic resources that cowboys had available to them
- C. To give the reader important context about the ecosystem of the American West
- D. To introduce the cowboy in the context of other disappearing figures of the American West

The author starts the paragraph by describing how the entire western landscape, including the variety of animals that live there, is changing. He then shifts to talking about cowboys with this transition: “The last figure to vanish is the cowboy, the animating spirit of the vanishing era.” In this way, the author puts the cowboy into context by comparing him to other classic—and disappearing—figures of the American West.

The next sample question also concerns the reasoning behind sequence, but instead of asking about one literary device, it asks about two sentences—specifically, why two lines of dialogue are placed immediately next to one another.



Practice Passage: Adapted from “The McWilliamses and the Burglar Alarm” in *The Mysterious Stranger and Other Stories* by Mark Twain (1898; 1916)

... Then one night we smelled smoke. I lit a candle, and started toward the stairs, and met a burglar coming out of a room with a basket of tinware, which he had mistaken for solid silver in the dark. He was smoking a pipe. I said, ‘My friend, we do not allow smoking in this room.’ He said he was a stranger, and could not be expected to know the rules of the house: said he had been in many houses just as good as this one, and it had never been objected to before.

“I said: ‘Smoke along, then. But what business have you to be entering this house in this furtive and clandestine way, without ringing the burglar alarm?’

He looked confused and ashamed, and said, with embarrassment: ‘I beg a thousand pardons. I did not know you had a burglar alarm, else I would have rung it. I beg you will not mention it where my parents may hear of it, for they are old and feeble, and such a seemingly wanton breach of the hallowed conventionalities of our civilization might all too rudely sunder the frail bridge which hangs darkling between the pale and evanescent present and the solemn great deeps of the eternities. May I trouble you for a match?’

“I said: ‘Your sentiments do you honor, but metaphor is not your best hold. Spare your thigh; this kind light only on the box. But to return to business: how did you get in here?’”

Sample Question

The underlined sentences are placed immediately next to each other in order to emphasize -----.

- A. the contrast in the formality of their language and their lengths
- B. the unexpected similarity in their subject matter
- C. the difference between how the thief speaks to his parents and how he speaks to Mr. McWilliams
- D. the fact that they are saying the same thing but in different ways

The first sentence is notably wordy, uses a complex phrase structure, and uses advanced (and archaic!) vocabulary like “darkling.” The second sentence, on the other hand, is something someone might say in everyday conversation, very different from the first sentence in terms of style and length. The best answer to this question is thus that the sentences are placed immediately next to each other to emphasize “the contrast in the formality of their language and their lengths.” As for the other answer choices, the sentences are talking about completely different subjects, so the sentences’ placement can’t be emphasizing “the unexpected similarity in their subject matter”; the speaker is addressing Mr. McWilliams the entire time, so “the difference between how the thief speaks to his parents and how he speaks to Mr. McWilliams” can’t be correct either; and “the fact that they are saying the same thing but in different ways” can’t be the right answer because they are saying completely different things.



Inferring the Previous Topic

What do you do when a question asks you to infer the topic of prose that's not even provided on the test—material that appears before or after the passage in the larger work from which it's excerpted? This may seem like an impossible task, but the ACT Reading section won't ask you to grasp at straws when making inferences. Enough context clues will be located in the passage for you to draw a reasonable conclusion about what material must precede or follow the passage at hand. If you're asked about material that preceded the passage, focus on its first few lines. You should be able to identify some transitional reference to what the author was talking about immediately before the passage's start.

Practice Passage: Adapted from *Volume Four of The Natural History of Animals: The Animal Life of the World in Its Various Aspects and Relations* by James Richard Ainsworth Davis (1903)

It has just been shown that compact, curved forms are often made to appear flattish by a particular distribution of color and shading, but it is clear that similar advantages might be secured in an entirely different manner—for example, by the body actually becoming flat. A striking case of this is that afforded by the flat-fishes, such as sole, turbot, plaice, and the rest. These, when in the condition of young fry, swim about like ordinary fish, and possess the same bilateral symmetry. Very soon, however, they become laterally flattened, and take to living on the sea floor with either the left or right side downwards, according to the species. This side remains white or pale, and the fish's eye migrates to the side which is kept upwards; otherwise it would be of no use. And, further, the upwardly-directed surface becomes darkly pigmented so as to harmonize with the sand or mud upon which the animal lives, the resemblance often being enhanced by the presence of spots and blotches of darker or different tint. Here, however, as in so many other cases, protection is only afforded by the coloration and marking when the animal remains at rest. Movement at once destroys the illusion.

Sample Question

Which of the following topics was discussed immediately before this passage in the book from which this passage is taken?

- A. Prey animals can use other methods of defense, like mimicry, effectively in conjunction with being flat.
- B. Being flat is a trait that has helped many species of organisms survive.
- C. Appearance can make something that is not flat look like it is flat.
- D. Appearing flat is not as useful a defense as actually being flat.

Since this question is about preceding material, let's take a close look at the passage's first line.

It has just been shown that compact, curved forms are often made to appear flattish by a particular distribution of color and shading, but it is clear that similar advantages might be secured in an entirely different manner—for example, by the body actually becoming flat.

We're given a major context clue in the introductory phrase "It has just been shown that." This is the equivalent of a neon sign saying "This is what the passage just talked about!" We just need to pick out the answer choice that best rephrases the topic of "compact, curved forms are often made to appear flattish by a particular distribution of color and shading." Let's consider them.



This question comes down to a “calibration” exercise—like in many main idea, summary, and paraphrase questions, we need to pick the answer that is just detailed enough: not too detailed to the point where it doesn’t apply to the whole phrase, and not too general to the point where points aren’t accurately conveyed. While the passage discusses a certain type of fish that uses its flatness as camouflage, the phrase we’re considering doesn’t mention organisms—specific ones or living things in general—at all. We can ignore answer choices A and B because of this. B is too general and mentions “organisms,” and while A may look like a potentially great answer choice because appearing flat might seem like a type of mimicry, the phrase doesn’t discuss prey animals or mimicry in conjunction with actually being flat—the two types of defense aren’t discussed in conjunction. The text moves from having discussed one, appearing flat, before the passage starts to discuss another, actually being flat.

This leaves us with C and D. Both are statements talking about appearance and flatness, so it looks like we’re on the right track. C and D mainly differ in that C is a statement of fact whereas D involves a comparison of the two types of defense, looking flat and actually being flat. The phrase in question doesn’t make any value judgment about these methods of defense—the author never states which one he considered to be the better one. Thus, D isn’t accurate, and C is the correct answer. Before the passage started, the author discussed how visual camouflage can make something that isn’t flat look as if it is.

Inferring the Next Topic

Some questions may ask you to predict the material that logically comes after a passage in the work from which it’s taken. Having a general idea of what’s discussed throughout the passage is important, but focus on the last few lines of the passage and keep on the lookout for any constructions that could signal a transition to a new topic. If you don’t see any, that’s fine—there might not be any, as the work from which the passage is taken may just continue discussing the topic at hand.

Practice Passage: Adapted from *Cowboy Songs and Other Frontier Ballads* by John A. Lomax (1910)

The big ranches of the West are now being cut up into small farms. The nester has come, and come to stay. Gone is the buffalo, the free grass of the open plain—even the stinging lizard, the horned frog, the centipede, the prairie dog, the rattlesnake, are fast disappearing. Save in some of the secluded valleys of southern New Mexico, the old-time round-up is no more; the trails to Kansas and to Montana have become grass-grown or lost in fields of waving grain; the maverick steer, the regal longhorn, has been supplanted by his unpoetic but more beefy and profitable Polled Angus, Durham, and Hereford cousins from across the seas. The changing and romantic West of the early days lives mainly in story and in song. The last figure to vanish is the cowboy, the animating spirit of the vanishing era. He sits his horse easily as he rides through a wide valley, enclosed by mountains, clad in the hazy purple of coming night—with his face turned steadily down the long, long road, “the road that the sun goes down.”



Sample Question

The next paragraph will most likely contain _____.

- A.** a description of cowboy songs
- B.** a comparison of cowboy and Arthurian legends
- C.** a discussion of profitable ranching
- D.** an introduction of the American cowboy

This paragraph is an introduction of the American cowboy, so it is unlikely that the next will be the same. More likely, the next paragraph will expound on what this one mentioned at the end: the cowboy song. Also, earlier in the paragraph, the author mentions that “The changing and romantic West of the early days lives mainly in story and in song.” If he wants to tell us more about the West, it would make sense that he discuss the songs to do so.



Practice Passages: Sequence

Adapted from *The Wind in the Willows* by Kenneth Grahame (1908)

The Mole had been working very hard all the morning, spring cleaning his little home. First with brooms, then with dusters, then on ladders and steps and chairs, with a brush and a pail of whitewash, 'till he had dust in his throat and eyes, and splashes of whitewash all over his black fur, and an aching back and weary arms. Spring was moving in the air above and in the earth below and around him, penetrating even his dark and lowly little house with its spirit of divine discontent and longing. It was small wonder, then, that he suddenly flung down his brush on the floor, said "Bother!" and "O blow!" and also "Hang spring cleaning!" and bolted out of the house without even waiting to put on his coat. Something up above was calling him imperiously, and he made for the steep little tunnel which answered in his case to the gaveled carriage-drive owned by animals whose residences are nearer to the sun and air. So he scraped and scratched and scrabbled and scrooged and then he scrooged again and scrabbled and scratched and scraped, working busily with his little paws and muttering to himself, "Up we go! Up we go!" 'till at last, pop! His snout came out into the sunlight, and he found himself rolling in the warm grass of a great meadow.

1. When does the mole use a brush to clean his home?
 - A. After he uses ladders
 - B. As he uses brooms
 - C. After he uses dusters
 - D. Before he uses chairs
2. Why does the author use synonyms of "scraped" sequentially in the underlined sentence?
 - A. To emphasize how long and how much energy it takes the mole to burrow to the surface
 - B. To emphasize how busily the mole worked while spring cleaning his home
 - C. To suggest that the mole gets lost on his way from his home to the surface
 - D. To confuse the reader, just as the mole is confused on his way to the surface

Adapted from *The Wealth of Nations* by Adam Smith (1776)

To take an example, therefore, from a very trifling manufacture, but one in which the division of labor has been very often taken notice of: the trade of a pin-maker. A workman not educated to this business (which the division of labor has rendered a distinct trade), nor acquainted with the use of the machinery employed in it (to the invention of which the same division of labor has probably given occasion), could scarce, perhaps, with his utmost industry, make one pin in a day, and certainly could not make twenty. But in the way in which this business is now carried on, not only the whole work is a peculiar trade, but it is divided into a number of branches, of which the greater part are likewise peculiar trades. One man draws out the wire; another straightens it; a third cuts it; a fourth points it; a fifth grinds it at the top for receiving the head; to make the head requires two or three distinct operations; to put it on is a peculiar business; to whiten the pins is another; it is even a trade by itself to put them into the paper; and the important business of making a pin is, in this manner, divided into about eighteen distinct operations, which, in some manufactories, are all performed by distinct hands, though in others the same man will sometimes perform two or three of them.

3. Which of the following is the first step in making a pin, according to the author?
 - A. Pointing the wire
 - B. Drawing out the wire
 - C. Cutting the wire
 - D. Straightening the wire
4. Discussion of which of the following topics likely preceded Passage 2?
 - A. How to apply machinery to one's trade
 - B. The importance of the pin-making trade
 - C. The varying profitability of different trades
 - D. The division of labor



Practice Passages: Sequence: Answer Key

1. When does the mole use a brush to clean his home?

- A. After he uses ladders
- B. As he uses brooms
- C. After he uses dusters
- D. Before he uses steps and chairs

We're told that the mole cleans his home in the following way:

First with brooms, then with dusters, then on ladders and steps and chairs, with a brush and a pail of whitewash, 'till he had dust in his throat and eyes, and splashes of whitewash all over his black fur, and an aching back and weary arms.

This sentence provides a sequential order for each of the items the mole uses. He uses the brush after he uses brooms, so B can't be correct. The brush and a pail he uses as he's using "ladders and steps and chairs," so A and D aren't correct, either. C is the correct answer; the mole uses dusters second and a brush along with "ladders and steps and chairs," which together form the third item in the initial list.

2. Why does the author use synonyms of "scraped" sequentially in the underlined sentence?

- A. To emphasize how long and how much energy it takes the mole to burrow to the surface
- B. To emphasize how busily the mole worked while spring cleaning his home
- C. To suggest that the mole gets lost on his way from his home to the surface
- D. To confuse the reader, just as the mole is confused on his way to the surface

The author's repetition of the words "scraped," "scratched," "scrabbled," and "scrooged" emphasizes the amount of work that the mole has to do to burrow to the surface. If the author only used one of these words, it would seem as if it didn't take the mole that much time or energy to burrow to the surface. Repetition draws out and highlights this moment in the text.

3. Which of the following is the first step in making a pin, according to the author?

- A. Pointing the wire
- B. Drawing out the wire
- C. Cutting the wire
- D. Straightening the wire

To correctly answer this question, you need to consider the third paragraph, specifically the part in which the author outlines the process of making a pin. The author begins discussing the specific tasks involved in pin-making by stating, "One man draws out the wire; another straightens it; a third cuts it; a fourth points it; a fifth grinds it at the top for receiving the head." While he continues after this, you can at this point tell that because the author begins by stating, "One man draws out the wire" and because the sequence is related in a logical, step-by-step order, the correct answer is B, "drawing out the wire."



4. Discussion of which of the following topics likely preceded Passage 2?
- A. How to operate the machinery involved in pin-making
 - B. The importance of the pin-making trade
 - C. The varying profitability of different trades
 - D. The division of labor

Considering the very beginning of passage 2 can help us get an idea of what was discussed immediately beforehand:

To take an example, therefore, from a very trifling manufacture, but one in which the division of labor has been very often taken notice of: the trade of a pin-maker.

The context clues given here are subtler than the example given in the lesson, but enough information is provided to help you figure out the correct answer. The phrase “To take an example” is the key part of the sentence. “The trade of a pin-maker” is introduced here as the example, so it is presumably being introduced for the first time, especially given the detail that the author goes into when describing the process involved. This means that B probably isn’t the best answer. Nothing suggests that the passage guides readers through the details of how to operate machinery involved in pin-making; the passage deals with the concept of machinery at an abstract level, not one you would find in how-to guides and instruction manuals.

This leaves us to pick between C and D. To identify the correct answer, we have to note why the pin-making trade is brought up as an example: the author points out that while one person working alone might make a pin a day, the industry is divided into specialized jobs that allow people to make pins more efficiently together. This, combined with the fact that profitability is never discussed in the passage, makes D the correct answer.



Why It's Said

Some ACT Reading questions ask you to ascertain what the author's likely purpose in writing the entire passage, or specific lines or paragraphs in it. Figuring out why the author most likely wrote what he or she did requires you to make use of context clues as well as a thorough understanding of the way the author progresses as he or she explains his or her ideas. In a prose fiction passage, questions about the purpose of characters' dialogue can be especially tricky, as statements may be able to mean different things depending on the situations in which they are spoken. Practicing questions about passage purpose is a great way to test your understanding of context and the subtleties of written arguments and dialogue.

Section Outline

[Purpose of Lines and Passage](#)

[Questions About the Purpose of a Single Word](#)

[Questions About the Purpose of a Sentence](#)

[Questions About a General Part of the Passage](#)

[Questions About the Purpose of the Passage as a Whole](#)

[Purpose of Dialogue](#)

[Practice Passage: Purpose of Dialogue](#)



Purpose of Lines and Passage

Questions that ask about the purpose of a passage or identified parts of it can seem difficult at first glance. They expect you to take in a great deal of information and distill from it not the main point of what it's saying, but why the author is making that particular point in the first place. Doing this requires analysis that moves beyond what is directly stated in the text, but with practice, you can start to put yourself in the author's shoes and gain insight on what each sentence, and the entire passage, is doing to create meaning.

Example Passage

Natural Science: Adapted from *Walden* by Henry Thoreau (1854)

Still we live meanly, like ants; it is error upon error, and clout upon clout, and our best virtue has for its occasion a superfluous and evitable wretchedness. Our life is frittered away by detail. An honest man has hardly need to count more than his ten fingers, or in extreme cases he may add his ten toes, and lump the rest. Simplicity, simplicity, simplicity! I say, let your affairs be as two or three, and not a hundred or a thousand; instead of a million count half a dozen, and keep your accounts on your thumbnail. In the midst of this chopping sea of civilized life, such are the clouds and storms and quicksands and thousand-and-one items to be allowed for, that a man has to live, if he would not founder and go to the bottom and not make his port at all, by dead reckoning, and he must be a great calculator indeed who succeeds. Simplify, simplify. Instead of three meals a day, if it be necessary eat but one; instead of a hundred dishes, five; and reduce other things in proportion.

Our life is like a German Confederacy, made up of petty states, with its boundary forever fluctuating, so that even a German cannot tell you how it is bounded at any moment. The nation itself, with all its so-called internal improvements, which, by the way are all external and superficial, is just such an unwieldy and overgrown establishment, cluttered with furniture and tripped up by its own traps, ruined by luxury and heedless expense, by want of calculation and a worthy aim, as the million households in the land; and the only cure for it, as for them, is in a rigid economy, a stern and more than Spartan

simplicity of life and elevation of purpose. It lives too fast. Men think that it is essential that the Nation have commerce, and export ice, and talk through a telegraph, and ride thirty miles an hour, without a doubt, whether they do or not, but whether we should live like baboons or like men is a little uncertain. If we do not get out sleepers, and forge rails, and devote days and nights to the work, but go to tinkering upon our lives to improve them, who will build railroads? And if railroads are not built, how shall we get to heaven in season? But if we stay at home and mind our business, who will want railroads? We do not ride on the railroad; it rides upon us. Did you ever think what those sleepers are that underlie the railroad? Each one is a man, an Irishman, or a Yankee man. The rails are laid on them, and they are covered with sand, and the cars run smoothly over them. They are sound sleepers, I assure you.



Purpose of Lines and Passage

A passage's purpose can be examined at many different levels of magnification. Certain purpose questions on the ACT Reading section may function at a microscopic level of single words, while others may ask you about the big picture of the purpose of an entire passage. Still, others fall elsewhere on the spectrum between these two extremes. In learning to face purpose questions, let's start with the small-scale, ultra-focused questions and zoom out to more general and far-reaching ones.

Questions About the Purpose of a Single Word

Questions that ask about the purpose of a single word may seem like they can ask about any word in the passage, but that's just not true. These questions will focus in on a word of some significance to the passage, so if you notice particular words that stick out to you as functioning in unique and overt ways as you are reading the passage, take note of them; such attention could make it easier for you to reorient yourself when answering questions about these terms.

Sample Question

The author repeats the words “simplicity” and “simplify” in the first paragraph in order to -----.

- A. mimic his critics in a derogatory way
- B. emphasize what he sees as the solution to a problem
- C. suggest that this is the only solution that has been put forward, and it is ineffective
- D. convey that working toward simplicity takes a lot of hard work

The author's repetition of “Simplicity, simplicity, simplicity!” and “Simplify, simplify” in the first paragraph add emphasis exhorting the audience to make their lives simpler, a change Thoreau sees as the solution to many of the excesses of modern life and the problems they cause. There is no mention of any critics in the passage, and his repetition does not suggest simplicity is an ineffective solution. He certainly does not imply that there is too much simplicity in modern society—one might read this into the line out of context, but throughout the passage, one can tell that the author is writing in favor of simplifying. Finally, while repetition can often make something seem as if it is difficult or takes time (e.g. “Digging, digging, digging—all that digging and they were only a few feet closer to the center of the earth.”), that is not what is going on here.



Questions About the Purpose of a Sentence

Purpose-of-sentence questions use entire lines as their building blocks, so when dealing with these, be sure to note any introductory, transitional, or concluding phrases as well as any other wording that interlocks specific lines together in particular ways.

Sample Question

The underlined sentence functions as _____ in context.

- A. a counterpoint to a critic's rebuttal
- B. a concrete example of how the reader can simplify
- C. a transition into the ideas discussed in the next paragraph
- D. a use of figurative language to emphasize the author's point

The underlined sentence is, "Instead of three meals a day, if it be necessary eat but one; instead of a hundred dishes, five; and reduce other things in proportion." Its position as the last line of the first paragraph may make it seem like "a transition into the ideas discussed in the next paragraph," but as the second paragraph begins on an entirely new point, this isn't the best answer. It is related to the author's discussion of simplicity, so we can't say that it is "an authorial aside only loosely related to the passage's main topic." No critics are mentioned in the passage, so we can't claim it to be "a counterpoint to a critic's rebuttal." While it is emphasizing the author's point, it's not using figurative language to do so, so "a use of figurative language to emphasize the author's point" can't be correct either. This leaves us with one answer choice, the correct one: the sentence is functioning as "a concrete example of how the reader can simplify." This is especially visible in that it directly follows the author's exhortation of "Simplify, simplify."

Questions About a General Part of the Passage

Certain purpose questions fall between sentence-level and passage-level on the specificity spectrum. The biggest issue with addressing these should not be figuring out which part of the passage is being indicated; that much should be relatively obvious. Figuring out how an argument, a listing of evidence, or a description is functioning in the context of the entirety of the presented text means identifying other large sectional building blocks in the passage and figuring out what makes the specified one function in relation to them.

Sample Question

Thoreau's discussion of "sleepers" at the end of the passage helps him _____.

- A. emphasize how the railroad is a burden upon people and their resources
- B. underscore the dangers to workers involved in building contemporary railroads
- C. suggest that the railroad is a great boon to the nation
- D. comment on how hard it is to fall asleep on a moving train



Thoreau discusses sleepers in the final lines of the passage:

We do not ride on the railroad; it rides upon us. Did you ever think what those sleepers are that underlie the railroad? Each one is a man, an Irishman, or a Yankee man. The rails are laid on them, and they are covered with sand, and the cars run smoothly over them. They are sound sleepers, I assure you.

The key line necessary to figuring out what Thoreau's purpose is in discussing sleepers is the line that precedes any mention of them: "We do not ride on the railroad; it rides upon us." With this wordplay, Thoreau is suggesting that the railway burdens us; the correct answer is thus that his discussion of "sleepers" helps him "emphasize how the railroad is a burden upon people and their resources." While he is decidedly anti-railroad in this passage, he focuses on the building of railroads instead of the use of them, weakening the argument that the point of his mentioning "sleepers" is to "urge readers never to use railroads to travel." Similarly, while the image of the railroad riding upon men's sleeping bodies may seem to "underscore the dangers to workers involved in building contemporary railroads," this is not the case either. In stating, "We do not ride on the railroad; it rides upon us," the author makes his purpose in mentioning "sleepers" clear.

Questions About the Purpose of the Passage as a Whole

Finally, certain questions encompass the entirety of the text and ask why the author would write it at all. While the most broad type of purpose question, these are not necessarily the most challenging of the species. When considering an entire passage's purpose, remember to treat it in the same vein as [picking main idea statements](#) or [looking for a good paraphrase or summary statement](#)—answer choices must be just specific enough, mentioning relevant details that apply to and/or together encompass the entire passage while not succumbing to generalities, irrelevant minutiae, or false information.

Sample Question

Which of the following best describes the author's purpose in writing this passage?

- A. to complain that people are oversimplifying their lives
- B. to point out that the United States has become like a German Confederacy and recommend it continue in this direction
- C. to discuss the pros and cons of developing railroads
- D. to discuss how his contemporary culture is overly complex and recommend people take steps to combat this

The passage talks about simplicity vs. complexity, and two of the answer choices have to do with this contrast: A and D. Let's look over B and C first, starting with B. Does the author point out that the United States has become like a German Confederacy? Yes, at the start of the second paragraph. This may be enough for certain students, who may be rushing through this question due to time restrictions, to choose B and move on to the next question, but those students would get this question wrong. The rest of the answer choice, "and recommend it continue in this direction," invalidates it, as Thoreau is not just pointing out the U.S.-German Confederacy similarity, he's complaining about it. Plus, even if that weren't the case, this comparison only appears at the start of the second paragraph. The author doesn't talk about it in the rest of the passage, which suggests that B is not a good reflection of the purpose of the entire text. A



purpose statement should encompass the entire passage, not just accurately describe the purpose of part of one paragraph.

Let's look at answer choice C. Again, the author does discuss the pros and cons of developing railroads, (though he focuses on the cons) in the passage's last paragraph. But he doesn't mention railroads in the previous paragraphs, making this answer choice unlikely to be the correct one. We can ignore it.

So, we've narrowed down the answer choices to A and D, the two options that discuss the author's views on simplicity and complexity. Before considering them in detail, consider the passage. How do these two terms relate in the author's opinion? He certainly seems to like simplicity and urges people to "simplify," while complaining about how modern life has gotten too complex. With that in mind, let's look over A and D. Is the purpose of the passage A "to complain that people are oversimplifying their lives?" Or is it D, "to discuss how his contemporary culture is overly complex and recommend people take steps to combat this?" A puts simplicity in a negative light, and D puts complexity in a negative light and simplicity in a positive light. Since the author urges people to "simplify" and considers simplicity to be a good thing throughout the passage, D is the correct answer.

Certain purpose questions may step out of the familiar "summary-esque" mode of the one presented above and instead ask you to describe the passage's purpose with a single word, much in the manner of [a direct tone question](#).

Sample Question

This is best described as a(n) _____ passage.

- A. persuasive
- B. informational
- C. scientific
- D. descriptive

This passage doesn't mention scientific principles, phenomena, or lab experiments, so we can't accurately call it "scientific." Furthermore, while the author uses the first-person perspective ("I"), he doesn't provide stories about himself, or tangential stories at all, so we can't call the passage "anecdotal." Its main purpose is not to describe something, so it's not "descriptive." This leaves us with "informational" and "persuasive." "Persuasive" is the correct option; the passage is not simply conveying information, but attempting to get readers to change their opinions, behaviors, and lifestyle. The persuasive element of the passage is best seen in the first paragraph, in the author's urging of "Simplify, simplify."



Purpose of Dialogue

Dialogue is a key component of many prose fiction passages. Understanding what goes on in the passage as a whole requires you to have a good sense of what the characters are saying to one another, and what that means in the context of the story. Certain statements might seem unrelated to what's going on when taken literally; it takes skill in reading comprehension to read between the lines and figure out why characters are saying what they're saying. By practicing your ability to understand the nuances of dialogue, you can be ready for whatever statements you face on test day.

Sample Passage

Prose Fiction: Adapted from *A Room With a View* by E. M. Forster (1908)

"The Signora had no business to do it," said Miss Bartlett, "no business at all. She promised us south rooms with a view close together, instead of which here are north rooms, looking into a courtyard, and a long way apart. Oh, Lucy!"

"And a Cockney, besides!" said Lucy, who had been further saddened by the Signora's unexpected accent. "It might be London."

"This meat has surely been used for soup," said Miss Bartlett, laying down her fork.

"I want so to see the Arno. The rooms the Signora promised us in her letter would have looked over the Arno. The Signora had no business to do it at all. Oh, it is a shame!"

"Any nook does for me," Miss Bartlett continued, "but it does seem hard that you shouldn't have a view."

Lucy felt that she had been selfish. "Charlotte, you mustn't spoil me; of course, you must look over the Arno, too. I meant that. The first vacant room in the front—" "You must have it," said Miss Bartlett, part of whose traveling expenses were paid by Lucy's mother—a piece of generosity to which she made many a tactful allusion.

"No, no. You must have it."

"I insist on it. Your mother would never forgive me, Lucy."

"She would never forgive me."

The ladies' voices grew animated, and—if the sad truth be owned—a little peevish. They were tired, and under the guise of unselfishness they wrangled. Some of their neighbors interchanged glances, and one of them—one of the ill-bred people whom one does meet abroad—leant forward over the table and actually intruded into their argument. He said:

"I have a view, I have a view."

Miss Bartlett was startled. Generally at a pension people looked them over for a day or two before speaking, and often did not find out that they would "do" till they had gone. She knew that the intruder was ill-bred, even before she glanced at him. He was an old man, of heavy build, with a fair, shaven face and large eyes. There was something childish in those eyes, though it was not the childishness of senility. What exactly it was Miss Bartlett did not stop to consider, for her glance passed on to his clothes. These did not attract her. He was probably trying to become acquainted with them before they got into the swim. So she assumed a dazed expression when he spoke to her, and then said: "A view? Oh, a view! How delightful a view is!"



Purpose of Dialogue

Purpose questions that ask you express why a certain character says what he or she says in a passage, or how a certain statement is intended to be taken, can require conversational reading-in-context skills. Briefly imagining yourself in a character's shoes can help you figure out how he or she might be feeling about his or her situation. Similarly, you can imagine how the passage might function were it to be rewritten as the script for a play or movie—if you were the director, what notes on tone and motivation would you include for the actors and actresses about how to deliver their lines? And as always, it's key to pay attention to the surrounding context of indicated moments—in this case, specifically how other characters respond to the speaker(s). Let's consider a few sample questions to practice these skills:

Sample Question

Miss Bartlett says “Oh, Lucy!” at the end of the first paragraph because _____.

- A. she pities Lucy's situation
- B. she has just been embarrassed by Lucy
- C. she is frustrated with their situation
- D. she is identifying and beginning to introduce Lucy in response to someone asking who her traveling companion is

Answering this question correctly requires you to understand the author's use of tone. Taken out of context, Miss Bartlett's statement of “Oh, Lucy!” could have been inspired by the sources covered in many of the incorrect answer choices. However, we need to focus on the context in which the statement appears in the passage. What precedes it? “The Signora had no business to do it,” said Miss Bartlett, “no business at all. She promised us south rooms with a view close together, instead of which here are north rooms, looking into a courtyard, and a long way apart.” The only answer choice that makes sense is that Miss Bartlett is frustrated with their situation. The idea that she pities Lucy's situation may look reasonable, but since neither of them have received the rooms they were promised, they are in the situation together, and it's not just Lucy who is in a frustrating situation.

Sample Question

When Miss Bartlett says “This meat has surely been used for soup,” the comment is intended to be _____.

- A. a compliment
- B. a criticism
- C. a statement of surprise
- D. an insult to the old man

Miss Bartlett's comment about the meat having been used for soup follows two paragraphs in which she and Lucy lament that they have not been given the rooms they were promised, and that the pension is distinctly English and familiar whereas they were expected it to seem more foreign. Thus, it makes sense that like the previous criticisms offered about the pension in the preceding paragraphs, Miss Bartlett's comment about the meat is also a criticism. None of the other answer choices make sense: the old man



hasn't been introduced in the passage when Miss Bartlett comments about the meat, so it can't be an insult to him; the meat having been used for soup would not be "a compliment" or "an instance of emphasizing a charming detail," especially coming on the heels of criticisms with no indication that it isn't a criticism too; and "a statement of surprise" doesn't make sense, as no indication is given that Miss Bartlett is surprised about the meat having been used for soup.

Sample Question

Which is the most logical reason for the man to state "I have a view, I have a view" at the end of the passage?

- A.** He is about to suggest that the young women trade rooms with him.
- B.** He is about to suggest that the young women try to find better rooms at another pension.
- C.** He wants to provide evidence that certain rooms in the pension do actually have a view.
- D.** He wants to support Lucy in her argument with Miss Bartlett.

Lucy and Miss Bartlett's argument concerns which of them will take the first room with a view to free up, so the man's comment doesn't support either one in this argument. Lucy and Miss Bartlett never doubt that certain of the pension's rooms do have views, so it doesn't make sense that the man would say "I have a view" in order "to provide evidence that certain rooms in the pension do actually have a view." There is no connection between the man stating he has a view and him suggesting that the women find rooms at another pension. However, there is a logical connection between him telling them he has a view and offering to switch rooms with them, so this is the correct answer.



Practice Passage: Purpose of Dialogue

Prose Fiction: Adapted from *The Wind in the Willows* by Kenneth Grahame (1908)

The Mole had been working very hard all the morning, spring cleaning his little home. First with brooms, then with dusters, then on ladders and steps and chairs, with a brush and a pail of whitewash, 'till he had dust in his throat and eyes, and splashes of whitewash all over his black fur, and an aching back and weary arms. Spring was moving in the air above and in the earth below and around him, penetrating even his dark and lowly little house with its spirit of divine discontent and longing. It was small wonder, then, that he suddenly flung down his brush on the floor, said "Bother!" and "O blow!" and also "Hang spring cleaning!" and bolted out of the house without even waiting to put on his coat. Something up above was calling him imperiously, and he made for the steep little tunnel which answered in his case to the gaveled carriage-drive owned by animals whose residences are nearer to the sun and air. So he scraped and scratched and scabbled and scrooged and then he scrooged again and scabbled and scratched and scraped, working busily with his little paws and muttering to himself, "Up we go! Up we go!" 'till at last, pop! His snout came out into the sunlight, and he found himself rolling in the warm grass of a great meadow.

"This is fine!" he said to himself. "This is better than whitewashing!" The sunshine struck hot on his fur, soft breezes caressed his heated brow, and after the seclusion of the cellarge he had lived in so long, the carol of happy birds fell on his dulled hearing almost like a shout. Jumping off all his four legs at once, in the joy of living and the delight of spring without its cleaning, he pursued his way across the meadow 'till he reached the hedge on the further side.

"Hold up!" said an elderly rabbit at the gap. "Sixpence for the privilege of passing by the private road!" He was bowled over in an instant by the impatient and contemptuous Mole, who trotted along the side of the hedge chaffing the other rabbits as they peeped hurriedly from their holes to see what the row was about. "Onion-sauce! Onion-sauce!" he remarked

jeeringly, and was gone before they could think of a thoroughly satisfactory reply. Then they all started grumbling at each other. "How STUPID you are! Why didn't you tell him—" "Well, why didn't YOU say—" "You might have reminded him—" and so on, in the usual way; but, of course, it was then much too late, as is always the case.

1. Why does the mole say "Bother," "O blow," and "Hang spring cleaning" in the passage's first paragraph?
 - A. He is enthusiastic about spring cleaning.
 - B. He is discussing spring cleaning with the elderly rabbit and is shocked that the elderly rabbit does not enjoy spring cleaning as much as he does.
 - C. He keeps dropping things while trying to clean his house.
 - D. He is sick of spring cleaning.
2. Why does the elderly rabbit say "'Sixpence for the privilege of passing by the private road?"
 - A. He wants the mole to stop to chat
 - B. He wants the mole to stop and pay a toll
 - C. He's telling the mole where the mole is headed
 - D. He's offering to pay the mole to help him clean his burrow
3. When the mole says "Onion-sauce! Onion-sauce!" to the rabbits, this statement is meant to be _____.
 - A. a taunt
 - B. a demand
 - C. a nickname
 - D. lyrics to a song



Practice Passage: Purpose of Dialogue: Answer Key

1. Why does the mole say “Bother,” “O blow,” and “Hang spring cleaning” in the passage’s first paragraph?

- ➔ A. He is enthusiastic about spring cleaning.
- B. He is discussing spring cleaning with the elderly rabbit and is shocked that the elderly rabbit does not enjoy spring cleaning as much as he does.
- C. He keeps dropping things while trying to clean his house.
- D. He is sick of spring cleaning.

2. Why does the elderly rabbit say “Sixpence for the privilege of passing by the private road?”

- A. He wants the mole to stop to chat
- ➔ B. He wants the mole to stop and pay a toll
- C. He’s telling the mole where the mole is headed
- D. He’s offering to pay the mole to help him clean his burrow

3. When the mole says “Onion-sauce! Onion-sauce!” to the rabbits, this statement is meant to be _____.

- ➔ A. a taunt
- B. a demand
- C. a nickname
- D. lyrics to a song

Let’s look at the rest of the passage to put the mole’s remarks in context. In the first paragraph, we are told that he’s doing a lot of spring cleaning: “The Mole had been working very hard all the morning, spring-cleaning his little home. First with brooms, then with dusters; then on ladders and steps and chairs, with a brush and a pail of whitewash; till he had dust in his throat and eyes, and splashes of whitewash all over his black fur, and an aching back and weary arms.” After he says the statements in question, he burrows out of his home and arrives in “a great meadow.” At this point (at the beginning of the passage’s second paragraph), he says “This is fine!” and “This is better than whitewashing!” So, looking at the statements “Bother,” “O blow,” and “Hang spring cleaning,” we can infer that the mole says these things because he is sick of spring cleaning and is going to stop soon, as this is just what happens in the rest of the passage.

At the beginning of the third paragraph, the mole encounters the elderly rabbit: “Hold up!” said an elderly rabbit at the gap. ‘Sixpence for the privilege of passing by the private road!’ “Pence” is an old British monetary unit; it is the plural of “penny.” So, we can paraphrase what the elderly rabbit says as “Stop! You need to pay six pennies to use the private road.” From this, we can tell that the elderly rabbit wants the mole to stop and pay a toll.

Let’s look at the context in which the mole says “Onion-sauce!” At this point in the passage, the mole has just ignored the elderly rabbit’s demand for him to pay a toll to use the private road. The passage then says that the mole “trotted along the side of the hedge chaffing the other rabbits as they peeped hurriedly from their holes to see what the row was about. “Onion-sauce! Onion-sauce!” he remarked jeeringly, and was gone before they could think of a thoroughly satisfactory reply.” We are told that he says “Onion-sauce!” “jeeringly,” or in a taunting way, and we know that the rabbits could not “think of a thoroughly satisfactory reply” in time. Also, he is said to be “chaffing” the rabbits. “Chaffing” means mocking or taunting. Given these context clues, we can safely say that the mole’s call of “Onion-sauce!” is a taunt, or an insult, as he says it “tauntingly” and the rabbits cannot think of a reply in time, and one might want to think of a reply to an insult.



Engaging with What's Said

ACT Reading questions can ask you to use your critical thinking skills to engage with what's said in a number of different ways. You might be asked to compare various features in the text, identify the cause or effect of a plot event, summarize or paraphrase a selection, or make a generalization, inference, or prediction based on what is said. Practicing your critical thinking skills before test day can help you approach each of these question types with confidence!

Section Outline

[Comparing and Contrasting](#)

[Identifying Comparisons and Contrasts](#)

[Comparing and Contrasting Two Textual Features](#)

[Comparing More than Two Textual Features](#)

[Comparing and Contrasting Passage Structures](#)

[Practice Passage: Comparing and Contrasting](#)

[Cause and Effect](#)

[Identifying Cause](#)

[Identifying Effect](#)

[Identifying Cause and Effect Simultaneously](#)

[Practice Passage: Cause and Effect](#)

[Summarizing and Paraphrasing](#)

[Paraphrasing an Excerpt](#)

[Summarizing an Excerpt](#)

[Summarizing a Passage](#)

[Practice Passage: Summarizing and Paraphrasing](#)

[Generalizations, Inferences, and Predictions](#)

[Making Inferences](#)

[Making Predictions](#)

[Making Generalizations](#)

[Practice Passage: Generalizations, Inferences, and Predictions](#)



Carefully weighing the similarities and differences between indicated things or characters in a passage requires an eye for detail and the ability to keep multiple details in mind at once. Comparison and contrast questions may involve overt comparisons already made in the text, or they may ask you to compare two things that aren't so explicitly related. Some questions may even involve structural components and ask you to compare various sentences or paragraphs. No matter what details a comparing and contrasting question concerns, you can conquer it with practiced critical thinking skills.

Sample Passage

Prose Fiction: Adapted from *The War of the Worlds* by H. G. Wells (1898)

No one would have believed in the last years of the nineteenth century that this world was being watched keenly and closely by intelligences greater than man's and yet as mortal as his own; that as men busied themselves about their various concerns they were scrutinized and studied, perhaps almost as narrowly as a man with a microscope might scrutinize the transient creatures that swarm and multiply in a drop of water. With infinite complacency men went to and fro over this globe about their little affairs, serene in their assurance of their empire over matter. It is possible that the infusoria under the microscope do the same. No one gave a thought to the older worlds of space as sources of human danger, or thought of them only to dismiss the idea of life upon them as impossible or improbable. It is curious to recall some of the mental habits of those departed days. At most terrestrial men fancied there might be other men upon Mars, perhaps ready to welcome a missionary enterprise. Yet across the gulf of space, minds that are to our minds as ours are to those of the beasts that perish, intellects vast and cool and unsympathetic regarded this earth with envious eyes, and slowly and surely drew their plans against us. And early in the twentieth century came the great disillusionment.

The planet Mars revolves about the sun at a mean distance of 140,000,000 miles, and the light and heat it receives from the sun is barely half of that received by this world. It must be, if the nebular hypothesis has any truth, older than our world; and long before this earth ceased to be molten, life upon its surface must have begun its course. The fact that it is scarcely one seventh of the volume of the earth must have accelerated its cooling to the temperature at which life could begin. It has air and water and all that is necessary for the support of animated existence.

Since Mars is older than our earth, it necessarily follows that it is not only more distant from time's beginning but nearer its end. The cooling that must someday overtake our planet has already gone far indeed with our neighbor. In its equatorial region, the midday temperature barely approaches that of our coldest winter. Its air is much more attenuated than ours; its oceans have shrunk until they cover but a third of its surface. That last stage of exhaustion, which to us is still incredibly remote, has become a present-day problem for the inhabitants of Mars. The immediate pressure of necessity has brightened their intellects, enlarged their powers, and hardened their hearts. And looking across space with instruments, and intelligences such as we have scarcely dreamed of, they see, at its nearest distance only 35,000,000 of miles sunward of them, a morning star of hope, our own warmer planet, green with vegetation and gray with water, with a cloudy atmosphere eloquent of fertility, with glimpses through its drifting cloud wisps of broad stretches of populous country and narrow, navy-crowded seas.

And we men, the creatures who inhabit this earth, must be to them at least as alien and lowly as are the monkeys and lemurs to us. The intellectual side of man already admits that life is an incessant struggle for existence, and it would seem that this too is the belief of the minds upon Mars. Their world is far gone in its cooling and this world is still crowded with life, but crowded only with what they regard as inferior animals. To carry warfare sunward is, indeed, their only escape from the destruction that, generation after generation, creeps upon them.

And before we judge of them too harshly we must remember what ruthless and utter destruction our own species has wrought, not only upon animals, such as the vanished bison and the dodo, but upon itself. The Tasmanians were entirely swept out of existence in a war of extermination waged by European immigrants in the space of fifty years. Are we such apostles of mercy as to complain if the Martians warred in the same spirit?



Comparing and Contrasting

Comparing and contrasting questions focus on the identification of similarities or differences between two or more things, people, or concepts in a passage. This presents the opportunity for a great deal of variation, and thus a wide variety of different levels of difficulty.

In this lesson, we'll consider questions that focus on overt comparisons and contrasts, as well as those that ask you to compare or contrast elements of the text that are not explicitly discussed together in the text. We'll also address how to handle questions that ask you to compare three or more things and take a closer look at those that look for similarities and differences in passage structure. Let's get going!

Identifying Comparisons and Contrasts

Let's begin by looking at a question that asks you to identify a contrast explicitly stated in the text.

Sample Question

It is clearly suggested in the passage that Mars, relative to Earth, _____.

- A. is older
- B. is larger
- C. has a thicker atmosphere
- D. has a warmer average temperature

The planets Mars and Earth are specifically compared in the second and third paragraphs, and the reader encounters the following lines at the beginning of the second:

The planet Mars revolves about the sun at a mean distance of 140,000,000 miles, and the light and heat it receives from the sun is barely half of that received by this world. It must be, if the nebular hypothesis has any truth, older than our world; and long before this earth ceased to be molten, life upon its surface must have begun its course.

The passage clearly states that Mars must be older than Earth if the nebular hypothesis is true, and the author goes on to explain how Earth looks luxurious compared to the conditions of Mars, which are described as worsening. The passage overall operates on the assumption that Mars is older than Earth, as it states in the second paragraph, so A is the correct answer. B can't be correct because in the second paragraph, the passage states, "The fact that it is scarcely one seventh of the volume of the earth . . ." where "it" refers to Mars. C and D can both be disproven by the third paragraph when it states, "In its equatorial region, the midday temperature barely approaches that of our coldest winter. Its air is much more attenuated than ours." ("Attenuated" may be an unfamiliar term, but you can gather that since the author is describing how Mars has lower temperatures and less water than Earth, and temperature and water make a planet habitable, and air also makes a planet habitable, the author is describing how the Martian atmosphere is becoming less habitable—that is, thinner.)

As you can see, questions that ask you to consider direct statements in the passage aren't necessarily straightforward. You may need to comb the passage looking for evidence that either supports or contradicts each answer choice. Approach each question with care, even if it initially seems uncomplicated!



Let's take a look at a similar question that asks you to identify a comparison instead of a contrast.

Sample Question

In the first paragraph, humanity is compared with which of the following?

- A. Monkeys and lemurs
- B. Infusoria
- C. Scientists
- D. Dodos

This time, we're directed to a particular paragraph, so we can concentrate our search. Skim over the first paragraph. Which of the answer choices are mentioned? Infusoria and scientists. While humans are compared to monkeys and lemurs and to dodos, these comparisons are located in the third and last paragraphs, respectively. Not missing that detail about location is key to answering this question correctly!

Now that we've identified two potentially correct answers, let's look at exactly what role each of their nouns plays in the passage. Which is compared to humans?

First, let's consider the mention of scientists:

No one would have believed in the last years of the nineteenth century that this world was being watched keenly and closely by intelligences greater than man's and yet as mortal as his own; that as men busied themselves about their various concerns they were scrutinized and studied, perhaps almost as narrowly as a man with a microscope might scrutinize the transient creatures that swarm and multiply in a drop of water.

Scientists are humans, so that may have clued you in that the comparison being made is not humans-scientists. Instead, the Martians are being compared to human scientists, while humanity is being compared to "the transient creatures that swarm and multiply in a drop of water." This comparison is echoed in later lines that use slightly different language:

With infinite complacency men went to and fro over this globe about their little affairs, serene in their assurance of their empire over matter. It is possible that the infusoria under the microscope do the same.

Aha, "infusoria"! If you didn't know what it meant, you can note that the passage is continuing along the same lines as the earlier comparison. This means that "infusoria" means "the transient creatures that swarm and multiply in a drop of water." That's exactly what the author is comparing humanity to, so B is the correct answer!

Comparing and Contrasting Two Textual Features

Both of the previous sample questions have asked you to work with information directly stated in the passage, but not all comparison and contrast questions do this. Some, like the next few, ask you to do the comparing and contrasting!



Sample Question

The passage suggests that as a whole, Martians are generally _____ humans.

- A. smarter than
- B. less technologically advanced than
- C. more interested in justice than
- D. more motivated by guilt and shame than

While we're still dealing with Martians and humans in this question, now we have to figure out how they differ. How does the passage characterize Martians in contrast to humans? The passage never mentions anything about "justice," "guilt," or "shame"—the closest it gets to those topics is in the last paragraph, but no definitive ranking of Martian and human ruthlessness can be drawn from the information it provides.

Near the beginning of the first paragraph, the author introduces Martians indirectly, referring to them as "intelligences greater than man's and yet as mortal as his own." While this may not be immediately clear when you encounter the line, the passage eventually provides the context that allows you to realize what the author is describing with that phrase. Later, near the end of the first paragraph, the author writes, "Yet across the gulf of space, minds that are to our minds as ours are to those of the beasts that perish, intellects vast and cool and unsympathetic regarded this earth with envious eyes." Here, the author is drawing an analogy: Martians are to humans as humans are to "the beasts that perish." Nothing found in the rest of the passage suggests that the Martians would be less technologically advanced than humans, and given that the Martians are characterized multiple times by their comparatively great intelligence, A is the best answer.

For questions like these in which the whole paragraph is fair game, it may help you to systemically account for each answer choice. Identifying three incorrect answer choices might provide a more direct route to the correct answer than rereading the passage looking for particular details.

Let's try another more abstract comparison and contrast question. This next one is focused on identifying similarities.

Sample Question

According to the passage, what do humans have in common with the Martians?

- A. Both humans and Martians are peaceful species
- B. Both humans and Martians have the capacity to be destructive when colonizing a place
- C. Both Martians and humans are interested in conserving natural resources
- D. Humans and Martians must both deal with dwindling natural resources in the near future

This time, we're looking for descriptions that apply to both Martians and humans equally. The Martians aren't set up to seem peaceful; a major point of the passage is that they want to take over Earth for its more abundant resources. Conserving natural resources is likely a Martian concern, given that their planet is becoming increasingly unlivable, but nothing is mentioned that suggests humans also want to conserve



natural resources. D is perhaps the trickiest answer to discount—Martians have to deal with dwindling natural resources in the near future, but you can infer that the inhabitants of Earth don't have to deal with this problem any time soon given that Mars' problems are due to its age, and Earth is a much younger planet, as the passage explains in its second paragraph. This leaves us with the correct answer, B—that humans and Martians have the capacity to be destructive when colonizing a place. You can arrive at this answer without accounting for the incorrect ones by considering the last paragraph, where this is suggested in the author's discussion of why it would be hypocritical for humans to complain about the Martians' insensitive treatment of them. The author provides examples of the "ruthless and utter destruction our own species has wrought" and then asks, "Are we such apostles of mercy as to complain if the Martians warred in the same spirit?" There is an implicit comparison being made here equating human and Martian ruthlessness, so B is the best answer.

Comparing More than Two Textual Features

Comparing and contrasting two passage concepts is difficult enough, but what do you do when the test asks you to identify commonalities between three or more concepts? You approach the question the same way as you would approach a two-concept comparison question, with the added bonus that it will perhaps be easier to identify incorrect answers, as there are more concepts involved. You can ignore an answer choice for a two-concept comparison question if one of the two items is contradicted, so the more options you have to compare, the more chances for those options to be contradicted by something in the passage.

Sample Question

What do monkeys, lemurs, infusoria, and "the beasts that perish" have in common in this passage?

- A. They are all sources of food for humans
- B. They are the organisms native to Earth that the Martians abducted before abducting a human
- C. They are all things Martians are afraid of
- D. They are all things compared to humans in the passage

"Monkeys and lemurs" get mentioned in the same breath in the passage's second-to-last paragraph:

And we men, the creatures who inhabit this earth, must be to them at least as alien and lowly as are the monkeys and lemurs to us.

"The beasts that perish" are mentioned in the first paragraph:

Yet across the gulf of space, minds that are to our minds as ours are to those of the beasts that perish, intellects vast and cool and unsympathetic regarded this earth with envious eyes, and slowly and surely drew their plans against us.

"Infusoria" are also mentioned in the first paragraph:

With infinite complacency men went to and fro over this globe about their little affairs, serene in their assurance of their empire over matter. It is possible that the infusoria under the microscope do the same.



So, what do these three topics have in common? While they are all “organisms native to Earth,” the passage doesn’t say anything about Martians abducting humans, so B can’t be the correct answer—it involves an unsubstantiated claim. Nothing is mentioned about humans eating any of the mentioned items, so A can’t be correct either, and similarly, nothing is pointed out as something the Martians particularly fear, so we can ignore C as well. D is the correct answer; all of these concepts are compared with humans in the passage. You can see these comparisons at work in the quotations above.

Comparing and Contrasting Passage Structures

For our last comparing and contrasting sample question, let’s consider one that only involves two elements, but can be tough because it asks about the passage’s structure (e.g. paragraphs) instead of specific concepts in the text.

Sample Question

The passage’s second paragraph provides _____, whereas the third paragraph uses that information _____.

- A.** an astronomical history of the planet Mars . . . to explain why humans attacked Mars
- B.** scientific information about Mars . . . to explain why the Martians wanted to colonize Earth
- C.** direct explanation of what happened during “the great disillusionment” . . . to explain the Martians’ motivation for abducting humans
- D.** comparisons between Martians and humans . . . to help readers see the Martians in a more positive light

Let’s look over the second paragraph. What are its main topics? How is it presenting them? Paragraph two concerns a comparison between the planets Earth and Mars. It is objective in tone and simply presenting scientific information.

Paragraph three continues in the same vein for a while before suggesting that since Mars is becoming difficult to inhabit, the Martians are considering Earth as a potentially habitable planet:

That last stage of exhaustion, which to us is still incredibly remote, has become a present-day problem for the inhabitants of Mars. The immediate pressure of necessity has brightened their intellects, enlarged their powers, and hardened their hearts. And looking across space with instruments, and intelligences such as we have scarcely dreamed of, they see, at its nearest distance only 35,000,000 of miles sunward of them, a morning star of hope, our own warmer planet . . .

Let’s look at the answer choices. D isn’t right; no part of paragraph two includes comparisons between Martians and humans, and the third paragraph doesn’t work to portray Martians in a more positive light. C can’t be right, either; paragraph two doesn’t explain what “the great disillusionment” was directly, and while the third paragraph does talk about the Martians’ motivations, it sets up their motivations for attacking Earth, not for abducting humans in particular. The passage doesn’t specify how abducting humans would help Martians take over Earth, so that’s a bit of a stretch to assume. Both A and B provide accurate descriptions of paragraph two, so we need to pick the one that most accurately describes



paragraph three. Does paragraph three “explain why humans attacked Mars,” or does it “explain why the Martians wanted to colonize Earth?” This question turns into one of understanding who is attacking whom in the passage; it is clearly the Martians who have set their sights on Earth. In addition, considering that the first paragraph suggests that humans had no idea the Martians were going to attack, it doesn’t make much sense to say that the humans attacked the Martians. B is the correct answer.



Practice Passage: Comparing and Contrasting

Natural Science: Adapted from “Birds in Retreat” in “Animal Defences—Active Defence” in Volume Four of *The Natural History of Animals: The Animal Life of the World in Its Various Aspects and Relations* by James Richard Ainsworth Davis (1903)

Among the large running birds are forms, like the African ostrich, in which the absence of powers of flight is largely compensated by the specialization of the legs for the purpose of rapid movement on the ground. For straightforward retreat in open country nothing could be more effective; but another kind of adaptation is required in birds like rails, which are deficient in powers of flight, and yet are able to run through thickly-growing vegetation with such rapidity as to commonly elude their enemies. This is rendered possible by the shape of their bodies, which are relatively narrow and flattened from side to side, so as to easily slip between the stems of grasses, rushes, and similar plants. Anyone who has pursued our native land-rail or corn-crake with intent to capture will have noted how extremely difficult it is even to get within sight of a bird of this sort.

Certain birds, unfortunately for themselves, have lost the power of flight without correspondingly increased powers of running, and have paid the penalty of extinction. Such an arrangement, as might be anticipated, was the result of evolution in islands devoid of any predatory ground-animals, and a classic example of it is afforded by the dodo and its allies, birds related to the pigeons. The dodo itself was a large and clumsy-looking species that at one time abounded in the island of Mauritius, which, like oceanic islands generally, possessed no native mammals, while its indigenous reptiles were only represented by lizards. The ubiquitous sailor, however, and the animals (especially swine) which he introduced, brought about the extinction of this helpless bird in less than a century after its first discovery in 1598. Its memory is now only kept green by a few contemporary drawings and descriptions, certain museum remains, and the proverb “as extinct as a dodo.” A similar fate must overtake any organism suddenly exposed to new and unfavorable conditions, if devoid of sufficient plasticity to rapidly accommodate itself to the altered environment.

1. One of the main differences between an ostrich and a rail, according to the passage, is _____.
 - A. ostriches are living things whereas rails are inanimate objects
 - B. ostriches run over open terrain, and rails run through thick grass
 - C. ostriches are smaller than rails
 - D. ostriches use their running abilities to catch prey, whereas rails use their running abilities to avoid predators
2. According to the passage, what does Mauritius have in common with other islands?
 - A. Mauritius has a very tropical environment
 - B. Rails are common on Mauritius
 - C. Mauritius has no native mammals
 - D. There are no lizards on Mauritius
3. According to the passage, what do dodos, ostriches, and rails have in common?
 - A. They are all birds of prey
 - B. They are all flightless birds
 - C. They all eat lizards
 - D. They can all be found on Mauritius
4. How does the second paragraph relate to the first?
 - A. The first paragraph describes flightless birds that learned to defend themselves from predators by fleeing them; the second paragraph describes a flightless bird that did not adapt in this way
 - B. The first paragraph describes birds people eat; the second describes birds people do not eat
 - C. The first paragraph describes a type of flightless bird that has gone extinct; the second describes species of flightless birds that are still living
 - D. The first paragraph provides a personal anecdote while the second paragraph provides historical information



Practice Passage: Comparing and Contrasting: Answer Key

1. One of the main differences between an ostrich and a rail, according to the passage, is:

A. ostriches are living things whereas rails are inanimate objects

- B. ostriches run over open terrain, and rails run through thick grass
- C. ostriches are smaller than rails
- D. ostriches use their running abilities to catch prey, whereas rails use their running abilities to avoid predators

Ostriches and rails are discussed in the first paragraph. The author says that “For straightforward retreat in open country nothing could be more effective; but another kind of adaptation is required in birds like rails, which are deficient in powers of flight, and yet are able to run through thickly-growing vegetation with such rapidity as to commonly elude their enemies.” The passage then goes on to detail how rails have thin bodies that allow them to dart through the grass. This means that the correct answer is “ostriches run over open terrain, and rails run through thick grass.”

The passage is describing the rail, a type of bird, and not using the word “rail” to describe part of a train track or a barrier on an elevated area, so “ostriches are living things whereas rails are inanimate objects” cannot be correct. Ostriches are never said to be smaller than rails or predators, so neither “ostriches are smaller than rails” nor “ostriches use their running abilities to catch prey, whereas rails use their running abilities to avoid predators” can be correct. Finally, rails are said to live in areas containing and have adapted to “thickly-growing vegetation” and ostriches are said to have developed a different adaptation suited to “open country,” so “ostriches live in grassy areas whereas rails live in the desert” cannot be correct either.

2. According to the passage, what does Mauritius have in common with other islands?

A. Mauritius has a very tropical environment

B. Rails are common on Mauritius

- C. Mauritius has no native mammals
- D. There are no lizards on Mauritius

This question is answered directly by the passage in the middle of the second paragraph:

The dodo itself was a large and clumsy-looking species that at one time abounded in the island of Mauritius, which, like oceanic islands generally, possessed no native mammals, while its indigenous reptiles were only represented by lizards.

Here, readers are told that “like oceanic islands generally,” Mauritius has “no native mammals.” Rails are only mentioned in the first paragraph and aren’t associated with Mauritius at all. While one might infer that Mauritius has a very tropical environment, the passage doesn’t say that it shares this feature with most islands. Be careful to only rely on what the passage says and not make use of any outside knowledge you might possess about passage topics! Finally, lizards are mentioned as living on Mauritius in the above sentence (“its indigenous reptiles were only represented by lizards”).



3. According to the passage, what do dodos, ostriches, and rails have in common?

- A. They are all birds of prey.
- B. They are all flightless birds.
- C. They all eat lizards.
- D. They can all be found on Mauritius.

4. How does the second paragraph relate to the first?

- A. The first paragraph describes flightless birds that learned to defend themselves from predators by fleeing them; the second paragraph describes a flightless bird that did not adapt in this way.
- B. The first paragraph describes birds people eat; the second describes birds people do not eat.
- C. The first paragraph describes a type of flightless birds that has gone extinct; the second describes species of flightless birds that are still living.
- D. The first paragraph provides a personal anecdote while the second paragraph provides historical information. (might need to adjust that answer choice)

Ostriches and rails are discussed in the first paragraph, and dodos are discussed in the second paragraph. What overarching passage topic ties these two paragraphs together? Ostriches are introduced as an example of a type of large running bird; rails are introduced later as being “deficient in powers of flight, . . . yet . . . able to run . . . with such rapidity as to commonly elude their enemies.” So at this point, we know that both rails and ostriches are running birds. How does the dodo fit into the picture? Dodos are introduced as “a classic example” of the following situation: “Certain birds, unfortunately for themselves, have lost the power of flight without correspondingly increased powers of running, and have paid the penalty of extinction.”

Examining the two paragraphs, one can find that the first one talks about ostriches and rails, two types of flightless birds that adapted in specific ways to be good at fleeing predators. The second paragraph tells the story of the dodo, a flightless bird that did not have time to develop such adaptations. From here, we can pick out the correct answer: “The first paragraph describes flightless birds that learned to defend themselves from predators by fleeing them; the second paragraph describes a flightless bird that did not adapt in this way.”

As for the other answer choices, nothing in the passage discusses people eating birds, so we can ignore the answer choice “The first paragraph describes birds people eat; the second describes birds people do not eat.” The answer choice “The first paragraph describes a type of flightless birds that has gone extinct; the second describes species of flightless birds that are still living” would be correct if it flipped around the paragraphs to which it refers, but as it is written, it is incorrect. The passage doesn’t say anything about the predators of flightless birds in the first paragraph or the prey of flightless birds in either paragraph, so “The first paragraph discusses predators of flightless birds, the second describes prey of flightless birds” cannot be correct, and since the first paragraph does not provide a personal anecdote, “The first paragraph provides a personal anecdote while the second paragraph provides historical information” cannot be correct either.



Identifying Cause and Effect

Cause-and-effect questions can be some of the trickiest on the ACT, as they strike a challenging middle ground between questions where the answer is directly stated somewhere in the passage and more abstract questions that require skills like making generalizations, inferences, and predictions. Cause-and-effect questions combine concrete and abstract reasoning skills and throw in a little logical thinking as well, but by understanding how these questions function, you can be ready for them on test day.

Practice Passage

Prose Fiction: Passage adapted from “The Sisters” in *Dubliners* by James Joyce (1914)

There was no hope for him this time: it was the third stroke. Night after night I had passed the house (it was vacation time) and studied the lighted square of window, and night after night I had found it lighted in the same way, faintly and evenly. If he was dead, I thought, I would see the reflection of candles on the darkened blind, for I knew that two candles must be set at the head of a corpse. He had often said to me, “I am not long for this world,” and I had thought his words idle. Now I knew they were true. Every night as I gazed up at the window I said softly to myself the word “paralysis.” It had always sounded strangely in my ears, like the word “gnomon” in the Euclid and the word “simony” in the Catechism. But now it sounded to me like the name of some maleficent and sinful being. It filled me with fear, and yet I longed to be nearer to it and to look upon its deadly work.

Old Cotter was sitting at the fire, smoking, when I came downstairs to supper. While my aunt was ladling out my stirabout he said, as if returning to some former remark of his:

“No, I wouldn’t say he was exactly . . . but there was something queer . . . there was something uncanny about him. I’ll tell you my opinion . . .”

He began to puff at his pipe, no doubt arranging his opinion in his mind. Tiresome old fool! When we knew him first he used to be rather interesting, talking of fairs and worms, but I soon grew tired of him and his endless stories about the distillery.

“I have my own theory about it,” he said. “I think it was one of those . . . peculiar cases . . . But it’s hard to say . . .”

He began to puff again at his pipe without giving us his theory. My uncle saw me staring and said to me:

“Well, so your old friend is gone, you’ll be sorry to hear.”

“Who?” said I.

“Father Flynn.”

“Is he dead?”

“Mr. Cotter here has just told us. He was passing by the house.”

I knew that I was under observation, so I continued eating as if the news had not interested me. My uncle explained to old Cotter.

“The youngster and he were great friends. The old chap taught him a great deal, mind you; and they say he had a great wish for him.”

“God have mercy on his soul,” said my aunt piously.

Old Cotter looked at me for a while. I felt that his little beady black eyes were examining me, but I would not satisfy him by looking up from my plate. He returned to his pipe and finally spat rudely into the grate.



Cause and Effect

Identifying Cause

Questions that ask you to identify the cause of something can range from those that ask you about causes presented rather directly in the passage, causes that you have to discern by using your critical thinking skills, or causes identified (perhaps incorrectly!) by certain characters. If there's a particular part of the passage that comes to mind when you're asked about the cause of something, by all means quickly skim it over, but be aware that many cause questions won't correlate precisely to any particular line or lines—they'll require you to ascertain a cause based on the passage as a whole.

Sample Question

To what does Old Cotter attribute the condition of the dying man?

- A. Paralysis
- B. Alcoholism
- C. Poisoning
- D. He doesn't provide a distinct opinion on the subject.

You can knock out answers B and C, "Alcoholism" and "Poisoning," immediately, as the passage never mentions either of them. Choice A, "Paralysis," is probably going to catch your eye immediately, not just because it's the first listed answer choice, but because you should remember having read about it specifically before, in the passage's first paragraph:

Every night as I gazed up at the window I said softly to myself the word "paralysis." It had always sounded strangely in my ears, like the word "gnomon" in the Euclid and the word "simony" in the Catechism. But now it sounded to me like the name of some maleficent and sinful being. It filled me with fear, and yet I longed to be nearer to it and to look upon its deadly work.

From this, we can gather that the dying man is paralyzed at the point at which the story is taking place. That might be enough for many test-takers to select A and move on to the next question—after all, the ACT Reading section isn't known for lavishing you with tons of extra time! But it's worth it to consider this question a bit more—it's not asking about the condition of the dying man at the point in the story. It's asking *what Old Cotter attributes it to*, and Old Cotter could very well be wrong in his opinion.

Glancing over the dialogue between the narrator and Old Cotter, we find this moment:

"I have my own theory about it," [Old Cotter] said. "I think it was one of those . . . peculiar cases . . . But it's hard to say . . ."

He began to puff again at his pipe without giving us his theory.

This swings the entire question. Old Cotter never specifies his opinion about what caused the dying man's condition, so the correct answer is D. Be sure to read each question carefully and not skim over important aspects of what, exactly, it is asking!



Identifying Effect

Like questions that ask you to identify a cause, questions that ask you to identify effects can range from easier ones grounded in provided passage details to more challenging ones that expect you to do a lot of critical thinking.

Let's start with a relatively easier example that is relatively direct, but still requires a bit of inference.

Sample Question

What effect has the third stroke had on the dying man described in the first paragraph?

- A.** It has blinded him.
- B.** It has made him unable to keep food down without being sick.
- C.** It has resulted in him being partially or completely unable to move.
- D.** It has given him amnesia.

The “third stroke” is mentioned in the passage’s opening lines: “There was no hope for him this time: it was the third stroke.” So, it shouldn’t be too difficult to figure out where to direct our focus: the first paragraph. We don’t hear anything about the man being blinded, unable to keep down food, or having lost his memory, but at the end of the paragraph, the narrator connects the concept of not being able to move—put in different terms, paralysis—with the stroke victim: “Every night as I gazed up at the window I said softly to myself the word ‘paralysis.’” Though this is not a directly stated connection, readers are expected to infer that the man who suffered from the third stroke is now paralyzed. The end of the paragraph offers context clues besides those conveyed by the sentence that initially mentions paralysis: “It filled me with fear, and yet I longed to be nearer to it and to look upon its deadly work.” If the narrator wants to “look upon [paralysis’s] deadly work” and is glancing up toward the stroke victim’s room, it’s safe to conclude that paralysis is one effect the stroke has had on its victim.

Let’s consider an effect question that requires more drawn-out reasoning:

Sample Question

How would the narrator know were the man described in the first paragraph to die?

- A.** The lighting seen through the window of the man’s house would be uneven.
- B.** Unlit candles would be placed at the man’s head.
- C.** An obituary would appear in the newspaper.
- D.** He would hear sobbing from the house next door.

Put in different language, this question is asking, if the dying man passes away, how will the narrator know? The passage mentions nothing about sobbing, a funeral parade, or an obituary as the sign that the dying man has passed away, so none of those answers can be correct. This leaves us with “Unlit candles would be placed at the man’s head” and “The lighting seen through the window of the man’s house would be uneven.” While “Unlit candles would be placed at the man’s head” may seem like the correct answer, especially if one is reading very quickly, it’s important to realize that the narrator thinks that lit candles would be the sign that the man has passed away, not unlit ones. We know this from the first paragraph,



where the narrator states,

If he was dead, I thought, I would see the reflection of candles on the darkened blind, for I knew that two candles must be set at the head of a corpse.

We can tell that the candles mentioned in this sentence must be lit, because the narrator is describing their “reflection . . . on the darkened blind.” We can also infer that lit candles would cause the lighting seen through the window of the man’s house to be uneven, because the narrator says, “night after night I had found it lighted in the same way, faintly and evenly,” describing his action of passing the house and assuming the man is still living. So, the correct answer is A, “The lighting seen through the window of the man’s house would be uneven.” As you answer this question, it’s important to consider the whole of the first paragraph and put several details together instead of relying solely on the one sentence about the candles, which may lead you to the incorrect answer about unlit candles. Make sure to consider the entire surrounding context of a point, not just the immediately adjacent lines!

Identifying Cause and Effect Simultaneously

Perhaps the most challenging cause and effect questions are those that ask you to identify both concepts in a single question. Consider the one below.

Sample Question

The narrator is _____ by Old Cotter’s speech because _____.

- A. flattered . . . Old Cotter is subtly complimenting him
- B. annoyed . . . he doesn’t complete his sentences
- C. frustrated . . . the narrator knows that Old Cotter’s theory is not true, but cannot correct him
- D. pleased . . . Old Cotter can’t figure out the situation he’s considering, and the narrator takes pleasure in seeing him confused

Old Cotter’s speech is described and takes place in paragraphs two through five. Some things that may stick out about it as you read it are that it is filled with ellipses (“ . . .”) and that it doesn’t form a complete thought; the narrator remarks upon this latter point at the end of the speech, saying, “He began to puff again at his pipe without giving us his theory.” The conversation then turns as the narrator’s uncle relays the news about Father Flynn’s death.

In looking at how the narrator reacts to Old Cotter’s speech, we can tell that he isn’t pleased by it, as the passage states from his perspective, “Tiresome old fool! When we knew [Old Cotter] first he used to be rather interesting, talking of fairs and worms, but I soon grew tired of him and his endless stories about the distillery.” This allows us to eliminate the answer choices that begin with “flattered” and “pleased,” leaving us with those that begin with “annoyed” and “frustrated.”

At this point, we have to pick out the correct reason why the narrator is annoyed with or frustrated by the speech: “[Old Cotter] doesn’t complete his sentences,” or “the narrator knows that Old Cotter’s theory is not true, but cannot correct him.” Old Cotter never provides the narrator or readers with his theory, so “the narrator knows that Old Cotter’s theory is not true, but cannot correct him” cannot be the correct answer. This leaves us with “annoyed . . . he doesn’t complete his sentences” as the correct answer.



Practice Passage: Cause and Effect

Natural Science: Adapted from “The Treatment of Rattlesnake Bite by Permanganate of Potassium, Based on Nine Successful Cases” by Amos W. Barber, M.D. in *Scientific American Supplement* No. 841, Vol. XXXIII (February 13th, 1892)

Poisoned wounds, inflicted by the fangs of the rattlesnake, are happily rarer each year, since, as the country is becoming more populated, the crotalus is rapidly being exterminated. Yet, considering the disregard that characterizes the cowboy in his treatment of this reptile, it is astonishing that this class of injury is not more common.

It is the invariable custom among the cattlemen to dismount and destroy these snakes whenever they are seen. This is readily accomplished, since a slight blow will break the back. This blow is, however, generally delivered by means of the quirt, a whip not over two and a half feet long, and hence a weapon which brings the one who wields it in unpleasant proximity to the fangs of the reptile. A still more dangerous practice, and one which I have frequently seen, is a method of playing with the rattlesnake for the humor of the cowboy at the expense of a “tenderfoot.” It is well known that unless a snake is coiled or in other specific positions, it cannot strike. On this theory, a mounted cowboy first puts a rattler to flight, then seizes it by the tail, and, swinging it so rapidly around his head that it is impossible for it to strike, sets off in pursuit of whoever has exhibited the most terror at the sight of the reptile. When within fair distance, he hurls the snake at the unfortunate victim, in the full assurance that even should it hit him it cannot bury its fangs in his flesh, since it cannot coil until it reaches the ground. This is a jest of which I have frequently been the victim, nor have I yet learned to appreciate it with unalloyed mirth.

The first case of rattlesnake wound to which I was called occurred in 1885. A cowboy was bitten on the foot, the fang penetrating through the boot. I saw him about twenty-four hours after he was struck. There was enormous swelling, extending up to the knee. There was no special discoloration about the wound; in fact, the swelling disguised this to such an extent that it was impossible to determine exactly where the fangs had entered. The patient was suffering great pain. His mind was clear, but he was oppressed with a dreadful anxiety.

1. Why was it difficult for the author to identify exactly where the cowboy had been bitten?
 - A. The area around the wound was discolored
 - B. The cowboy was refusing to cooperate
 - C. The wound was extremely swollen
 - D. It wasn't; the cowboy was bitten in the foot
2. Why does the author believe the number of rattlesnake poisonings is decreasing each year?
 - A. Changes in climate have reduced the rattlesnake population
 - B. People are being more careful around rattlesnakes
 - C. Treatment of rattlesnake bites is improving
 - D. The population of people is growing
3. Which of the following best explains why cowboys might often be bitten by rattlesnakes?
 - A. Cowboys usually attempt to kill rattlesnakes when they see them
 - B. Cowboys attack rattlesnakes in a way that brings them near the animal
 - C. Cowboys often tease members of their group by throwing rattlesnakes at them
 - D. All of the above are reasons identified by the passage as to why cowboys might often be bitten by rattlesnakes
4. Why won't a thrown rattlesnake bite anyone?
 - A. Rattlesnakes are too scared to bite anyone just after having been thrown
 - B. Rattlesnakes only bite other rattlesnakes
 - C. Rattlesnakes can't strike unless coiled in specific positions
 - D. A thrown rattlesnake will usually bite the person at which it is thrown
5. Which of the following is an effect of rattlesnake bites as conveyed by the passage?
 - A. Swelling
 - B. Discoloration of the wound
 - C. Nausea
 - D. Mental confusion



Practice Passage: Cause and Effect: Answer Key

1. Why was it difficult for the author to identify exactly where the cowboy had been bitten?

- A. The area around the wound was discolored.
- B. The cowboy was refusing to cooperate.
- C. The wound was extremely swollen.
- D. It wasn't; the cowboy was bitten in the foot.

This question is a little difficult if you do not read carefully. You might be inclined to answer "It wasn't; the cowboy was bitten in the foot," based on the fact that the author says "A cowboy was bitten on the foot, the fang penetrating through the boot." However, hopefully the use of the word "exactly" in the question encouraged you to read more carefully. The author goes on to say "The whole limb was bronzed in appearance. There was no special discoloration about the wound; in fact, the swelling disguised this to such an extent that it was impossible to determine exactly where the fangs had entered." So, it is because the wound was "extremely swollen."

2. Why does the author believe the number of rattlesnake poisonings is decreasing each year?

- A. Changes in climate have reduced the rattlesnake population
- B. People are being more careful around rattlesnakes
- C. Treatment of rattlesnake bites is improving
- D. The population of people is growing

In the first paragraph, the author directly states, "Poisoned wounds, inflicted by the fangs of the rattlesnake, are happily more rare each year, since, as the country is becoming more populated, the crotalus is rapidly being exterminated." So, bites are more rare because the country is "becoming more populated." The author certainly would not say "more careful behavior" is contributing to the decline in rattlesnake bites, nor does he mention climate. It is perhaps reasonable to infer that he might say "better treatment," as he is advocating for one such procedure, but since those treatments that came before had little success, this might be better seen as why the author would think rattlesnake poisonings would continue to decrease.

3. Which of the following explains why cowboys might often be bitten by rattlesnakes?

- A. Cowboys usually attempt to kill rattlesnakes when they see them.
- B. Cowboys attack rattlesnakes in a way that brings them near the animal.
- C. Cowboys often tease members of their group by throwing rattlesnakes at them.

→ D. All of the above are reasons identified by the passage as to why cowboys might often be bitten by rattlesnakes.

The author mentions answer choices A, B, and C in the second paragraph, beginning with "It is the invariable custom among the cattlemen to dismount and destroy these snakes whenever they are seen" (A). He adds that the killing blow is "generally delivered by means of the quirt, a whip not over two and a half feet long, and hence a weapon which brings the one who wields it in unpleasant proximity to the fangs of the reptile" (B). At this point, he transitions to discussing answer choice C: "A still more dangerous practice, and one which I have frequently seen, is a method of playing with the rattlesnake for the humor of the cowboy at the expense of a 'tenderfoot.' He goes on to detail how cowboys tease "tenderfoot" individuals by throwing rattlesnakes at them. Even though the author's point about throwing rattlesnakes is qualified with the information that they can only strike when in certain positions, he nevertheless calls the practice "dangerous." This makes the best answer choice D, all of the above. Each of the other listed answer choices is a reason why a cowboy might be close enough to a rattlesnake to be bitten by one.



4. Why won't a rattlesnake that has just been thrown bite anyone?
- A. Rattlesnakes are too scared to bite anyone just after having been thrown.
 - B. Rattlesnakes only bite other rattlesnakes.
 - C. Rattlesnakes can't strike unless coiled in specific positions.
 - D. A thrown rattlesnake will usually bite the person at which it is thrown.
5. Which of the following is an effect of rattlesnake bites as conveyed by the passage?
- A. Swelling
 - B. Discoloration of the wound
 - C. Nausea
 - D. Mental confusion

The author answers this question directly in the second paragraph when he states, "It is well known that unless a snake is coiled or in other specific positions, it cannot strike. On this theory . . ." At this point, he describes the cowboy custom of throwing a rattlesnake at a "tenderfoot." It is not suggested anywhere in the passage that rattlesnakes will only bite other rattlesnakes or that they are too scared just after having been thrown to bite people. In addition, we are not told that a thrown rattlesnake will usually bite the person at which it is thrown, merely that the practice of throwing rattlesnakes is "dangerous," likely because the snakes can strike before they are grabbed or once they are back on the ground and angry. The correct answer is C.

To answer this question, we must consider the passage's final paragraph, in which the author discusses the first time he treated someone for a rattlesnake bite. Swelling is certainly an effect of rattlesnake bites, as we can tell by "There was enormous swelling, extending up to the knee." A lack of discoloration is mentioned in the passage ("There was no special discoloration about the wound"), as is a lack of mental confusion ("His mind was clear"). Nausea isn't mentioned at all. The correct answer is thus A.



Summarizing and Paraphrasing

Summarizing and paraphrasing—two concepts often drastically underestimated in how difficult questions about them can be! Coming up with a good paraphrase or summary of a sentence, section, or passage involves discerning the perfect amount of detail to include. This gets more difficult as the focus of a question grows wider in scope; paraphrasing a sentence might be much easier than summarizing a passage. Read on to practice putting passage content into your own words with confidence!

Practice Passage

Prose Fiction: Adapted from “The McWilliamses and the Burglar Alarm” in *The Mysterious Stranger and Other Stories* by Mark Twain (1898; 1916)

The conversation drifted along from weather to crops, from crops to literature, from literature to scandal, from scandal to religion; then took a random jump, and landed on the subject of burglar alarms. And now for the first time Mr. McWilliams showed feeling. Whenever I perceive this sign on this man’s dial, I comprehend it, and lapse into silence, and give him opportunity to unload his heart. Said he, with but ill-controlled emotion:

“I do not go one single cent on burglar alarms, Mr. Twain—not a single cent—and I will tell you why. When we were finishing our house, we found we had a little cash left over. I was for donating it to charity; but Mrs. McWilliams said no, let’s have a burglar alarm. I agreed to this compromise. Whenever I want a thing, and Mrs. McWilliams wants another thing, and we decide upon the thing that Mrs. McWilliams wants—as we always do—she calls that a compromise. Very well: the man came up from New York and put in the alarm, and charged three hundred and twenty-five dollars for it, and said we could sleep without uneasiness now. So we did for awhile—say a month. Then one night we smelled smoke. I lit a candle, and started toward the stairs, and met a burglar coming out of a room with a basket of tinware, which he had mistaken for solid silver in the dark. He was smoking a pipe. I said, ‘My friend, we do not allow smoking in this room.’ He said he was a stranger, and could not be expected to know the rules of the house: said he had been in many houses just as good as this one, and it had never been objected to before.

“I said: ‘Smoke along, then. But what business have you to be entering this house in this furtive and clandestine way, without ringing the burglar alarm?’

He looked confused and ashamed, and said, with embarrassment: ‘I beg a thousand pardons. I did not know you had a burglar alarm, else I would have rung it. I beg you will not mention it where my parents may hear of it, for they are old and feeble, and such a seemingly wanton breach of the hallowed conventionalities of our civilization might all too rudely sunder the frail bridge which hangs darkling between the pale and evanescent present and the solemn great deeps of the eternities. May I trouble you for a match?’

“I said: ‘Your sentiments do you honor, but metaphor is not your best hold. Spare your thigh; this kind light only on the box. But to return to business: how did you get in here?’”



Summarizing and Paraphrasing

Summarizing and paraphrasing are two different but closely related tasks that vary at the level of the material being considered. One is usually asked to paraphrase a sentence or a short paragraph, while material indicated by questions asking you to summarize can span from one paragraph to the entire passage. Because of its focus on relatively short amounts of text, paraphrasing requires you to pay attention to retaining the specific content of the sentence(s) at hand, whereas in summarizing, you are working with lots of material and should instead make sure your statement refers to all of the major units (e.g. paragraphs, sentences) that you are being asked to encompass while remaining relevant and not erring on the side of being too general in a way that doesn't represent the content you're summarizing. Let's work through a few sample questions so that you can understand how paraphrasing and summarizing slightly differ by trying out each task yourself.

Paraphrasing an Excerpt

As usual, we'll work from specific to general, so let's begin with paraphrasing. While some paraphrasing questions may focus on relatively simple sentences, their difficulty increases with the complexity of the sentence that you have to put into your own words, and literary devices like metaphor and figurative language can also present potential obstacles between you and the correct answer. For example, here's a rather challenging paraphrasing question:

Sample Question

Which of the following is the BEST paraphrase of the underlined and bolded excerpt?

- A. They might learn that their son is a burglar.
- B. They might fall off of the bridge on which they are traveling.
- C. They may not be able to forgive the speaker for his breach of etiquette.
- D. The shock of the news might kill them.

The language used in the indicated part of the passage is much more complex and full of rhetorical flourishes than the rest of the passage, which may throw you off when answering this question. Let's consider the context of the entire sentence first, as this can help us figure out which of the answer choices' meanings don't make sense. The sentence is spoken by the burglar, who is replying to Mr. McWilliams; Mr. McWilliams has just asked him why the burglar didn't ring the burglar alarm, and the burglar has apologized for not doing so. The burglar then asks Mr. McWilliams to "not mention it where my parents may hear of it," so we know he is talking about his parents. He says his parents are "old and feeble," and then we run into the very complex part of the sentence. At this point we know the complex part is referring to the burglar's older parents. While there is a bridge mentioned, it is mentioned figuratively—there is not an actual bridge in the story being referred to here; this means that B, "they might fall off of the bridge on which they are traveling," cannot be correct.

This leaves us to choose between three remaining answer choices: "they might learn that their son is a burglar," "they may not be able to forgive the speaker for his breach of etiquette," and "the shock of the news might kill them." While the first two of these answer choices may look correct, the third one is actually the best paraphrase. If Mr. McWilliams mentioned the burglar not ringing the burglar alarm where the burglar's parents could hear of it, they "might learn their son is a burglar" and "they might not be able to forgive their son for his breach of etiquette." Both of those are reasonable, even almost necessary conclusions. However, the underlined portion of the sentence is discussing the idea that the shock of this



information might kill the burglar's parents; the burglar is describing how the metaphorical "bridge" between "the pale and evanescent present" (in other words, the fleeting present) and "the solemn great deeps of the eternities" (a phrase that evokes the future, the passage of time, and most relevantly for our purposes in answering this question, death) would be broken. The underlined portion is thus saying that the shock of the news might kill the burglar's parents, making D, "the shock of the news might kill them," the correct answer.

When paraphrasing, it's crucial to stick to what's said in the passage, even if other answer choices make sense. This will help you to not introduce ideas that appear later or not at all in the passage into your answer choices. And watch out, because you may be asked to paraphrase some unique or unexpected sentences, and the test-writers may expect you to answer based on an assumption of what sounds logical or correct, as so much of the rest of the ACT is based on that set of expectations. Not in this case! If the passage is completely objectively wrong about something and/or not making logical sense, it doesn't matter for this type of problem: the indicated part of the passage is still what you have to paraphrase!

Summarizing an Excerpt

Let's take a look at summary now, zooming out to consider larger excerpts that encompass several sentences or more. The next question concerns a five-sentence excerpt. Can you condense it into a single line of summary?

Sample Question

Which of the following best summarizes the underlined selection in the second paragraph?

- A. After some debate, Mr. and Mrs. McWilliams agree to buy a burglar alarm.
- B. When Mr. and Mrs. McWilliams disagree, Mrs. McWilliams usually gets her way.
- C. Mr. McWilliams does not trust burglar alarms.
- D. The McWilliamses paid \$325.00 for their burglar alarm, and a man from New York installed it.

The trick to a good summary is all in the level of detail it conveys. We need the detail to cover what happens in all of the indicated sentences, so it can't just focus on one aspect of them while ignoring what happens in the rest of the excerpt. Also, a good summary won't introduce new information or generalities; this helps it stick closely to the meaning of the material being summarized.

Consider the indicated excerpt. If you were telling a friend what happens in this part of the story, what might you say? "Well, Mr. and Mrs. McWilliams had some extra money, and Mr. McWilliams wanted to give the money to charity, but Mrs. McWilliams wanted a burglar alarm, so they got one. It sounded expensive." That's a perfectly good, if casual and long, summary of the section. Quickly summarizing the indicated text for yourself before considering the answer choices can help you calibrate your thinking to the level of detail the correct answer will use, and maybe help incorrect answers that are too general or too specific stick out as incorrect.

For instance, in the summarizing-to-a-friend quotation above, the price of the burglar alarm and details about the person who installed it weren't included, yet we got a good sense of the events in the text. This (correctly) suggests that D isn't the correct answer—it's too detailed. The same thing can be said for B—while this forms a larger part of the excerpt, just relating how Mr. and Mrs. McWilliams handle compromises is also not an summary of the excerpt. Look at all sentences that are just about burglar



alarms that follow the part about their compromises—B doesn't convey any information about those sentences, and when entire sentences have nothing to do with a statement that's supposed to include them in a summary, that's a pretty good indication that the summary isn't accurate to the text it's describing. B isn't correct either. C seems appropriately general, and might seem a little familiar; that's because it's describing the wrong part of the passage at the right level of specificity.

Summarizing a Passage

Time to look at the largest-scale question of this type and attempt to summarize the entire passage! Passage-level summary questions can take a few different forms, from the directive to just pick out the best summary sentence to others that focus on other, more general summaries about what kind of story or text you've read or even complex logical underpinnings of the text. Let's have a look at a few examples.

Sample Question

Which of the following best describes this passage?

- A.** An educational story about why people shouldn't smoke
- B.** A humorous story about burglar alarms
- C.** A serious story about a man being robbed
- D.** A funny story about purchases people regret

This story is best described as humorous because of several illogical details meant to strike the reader as funny: the fact that Mr. McWilliams meets a burglar and asks him why the burglar didn't ring the burglar alarm is a humorous detail, and the fact that the burglar asks for a match and we can infer that Mr. McWilliams provides him with one (from the statement "Spare your thigh; this kind light only on the box") is also meant to be a funny, unexpected interaction. So, we can narrow down our answer choices to "a humorous story about burglar alarms" or "a funny story about purchases people regret." The best answer is "a humorous story about burglar alarms" because it is the more specific answer choice; in the first paragraph, the story begins with a specific focus on burglar alarms, saying, "[The conversation] landed on the subject of burglar alarms. And now for the first time Mr. McWilliams showed feeling." The rest of the passage maintains this initial focus on burglar alarms.

The next question asks you not to summarize the text, but the logic at work in the story being told.

Sample Question

Which of the following accurately describes how the narrator believes the burglar alarm to work in the passage?

- A.** A thief entering the McWilliams' house causes the burglar alarm goes off.
- B.** The burglar alarm warns other burglars to avoid the McWilliams' house before any potential theft has occurred.
- C.** If a theft is thought to have occurred, the McWilliams' are to ring the burglar alarm and ask the burglars if they have stolen anything.
- D.** The burglar alarm is set off when a thief rings it.

A major part of this passage hinges on the faulty but humorous way in which Mr. McWilliams thinks the burglar alarm works. While "A thief entering the McWilliams' house causes the burglar alarm goes off"



may look like the correct answer because that is how burglar alarms actually work, that is not the case in this passage. The line most crucial for understanding how Mr. McWilliams thinks the burglar alarm works is when he addresses the burglar and says, “But what business have you to be entering this house in this furtive and clandestine way, without ringing the burglar alarm?” Based on this sentence, Mr. McWilliams expected the burglar to ring the burglar alarm on his way into the house, so the correct answer is “The burglar alarm is set off when a burglar rings it.”



Practice Passage: Summarizing and Paraphrasing

Prose Fiction: Adapted from the Advertisement by William Wordsworth to *Lyrical Ballads* by William Wordsworth and Samuel Taylor Coleridge (1798)

It is the honorable characteristic of Poetry that its materials are to be found in every subject which can interest the human mind. The evidence of this fact is to be sought, not in the writings of Critics, but in those of Poets themselves.

The majority of the following poems are to be considered as experiments. They were written chiefly with a view to ascertain how far the language of conversation in the middle and lower classes of society is adapted to the purposes of poetic pleasure. Readers accustomed to the gaudiness and inane phraseology of many modern writers, if they persist in reading this book to its conclusion, will perhaps frequently have to struggle with feelings of strangeness and awkwardness: they will look round for poetry, and will be induced to enquire by what species of courtesy these attempts can be permitted to assume that title. It is desirable that such readers, for their own sakes, should not suffer the solitary word Poetry, a word of very disputed meaning, to stand in the way of their gratification; but that, while they are perusing this book, they should ask themselves if it contains a natural delineation of human passions, human characters, and human incidents; and if the answer be favorable to the author's wishes, that they should consent to be pleased in spite of that most dreadful enemy to our pleasures, our own pre-established codes of decision.

Readers of superior judgment may disapprove of the style in which many of these pieces are executed it must be expected that many lines and phrases will not exactly suit their taste. It will perhaps appear to them, that wishing to avoid the prevalent fault of the day, the author has sometimes descended too low, and that many of his expressions are too familiar, and not of sufficient dignity. It is apprehended, that the more conversant the reader is with our elder writers, and with those in modern times who have been the most successful in painting manners and passions, the fewer complaints of this kind will he have to make.

An accurate taste in poetry, and in all the other arts, Sir Joshua Reynolds has observed, is an acquired talent, which can only be produced by severe thought, and a long continued intercourse with the best models of composition. This is mentioned not with so ridiculous a purpose as to prevent the most inexperienced reader from judging for himself; but merely to temper the rashness of decision, and to suggest that if poetry be a subject on which much time has not been bestowed, the judgment may be erroneous, and that in many cases it necessarily will be so.

1. Which of the following sentences best summarizes the first paragraph?
 - A. The longer the poem, the more materials a poet can use, although only critics can certify this.
 - B. The essence of poetry is a secret only poets know, and to ask a critic where the inspiration for a poem comes from is a foolish thing to do.
 - C. Poetry is derived from all things and the work of critics proves this.
 - D. Inspiration for poetry is found in anything the poet cares to think about and their poetry attests to this being a fact.

2. Which of the following most fully lists solutions considered by Sir Joshua Reynolds to the acquiring of "an accurate taste in poetry?"
 - A. Severe focus and unerring attention to detail
 - B. Several hours of dialogue with a piece followed by the prevention of others from reading it
 - C. Intense contemplation and extended periods of reading the highest quality pieces
 - D. Preoccupation with the facts of a piece and a careful consideration of the popularity of it

3. Which of the following best describes the passage as a whole?
 - A. A discussion of the author's work and a justification of it in a contemporary context
 - B. A list of the honorable features of poetry and the poems in which these features appear
 - C. An advertisement for a collection of poetry written for a bookshop sales magazine
 - D. A criticism of the literary world and a manifesto for change in poetry



Practice Passage: Summarizing and Paraphrasing: Answer Key

1. Which of the following sentences best summarizes the first paragraph?
- A. The longer the poem, the more materials a poet can use, although only critics can certify this.
 - B. The essence of poetry is a secret only poets know, and to ask a critic where the inspiration for a poem comes from is a foolish thing to do.
 - C. Poetry is derived from all things and the work of critics proves this.
 - D. Inspiration for poetry is found in anything the poet cares to think about and their poetry attests to this being a fact.

2. Which of the following most fully lists solutions considered by Sir Joshua Reynolds to the acquiring of “an accurate taste in poetry?”
- A. Severe focus and unerring attention to detail
 - B. Several hours of dialogue with a piece followed by the prevention of others from reading it
 - C. Intense contemplation and extended periods of reading the highest quality pieces
 - D. Preoccupation with the facts of a piece and a careful consideration of the popularity of it

3. Which of the following best describes the passage as a whole?
- A. A discussion of the author’s work and a justification of it in a contemporary context
 - B. A list of the honorable features of poetry and the poems in which these features appear
 - C. An advertisement for a collection of poetry written for a bookshop sales magazine
 - D. A criticism of the literary world and a manifesto for change in poetry

The author is suggesting in the first paragraph that inspiration or “the materials” with which poetry is made are found in everything and that it is better to seek validation of this as a fact within the work of poets instead of in the work of critics. Therefore, only poetry can attest that it can be inspired by anything. The work of critics is misleading, as it could suggest that poetry should only be about certain subjects.

Of the presented answer choices, several are quite similar. We can eliminate the answer that reads “Severe focus and unerring attention to detail,” as it does not quite fit with the text since the passage does not discuss an attention to detail, only a prolonged exposure to a piece. The answer choice “Preoccupation with the facts of a piece and a careful consideration of the popularity of it” negates itself by suggesting a piece has to be popular, while the passage only suggests it has to be of high quality. Therefore, the answer “Intense contemplation and extended periods of reading the highest quality pieces” most fully lists the solutions considered.

It is obvious from the passage that the author is justifying the “following poems” and that they are trying to justify them in a “modern,” or contemporary, context. The author goes to some length to discuss the work and how readers might react to it.



Generalizations, Inferences, and Predictions

Inferences and predictions are two types of questions that ask you to think beyond the passage, taking the information provided and attempting to use it to make informed but unconfirmed guesses about topics beyond the passage's scope. These questions can be tricky, as you need to read ACT Reading passages as if encountering their topics for the first time while focusing on employing sturdy logical reasoning. With a bit of practice, you can become comfortable with these questions and confident in the way you approach them.

Practice Passage

Humanities: Adapted from "Utopia" by Thomas More (1516) in *Ideal Commonwealths: Comprising More's Utopia, Bacon's New Atlantis, Campanella's City of the Sun, and Harrington's Oceans* (1901)

Thus have I described to you, as particularly as I could, the constitution of that commonwealth, which I do not only think the best in the world, but indeed the only commonwealth that truly deserves that name. In all other places it is visible that, while people talk of a commonwealth, every man only seeks his own wealth; but there, where no man has any property, all men zealously pursue the good of the public, and, indeed, it is no wonder to see men act so differently, for in other commonwealths every man knows that unless he provides for himself, how flourishing soever the commonwealth may be, he must die of hunger, so that he sees the necessity of preferring his own concerns to the public; but in Utopia, where every man has a right to everything, they all know that if care is taken to keep the public stores full no private man can want anything, for among them there is no unequal distribution, so that no man is poor, none in necessity, and though no man has anything, yet they are all rich; for what can make a man so rich as to lead a serene and cheerful life, free from anxieties, neither apprehending want himself, nor vexed with the endless complaints of his wife? He is not afraid of the misery of his children, nor is he contriving how to raise a portion for his daughters, but is secure in this, that both he and his wife, his children and grandchildren, to as many generations as he can fancy, will all live both plentifully and happily, since, among them, there is no less care taken of those who were once engaged in labor, but

grow afterwards unable to follow it, than there is, elsewhere, of these that continue still employed. I would gladly hear any man compare the justice that is among them with that of all other nations; among whom may I perish if I see anything that looks either like justice or equity; for what justice is there in this: that a nobleman, a goldsmith, a banker, or any other man, who either does nothing at all, or, at best, is employed in things that are of no use to the public, should live in great luxury and splendor upon what is so ill acquired, and a mean man, a carter, a smith, or a plowman, who works harder even than the beasts themselves, and is employed in labors so necessary, that no commonwealth could hold out a year without them, can only earn so poor a livelihood and must lead so miserable a life, that the condition of the beasts is much better than theirs? For as the beasts do not work so constantly, so they feed almost as well, and with more pleasure, and have no anxiety about what is to come, whilst these men are depressed by a barren and fruitless employment, and tormented with the apprehensions of want in their old age; since that which they get by their daily labor does but maintain them at present, and is consumed as fast as it comes in, there is no overplus left to lay up for old age.



Generalizations, Inferences, and Predictions

Making accurate and reasonable inferences and predictions based on what you read takes a certain discerning perspective. Both need to be firmly rooted in specific evidence you can point to in the passage. If you can't articulate what material forms the basis of an inference or prediction, it might not be a reliable or reasonable one. Just like quoting material to back up your claims when writing a paper, inferences and predictions both need evidence to justify them. Even if questions aren't asking you about this evidence directly, it forms a crucial middle step in coming up with correct answers.

It's also crucial to base your inferences in the passage in the sense that you should never rely on any information you may know about a topic that is not mentioned in the passage. While it's not likely that the test will ask you to work with incorrect or outdated information, it may provide only certain, limited views of subjects and then ask you what inferences and predictions can be supported by the material at hand. If you incorporate outside knowledge into your responses, you're potentially no longer doing what the test is asking you to—determine information using critical thinking skills based on a passage.

For instance, let's say you are reading a passage about the first moon landing and asked to infer why the U.S. wanted to be the first to the moon. Let's say you're an expert in the history of NASA, Kennedy's speech, and the Russian-U.S. "space race"—if any of these are presented as answer options, they might look pretty appealing; however, perhaps the passage is written to include support for the more general idea that the United States wanted to be the first to put a person on the moon because no one had done it before. Perhaps the passage conveys one author's personal analysis of the cause, and doesn't talk about a generally accepted historical cause. If one answer choice mentioned Kennedy's speech and the passage didn't talk about that speech at all, you'd get the question wrong simply because you knew about the topic! In sum, if you recognize that you know about the topic of a passage, pretend you're learning about it for the first time. Analyzing the information presented in ACT Reading passage this way can help you focus on making well-supported inferences and predictions. With that warning out of the way, let's tackle some practice problems!

Making Inferences

Let's look over a question that asks you which information you can infer based on what the passage says.

Sample Question

Which of the following can be inferred from the passage?

- A. Academic work is prized above manual labor in Utopia
- B. Utopia is just while other countries are not
- C. The inhabitants of Utopia are extremely insular and don't know that other countries exist
- D. Every inhabitant of Utopia performs physical labor every day, no matter his or her age.

We can figure out the correct answer by systematically disproving the other choices. The idea that "academic work is prized above manual labor in Utopia" cannot be inferred from the passage because the passage never mentions academic work specifically. Similarly, the answer choice "the inhabitants of Utopia are extremely insular and don't know that other countries exist" isn't supported either because the passage tells us nothing about what Utopians know of other countries. The answer "Every inhabitant



of Utopia performs physical labor every day, no matter his or her age” is incorrect because the passage contradicts it when the narrator states “there is no less care taken of those who were once engaged in labor, but grow afterwards unable to follow it, than there is, elsewhere, of these that continue still employed.” If it’s possible that there are people “once engaged in labor” but now “unable to follow it,” this can be taken to mean that not everyone works every day, as some people cannot. The idea that “the narrator believes that the Utopian constitution could be improved upon by incorporating features of other countries’ founding documents” is similarly contradicted by the passage’s first sentence, in which the narrator states that he thinks the Utopian constitution is “the best in the world.” It wouldn’t make sense to incorporate aspects of what the narrator considers less-than-perfect constitutions into a perfect one in order to try to improve it; it would most likely make it worse. The last answer choice remaining is the correct one: B, “Utopia is just while other countries are not.” This statement is supported in the passage when the narrator says, “I would gladly hear any man compare the justice that is among [the Utopians] with that of all other nations; among whom may I perish if I see anything that looks either like justice or equity.”

Making Predictions

Let’s take a step away from inferences and toward predictions, which involve larger logical leaps.

Sample Question

Based on the information presented in the passage, which of the following is most likely to be true concerning theft of personal property in Utopia?

- A. This type of theft likely cannot occur in Utopia because no one owns any personal property.
- B. Such theft is likely rare in Utopia because the inhabitants are much more concerned with the public good than with private possessions.
- C. Such theft is likely rare in Utopia because everyone owns an identical set of personal possessions.
- D. Theft is likely much more prevalent in Utopia because its inhabitants spend so much time working outside of their homes.

The narrator tells us near the beginning of the passage that Utopia is a place “where no man has any property, [and] all men zealously pursue the good of the public.” He goes on to describe it as a place “where every man has a right to everything,” and says “[its inhabitants] all know that if care is taken to keep the public stores full no private man can want anything, for among them there is no unequal distribution, so that no man is poor, none in necessity, and though no man has anything, yet they are all rich.” Considering the question, the key phrases from these quotations are that “no man has any property” and “no man has anything, yet they are all rich.” This tells us that no one owns any personal property in Utopia, supporting answer choice A, “this type of theft likely cannot occur in Utopia because no one owns any personal property.”



Sample Question

Imagine you meet a Utopian who knows nothing of countries other than his or her own. Based on the passage, which of the following would likely be a new concept to him or her?

- A. Going to a hospital when feeling ill
- B. Taking off work for a national holiday
- C. Saving money for retirement
- D. Clearing a forest in order to make paper from the trees

Saving money for retirement would most likely be a new concept to a Utopian based on the passage, as it states that “among them, there is no less care taken of those who were once engaged in labor, but grow afterwards unable to follow it, than there is, elsewhere, of these that continue still employed.” This quotation tells us that Utopians don’t have to worry about saving money to live on when they are too old to work—or in other words, saving money for retirement.

Making Generalizations

Some ACT Reading questions will ask you to take a step back from the passage and answer a question that requires you to make a generalization about the whole thing. Questions about the theme of a passage would fall into this category, as would questions like the following, which asks you to identify which terms describe the passage accurately—and which one does not.

Sample Question

Of the following choices, which does NOT describe an aspect of the passage?

- A. Apolitical
- B. Concerned with economics
- C. Comparative
- D. Critical

Given that the passage concerns a country, its organizational practices, and its citizens, it would be incorrect to call the passage “apolitical,” a word that literally means not political, or more specifically, not considering or concerned with politics. The passage compares Utopia with other countries with a specific focus on the distribution of wealth, so we can certainly say it is “comparative” and “concerned with economics.” The passage criticizes other countries, so it includes “critical” aspects.



Prose Fiction: Adapted from “The Cask of Amontillado” by Edgar Allan Poe (1846)

The thousand injuries of Fortunato I had borne as I best could, but when he ventured upon insult I vowed revenge. You, who so well know the nature of my soul, will not suppose, however, that I gave utterance to a threat. At length I would be avenged; this was a point definitively settled — but the very definitiveness with which it was resolved precluded the idea of risk. I must not only punish but punish with impunity. A wrong is unredressed when retribution overtakes its redresser. It is equally unredressed when the avenger fails to make himself felt as such to him who has done the wrong.

It must be understood that neither by word nor deed had I given Fortunato cause to doubt my good will. I continued, as was my wont, to smile in his face, and he did not perceive that my smile now was at the thought of his immolation.

He had a weak point — this Fortunato — although in other regards he was a man to be respected and even feared. He prided himself upon his connoisseurship in wine. In this respect I did not differ from him materially; — I was skillful in the Italian vintages myself, and bought largely whenever I could.

It was about dusk, one evening during the supreme madness of the carnival season, that I encountered my friend. He accosted me with excessive warmth, for he had been drinking much. The man wore motley. He had on a tight-fitting parti-striped dress, and his head was surmounted by the conical cap and bells. I was so pleased to see him that I thought I should never have done wringing his hand.

I said to him — “My dear Fortunato, you are luckily met. How remarkably well you are looking to-day. But I have received a pipe of what passes for Amontillado, and I have my doubts.”

“How?” said he. “Amontillado? A pipe? Impossible! And in the middle of the carnival!”

“I have my doubts,” I replied; “and I was silly enough to pay the full Amontillado price without consulting you in the matter. You were not to be found, and I was fearful of losing a bargain.”

“Amontillado!”

“I have my doubts.”

“Amontillado!”

“And I must satisfy them.”

“Amontillado!”

“As you are engaged, I am on my way to Luchresi. If any one has a critical turn it is he. He will tell me —”

“Luchresi cannot tell Amontillado from Sherry.”

“And yet some fools will have it that his taste is a match for your own.”

“Come, let us go.”

1. What can you predict is MOST likely to happen later in the story?
 - A. The narrator will get revenge on Fortunato
 - B. The narrator will sell the amontillado to Fortunato
 - C. The narrator will reveal that he stole the amontillado
 - D. Fortunato will steal the amontillado
2. Given the passage as a whole, Fortunato’s expression of disbelief about the pipe of Amontillado implies all of the following EXCEPT:
 - A. Amontillado is hard to get in large quantities
 - B. Amontillado is hard to get during Carnival season
 - C. Amontillado can be gotten for a bargain
 - D. Amontillado is very expensive
3. Judging from the details in the passage, all of the following can be inferred about the Carnival season EXCEPT:
 - A. People dress up in costume during Carnival
 - B. Carnival is a religious holiday
 - C. Carnival is a chaotic holiday
 - D. Carnival lasts for multiple days
4. To where does Fortunato most likely want to go with the narrator at the end of the passage?
 - A. To somewhere quiet where he can apologize to the narrator
 - B. To a shop having a sale during Carnival
 - C. To somewhere where he can get revenge on the narrator
 - D. To where the narrator is keeping the amontillado
5. Aside from having an ulterior motive, why does the narrator mention the amontillado to Fortunato?
 - A. Because Fortunato can confirm if the amontillado is authentic.
 - B. Because Fortunato knows the best way to serve amontillado
 - C. Because Fortunato is a trader and knows to whom the narrator can sell the amontillado for the best price
 - D. Because he wants to make Fortunato jealous of the great deal he got on the amontillado



Practice Passage: Inferences and Predictions: Answer Key

1. What can you predict is MOST likely to happen later in the story?

- ➔ A. The narrator will get revenge on Fortunato.
- B. The narrator will sell the amontillado to Fortunato.
- C. The narrator will reveal that he stole the amontillado.
- D. Fortunato will steal the amontillado.

2. Given the passage as a whole, Fortunato's expression of disbelief about the pipe of Amontillado implies all of the following EXCEPT _____.

- A. Amontillado is hard to get in large quantities
- B. Amontillado is hard to get during Carnival season
- ➔ C. Amontillado can be gotten for a bargain
- D. Amontillado is very expensive

3. Judging from the details in the passage, all of the following can be inferred about the Carnival season EXCEPT:

- A. People dress up in costume during Carnival
- ➔ B. Carnival is a religious holiday
- C. Carnival is a chaotic holiday
- D. Carnival lasts for multiple days

The story begins with the narrator vowing to get revenge on Fortunato: "The thousand injuries of Fortunato I had borne as I best could, but when he ventured upon insult I vowed revenge." Since the narrator does not get revenge on Fortunato in the passage, but it seems as if the story isn't over when the passage concludes, this tells us (rather directly) that among the presented options, it is most likely that "the narrator will get revenge on Fortunato."

While the narrator says that he bought the cask of Amontillado because he feared losing a bargain, there is nothing to suggest that Amontillado (a kind of sherry or fortified wine—more generally, an alcoholic beverage) is anything but expensive, rare, and hard to obtain during the Carnival season, especially an entire cask (or pipe) of it.

Although Carnival—also known as Mardi Gras—actually refers to the period before the religious holiday Ash Wednesday, nothing in the passage tells us this. Instead, the narrator describes the season as "supreme madness" and implies that Fortunato's costume (he is dressed in motley) and drunkenness are common among celebrants of this holiday. Additionally, the fact that he refers to "the Carnival season" suggests that the holiday lasts for more than one day.



4. To where does Fortunato most likely want to go with the narrator at the end of the passage?
- A. To somewhere quiet where he can apologize to the narrator
 - B. To a shop having a sale during Carnival
 - C. To somewhere where he can get revenge on the narrator
 - D. To where the narrator is keeping the *amontillado*
5. Aside from having an ulterior motive, why does the narrator mention the *amontillado* to Fortunato?
- A. Because Fortunato can confirm if the *amontillado* is authentic.
 - B. Because Fortunato knows the best way to serve *amontillado*
 - C. Because Fortunato is a trader and knows to whom the narrator can sell the *amontillado* for the best price
 - D. Because he wants to make Fortunato jealous of the great deal he got on the *amontillado*

When Fortunato declares “Come, let us go” at the end of the passage, you have to rely on inferential evidence to figure out where it is that he wants to go with the narrator. No mention of a shop having a sale during Carnival is made, and no suggestion is given that that is their intended destination. We’re not given any clues to suggest that Fortunato wants to go somewhere quiet to apologize to the narrator, as Fortunato doesn’t appear to realize that he’s insulted the narrator to the point where the narrator is plotting revenge. Furthermore, it is not Fortunato who wants to get revenge on the narrator—it is the narrator who wants to get revenge on Fortunato. The remaining answer choice is the best one: given the context of the characters’ conversation leading up to Fortunato declaring that they should go somewhere particular, you can infer that Fortunato wants to go to where the narrator is keeping the *amontillado*. Even if you did not reach this answer at first, realizing that the other answers are incorrect can help you reach it by process of elimination.

Answering this question requires that you pick up on some subtleties conveyed in the characters’ dialogue.

I said to him — “My dear Fortunato, you are luckily met. How remarkably well you are looking to-day. But I have received a pipe of what passes for *Amontillado*, and I have my doubts.”

“How?” said he. “*Amontillado*? A pipe? Impossible! And in the middle of the carnival!”

“I have my doubts,” I replied; “and I was silly enough to pay the full *Amontillado* price without consulting you in the matter. You were not to be found, and I was fearful of losing a bargain.”

First, notice that the narrator describes the *amontillado* as “a pipe of what passes for *Amontillado*,” and that he follows this statement with, “and I have my doubts.” He repeats this two paragraphs later. Doubts about what? About whether the liquor that he has received is really *amontillado* or not. The use of “passes for” conveys that the narrator thinks the *amontillado* might not actually be *amontillado*. Yet, the narrator says he “was silly enough to pay the full *Amontillado* price without consulting [Fortunato] in the matter.” Why would the narrator be silly for not consulting Fortunato in this situation? This suggests that Fortunato is an *amontillado* expert who can tell if the *amontillado* the narrator purchased is authentic or not. This matches up with answer choice A, the correct answer.



Comparing What's Said: Paired Passages

On the reading comprehension section of a standardized test, paired passages may seem to be the most intimidating type of problem you can face. Having to deal with two passages in conjunction opens up new possibilities for questions. While some questions may ask about one passage or another, others may test your awareness of the passages' unique and common features and ask you to make inferences about how their authors would interact based on what you've read. The questions that rely on both passages in conjunction may seem like the toughest, but with a bit of practice with common question types, you can feel ready for them.

Section Outline

[Comparing and Contrasting Ideas in Paired Passages](#)

[Identifying the Main Idea of Each Passage](#)

[Comparing the Main Ideas of Paired Passages](#)

[Practice Passages: Comparing the Main Ideas of Paired Passages](#)

[Comparing and Contrasting Arguments in Paired Passages](#)

[Comparing Arguments: Where Do the Authors Agree?](#)

[Contrasting Arguments: Where Do the Authors Disagree?](#)

[Hypothetical Changes to Arguments](#)

[Making Inferences Based on Newly Introduced Information](#)

[Choosing Which Passage Best Fits a Description](#)

[Comparing and Contrasting Language and Rhetoric in Paired Passages](#)

[Direct Comparisons of Style Descriptions](#)

[Direct Comparisons of Textual Features](#)

[Making Inferences about Passage Genre](#)

[Making Inferences about the Authors](#)

[Multi-Passage Critical Thinking: Inferences, Conclusions, and Predictions](#)

[Drawing Inferences and Conclusions from Paired Passages](#)

[Making Predictions Based on Paired Passages](#)

[Practice Passages: Multi-Passage Critical Thinking](#)

[Practice Passage: Comparing What's Said](#)



Comparing and Contrasting Ideas in Paired Passages

Some paired passages will be accompanied by questions that ask you not only to identify the main ideas of both passages, but also to compare and discuss the differences and similarities between these central ideas.

The process for [identifying the main idea](#) of paired passages is fundamentally similar to the process for identifying the main idea of a single passage, but rather than making the process twice as much work, the pairing of passages can actually work to help you understand the main idea of both individual passages more easily. Passages selected for pairing in the ACT Reading section will have been selected, and paired, for specific reasons, and they will always be relevant to one another or have some sort of thematic relationship. Understanding how the two passages relate to one another can offer many clues about the main idea of each passage.

Practice Passages

Passage 1

Adapted from Samuel Johnson's "Preface to Shakespeare" (1765)

Shakespeare is above all writers, at least above all modern writers, the poet of nature; the poet that holds up to his readers a faithful mirror of manners and of life. His characters are not modified by the customs of particular places, unpracticed by the rest of the world; by the peculiarities of studies or professions, which can operate but upon small numbers; or by the accidents of transient fashions or temporary opinions: they are the genuine progeny of common humanity, such as the world will always supply, and observation will always find. His persons act and speak by the influence of those general passions and principles by which all minds are agitated, and the whole system of life is continued in motion. In the writings of other poets a character is too often an individual; in those of Shakespeare it is commonly a species.

Passage 2

Adapted from John Dryden's "Essay of Dramatick Poesie" (1668)

To begin, then, with Shakespeare. He was the man who of all modern, and perhaps ancient poets, had the largest and most comprehensive soul. All the images of Nature were still present to him and he drew them, not laboriously, but luckily; when he describes any thing, you more than see it, you feel it too. Those who accuse him to have wanted learning, give him the greater commendation: he was naturally learned; he needed not the spectacles of books to read nature; he looked inwards, and found her there. I cannot say he is everywhere alike; were he so, I should do him injury to compare him with the greatest of mankind. He is many times flat, insipid; his comic wit degenerating into clenches [puns], his serious swelling into bombast. But he is always great, when some occasion is presented to him; no man can say he ever had a fit subject for his wit, and did not then raise himself as high above the rest of poets.

Identifying the Main Idea of Each Passage

The first step in approaching paired passages is to read each of the passages on their own and try to decide what you think the main ideas are. The first step is to read the entire passage through, noting any clues about the tone, style, or attitude, as well as reading for the literal content the passage provides. In the case of these two passages, we can clearly identify the main subject as the English dramatist William Shakespeare. We can also tell from the analytical tone of both passages that they are expressing judgments about the quality of Shakespeare's literary work, and the nature of his talent in representing "Nature" in art. So, in order to understand the main idea of the passages, we must determine what kind of judgments each passage is passing on their subject.



Comparing the Main Ideas of Paired Passages

Sample Question

The main difference between the two authors' opinions of William Shakespeare is that:

- A.** the author of passage 1 expresses some harsh critiques of Shakespeare's work, while the author of passage 2 is exclusively laudatory.
- B.** the author of passage 2 is mostly dismissive of Shakespeare's work, while the author of passage 1 is exclusively laudatory.
- C.** the author of passage 1 praises Shakespeare as the best writer at portraying the complexities of humanity, while the author of passage 2, while praising Shakespeare's natural talent, feels Shakespeare to be fundamentally below the greatest authors in history.
- D.** Both authors are exclusively laudatory of Shakespeare and his work. There is no fundamental difference between their opinions.

From reading them separately, we can tell that both passages praise Shakespeare, but that Passage 2 certainly is not “exclusively laudatory,” so we can eliminate option D immediately. Reading the passages together, we can see that the author of passage 1 is almost exclusively laudatory; however, in Passage 2 the author praises Shakespeare as having “the largest and most comprehensive soul,” which is hardly dismissive. So, we can eliminate A and B based on these understandings.

Based on this process of elimination, we know that the correct answer will be C, but let's further interrogate the truth of the claim made in this answer choice.

The main difference between the two passages is in the nature and intensity of the praise or criticism. It is not enough to know that the authors are discussing the work of William Shakespeare, you must also understand the nature of and basis for each author's treatment of that subject, and for this purpose, it is often helpful to think about the passages as they relate to each other.

Passage 1 praises both Shakespeare's talent and his actual written work product. Passage 2 is fundamentally focused on Shakespeare's natural talent and “soul,” defending him from the apparently common critique that he was not sufficiently “learned,” while being strongly critical of some aspects of Shakespeare's actual work product, even going so far as to say that it would “do [Shakespeare] injury to compare him with the greatest of mankind,” and saying that his work is often “flat [and] insipid.”

The key to answering any question about the main ideas is to understand the basis by which the passages were analyzing the work and artistic talent of William Shakespeare. The clear difference here is that the main idea of Passage 1 is that Shakespeare's individual talent and work are unique and better in at least one specific quality than most, if not all, other writers. Passage 2, on the other hand, focuses almost exclusively on the level of “natural” talent that can be ascribed to Shakespeare, while also outlining, in fairly harsh terms, some of the areas in which the author believes he does not excel.

While both of these passages are very old and written in somewhat antiquated language, and as such, possibly a bit difficult to understand, the fact that these two passages were paired created a clear, noticeable contrast in their treatment of a shared topic, and helped to make the main points of both passages more clear.



Passage 1

Adapted from *Manual of Gardening* by Liberty Hyde Bailey (ed. 1910)

Having now discussed the most essential elements of gardening, we may give attention to such minor features as the actual way in which a satisfying garden is to be planned and executed.

Speaking broadly, a person will get from a garden what he puts into it; and it is of the first importance, therefore, that a clear conception of the work be formulated at the outset. I do not mean to say that the garden will always turn out what it was desired that it should be; but the failure to turn out properly is usually some fault in the first plan or some neglect in execution.

Sometimes the disappointment in an ornamental garden is a result of confusion of ideas as to what a garden is for. One of my friends was greatly disappointed on returning to his garden early in September to find that it was not so full and floriferous as when he left it in July. He had not learned the simple lesson that even a flower-garden should exhibit the natural progress of the season. If the garden begins to show ragged places and to decline in late August or early September, it is what occurs in all surrounding vegetation. The year is maturing. The garden ought to express the feeling of the different months. The failing leaves and expended plants are therefore to be looked on, to some extent at least, as the natural order and destiny of a good garden.

Passage 2

Adapted from “Plant Food Elements” in *The Chemistry of Plant Life* by Roscoe Wilfred Thatcher (1921)

The raw materials from which the food and tissue building compounds of plants are synthesized include carbon dioxide, oxygen, water, nitrogen, phosphorus, sulfur, potassium, calcium, magnesium, and iron. The two gases first mentioned are derived directly from the air through the respiratory organs of the plant. Water is taken into the plant chiefly from the soil through its fibrous roots. All the other elements in the list are taken from the soil, nitrogen being derived from decaying organic matter. The original source of the nitrogen is however the atmosphere, from which the initial supply of nitrogen is obtained by direct assimilation by certain bacteria and perhaps other low forms of plant life and the remaining ones from the mineral compounds of the soil.

Carbon dioxide and oxygen, being derived from the air, are always available to the leaves and stems of growing plants in unlimited supply, but the supply available to a seed when germinating in the soil or to the roots of a growing farm crop may sometimes become inadequate, especially in soils of a very compact texture or “water-logged” soils. In such cases the deficiency of these gaseous food elements may become a limiting factor in plant growth.

Water is often a limiting factor in plant growth. Experiments which have been repeated many times and under widely varying conditions show that when water is supplied to a plant in varying amounts by increasing the percentage of water in the soil in which the plant is growing by regular increments up to the saturation point, the growth of the plant or yield of the crop increases up to a certain point and then falls off because the excess of water reduces the supply of air which is available to the plant roots. Hence, abundance of water is in general a most essential factor in plant growth. Under normal conditions of air and moisture supply, however, the plant food elements which may be considered to be the limiting factors in the nutrition and growth of plants are the chemical elements mentioned in the list above.



1. The primary difference between these two passages is that:
 - A. the two passages give contradictory advice about watering
 - B. Passage 1 is concerned with the emotional impact and rewards of keeping a garden; Passage 2 is a scientific text that explains the basic elements of fertile soil
 - C. Passage 1 is primarily concerned with the author's friend's experience with gardening; Passage 2 is a scientific text that explains the basic elements of fertile soil
 - D. There is no substantive difference between the two texts; they are both about gardening.
2. Which of the following statements accurately reflects the role of evidence in the passages?
 - A. Passage 1 uses anecdotal evidence; Passage 2 uses scientific evidence.
 - B. Passage 1 uses quantitative evidence; Passage 2 uses anecdotal evidence.
 - C. Passage 1 does not use evidence to support its assertions; Passage 2 uses scientific evidence.
 - D. Neither passage uses evidence to support its statements.
3. With which of the following actions would the authors of these passages most disagree?
 - A. Digging a trench around a garden to collect water
 - B. Using fertilizer in one's garden
 - C. Excessively watering one's garden in an effort to extend the growing season
 - D. Based on the passages, both authors would agree with all of these statements.

Reveal Answers and Explanations



Comparing and Contrasting Arguments in Paired Passages

If you notice that one or both of the passages presented are argumentative in nature, there's a good chance that you will encounter some questions about how the authors' arguments interact.

Tip #1: Paraphrase and Summarize

Distilling the argument from a densely worded passage can make answering questions about it a lot easier. You don't need to take much time to jot down notes, but scrawling a few nouns and verbs beside each paragraph reflecting its main points can help you reorient yourself when comparing passages. You don't want to let archaic language get in the way of your understanding.

Tip #2: Give Passages Titles If They Don't Have Them

Unlike in sections based on one passage, you may not receive a title for the passages in paired sections; this might give away too much of the author's opinion overtly when the test writers want you to infer it. Instead, you may have to use context clues to sort out what is being discussed in each passage. Making up a title for each passage may help you to keep them distinct in your mind as you answer questions, and it may even help you identify the main point on which they converge.

Tip #3: Consider Main Ideas After Reading the First Passage

You just finished reading the first of two paired passages. Jumping to the second one immediately might seem like the thing to do, but this may not be as helpful as taking a moment to consider the main points of what you just read. While the passage's argument may be specific to it, the main topics it discusses are likely to reappear in the second passage. If you take a moment to consider them, you can prepare yourself to be on the lookout for those topics in the second passage and to note how the authors' views on these main ideas differ.



Consider the following paired passages:

Passage 1

Adapted from *Federalist No. 25: The Powers Necessary to the Common Defense Further Considered* by Alexander Hamilton (1787)

It may perhaps be urged that [standing armies] ought to be provided for by the State governments, under the direction of the Union. But this would be, in reality, an inversion of the primary principle of our political association, as it would in practice transfer the care of the common defense from the federal head to the individual members: a project oppressive to some States, dangerous to all, and baneful to the Confederacy.

The territories of Britain, Spain, and of the Indian nations in our neighborhood do not border on particular States, but encircle the Union from Maine to Georgia. The danger, though in different degrees, is therefore common. And the means of guarding against it ought, in like manner, to be the objects of common councils and of a common treasury. It happens that some States, from local situation, are more directly exposed. New York is of this class. Upon the plan of separate provisions, New York would have to sustain the whole weight of the establishments requisite to her immediate safety, and to the mediate or ultimate protection of her neighbors. This would neither be equitable as it respected New York nor safe as it respected the other States.

Passage 2

Adapted from *Brutus No. 10* by Brutus (1788)

The liberties of a people are in danger from a large standing army, not only because the rulers may employ them for the purposes of supporting themselves in any usurpations of power, which they may see proper to exercise, but there is great hazard, that an army will subvert the forms of the government, under whose authority, they are raised, and establish one, according to the pleasure of their leader.

We are informed, in the faithful pages of history, of such events frequently happening. Two instances have been mentioned in a former paper: Rome and Britain. In the first, the liberties of the commonwealth were destroyed, and the constitution overturned, by an army lead by Julius Caesar, who was appointed to the command by the constitutional authority of that commonwealth. He changed it from a free republic, whose fame had sounded and is still celebrated by all the world, into that of the most absolute despotism. A standing army effected this change. The same army that in Britain vindicated the liberties of that people from the encroachments and despotism of a tyrant king assisted Cromwell, their General, in wresting from the people that liberty they had so dearly earned.

The advocates for power, in support of this right in the proposed government, urge that a restraint upon the discretion of the legislatures, in respect to military establishments in time of peace, would be improper to be imposed, because they say it will probably be necessary to keep up a small body of troops to garrison a few posts in order to guard against the sudden encroachments of the Indians, or of the Spaniards and British; and therefore, the general government ought to be invested with power to raise and keep up a standing army in time of peace, without restraint, at their discretion.

I confess, I cannot perceive that the conclusion follows from the premises. Logicians say, it is not good reasoning to infer a general conclusion from particular premises; though I am not much of a logician, it seems to me this argument is very like that species of reasoning.



Comparing Arguments: Where Do the Authors Agree?

Figuring out where the authors agree means considering the arguments being made, not just the topics being discussed. Both example passages mention the possibility of being attacked, for instance, but this information doesn't have the same importance in each passage, and it is used a different way in each as well. If the authors have differing opinions on a subject, as in this case, it may seem as if they don't agree about anything; however, certain questions can ask you to find the point that they both assert—perhaps a very general one, or the starting point of the debate at hand.

Sample Question

With which of the following assertions would both authors agree?

- A.** No limitations should be placed on the federal government's ability to raise a standing army.
- B.** If a country has a standing army, it can endanger the freedoms of its people.
- C.** Whether or not the federal government can raise a standing army in peacetime is important to its well-being.
- D.** The Union shouldn't have a standing army because it would cost too much.

Three of the incorrect answer choices state points with which only one author clearly agrees: "The Union shouldn't place any limitations on the federal government's ability to raise a standing army" is only supported by Passage 1; "If a country has a standing army, it can endanger the freedoms of its people" and "If a general conclusion is drawn from specific premises, we shouldn't consider that general conclusion at all" are each taken from Passage 2. We cannot say that the authors agree with each other if there is no evidence in each text that this is so, so none of these answers can be the correct one. "The Union shouldn't have a standing army because it would cost too much" is incorrect because it misquotes a point from Passage 1: that the costs of a standing army should be shared by all of the states it protects.

The correct answer is "Whether or not the Union can raise a standing army in peacetime is important to its well-being." The author of Passage 1 is arguing that letting the federal government have a standing army in peacetime is a prudent idea; the author of Passage 2 is arguing against this point and saying it is a dangerous idea. The "whether or not" in this answer captures their disagreement about the specific effects of a standing army, but both authors clearly agree that the issue of the Union being able to raise a standing army in peacetime is important to its well-being: the author of Passage 1 thinks that if it cannot raise a standing army, it will be poorly defended against attacks, while the author of Passage 2 thinks that if it can raise a standing army, it might go the way of Rome and Britain and have negative changes made to its organization.



Contrasting Arguments: Where Do the Authors Disagree?

Questions that ask you to identify where the authors disagree can be tricky, as mixing up the passages becomes a distinct possibility. In addition, you don't want to infer too much, even in passages where the authors are overtly opposed to one another. Try to make sure each answer you select as correct has textual evidence in both passages; if it doesn't, you may want to consider the other answer choices to see if another is better supported.

Sample Question

Which of the following most accurately describes the difference of opinion found in the above passages?

- A.** Passage 1 is in favor of a federally controlled standing army, whereas Passage 2 opposes one.
- B.** The author of Passage 2 thinks that a state should not be liable for protecting other states; the author of Passage 1 disagrees.
- C.** The author of Passage 1 thinks that historical examples are relevant to an argument, but the author of Passage 2 does not.
- D.** The author of Passage 2 thinks that protecting the country from attacks is more important than a potential misuse of federal power, but the author of Passage 1 disagrees.

Questions like these make it really easy to mix up the passages and to accidentally infer more than you should, two of the major pitfalls of paired-passage questions. Considering each answer choice one at a time can help you narrow down your options. Remember, for the authors to disagree about something, it at least has to be mentioned in each passage!

"The author of Passage 2 thinks that protecting the country from attacks is more important than a potential misuse of federal power, but the author of Passage 1 disagrees." Ok, who is afraid of the misuse of federal power? The author of Passage 2, with his examples of Rome and Britain. The author of Passage 2 does not think that protecting the country from potential attacks is more important than a misuse of federal power, so this answer choice cannot be correct.

"The author of Passage 1 thinks that historical examples are relevant to an argument, but the author of Passage 2 does not." Historical examples are only brought up in Passage 2, so this answer choice is flipped around incorrectly, and even if it were reversed, it wouldn't be true because the author of Passage 1 doesn't use or mention historical examples at all!

"The author of Passage 2 thinks that a state should not be liable for protecting other states; the author of Passage 1 disagrees." Ok, in which passage is the liability of states to protect other states mentioned? Passage 1, where it says, "Upon the plan of separate provisions, New York would have to sustain the whole weight of the establishments requisite to her immediate safety, and to the mediate or ultimate protection of her neighbors. This would neither be equitable as it respected New York nor safe as it respected the other States." This answer choice is incorrect.

"Passage 1 is in favor of a federally controlled standing army, whereas Passage 2 opposes one." - This is the correct answer! Passage 1 is arguing against a state-controlled standing army, and we can tell that the author supports a federally controlled one when he says, "The danger, though in different degrees, is therefore common. And the means of guarding against it ought, in like manner, to be the objects of common councils and of a common treasury." Passage 2 is arguing against a federally controlled standing army.



Hypothetical Changes to Arguments

Some questions on certain tests may present hypothetical “if” statements and ask how this could affect the authors’ arguments. This information will relate in some way to something you have just read, so even if it initially seems unrelated, skim the passages and try to figure out where it lines up with the arguments. Information that goes against what an author is arguing will weaken an argument, while information that provides more evidence to an argument will support it. Note that more information is enough to support an argument, even if the author has already made that point—giving more examples in support of a point already made is just fine; the new information doesn’t have to make a brand new point.

Sample Question

If it were demonstrated that a nation that allowed its city-states to keep their own standing armies was destroyed by a rebellion started by the strongest city-state, how would this affect the preceding arguments?

- A.** It would support the argument of Passage 1.
- B.** It would weaken the argument of Passage 2.
- C.** It would support the argument of Passage 2.
- D.** It would weaken the argument of Passage 1.

Armies causing rebellions sounds like a bad thing for a government. Which passage deals with the potential negative consequences of a federally controlled standing army? Passage 2. You can find supporting evidence that this point would support the argument of Passage 2 in that the author already provides two examples of similar historical events in Rome and Britain. Adding a third event to the author’s list would only strengthen his argument.



Making Inferences Based on Newly Introduced Information

Some questions may move out of the hypothetical realm and provide new information. This information could be a “newly discovered” point, a revelation that a point is actually false, or, like the following question, a third opinion.

Sample Question

“The means of defense against foreign danger have been always the instruments of tyranny at home.”

- James Madison in Volume I of *Records of the Federal Convention* by Max Farand
(1787, ed. 1911)

Based on what you’ve read in the passages, with which author would Madison most likely side?

- A.** Madison would likely side with the author of Passage 1.
- B.** Madison would likely side with the author of Passage 2.
- C.** Madison would likely oppose both authors.
- D.** Madison would likely side with both authors.

We can infer that Madison could be talking about standing armies and calling them “means of defense against foreign danger” and “the instruments of tyranny at home.” Since the passages are arguing about the effects of having a federally controlled standing army, it’s highly unlikely that Madison would side with both or neither of the authors—he’ll side with one, since he’s talking about what they’re debating.

While “the means of defense against foreign danger” may seem to support Passage 1 and “the instruments of tyranny at home” may seem to support Passage 2, consider that the standing armies discussed in the passage are always discussed in terms of what changes they will cause “at home,” in times of peace. Neither author ever discusses the standing army going outside of the country to fight. Plus, the structure of Madison’s statement puts emphasis on the second phrase. If Madison thinks that standing armies in a country at peace are “instruments of tyranny,” he would most likely side with the author of Passage 2, who cites Rome and Britain as having developed tyrannical governments through the misuse of a standing army.



Comparing and Contrasting Language and Rhetoric in Paired Passages

Certain questions about paired passages may focus on the language, tone, and rhetoric involved in each of the paired passages—the way in which each is written, rather than their content, which may be a single topic considered in vastly different ways. Looking at the effect that the form of the text, rather than its content, has on you as the reader will be key to answering these questions correctly.

Tip #1: How is the Passage Saying What It's Saying?

It's easy to focus on what a passage is saying only to encounter questions about how it is saying it. While reading a passage, make note of any overt styles that you encounter. Paying attention to style as well as content can save you time if you encounter questions that ask you to make those kinds of comparisons. Jotting down an adjective or underlining a phrase that exemplifies the style of the whole passage can help.

Tip #2: Watch Out for Rhetorical Devices in Argumentative Passages

Applying the last tip to an argumentative passage means looking out for rhetorical strategies. If you spot any rhetorical questions, straw arguments, or any particular technique that the writer is using to make his or her point, note it. It could be important in comparative questions.

Tip #3: Consider Where it Came From

Certain questions may ask you to infer a passage's genre or the occupation of its author. After you finish a passage, consider where it most likely came from and how you can tell. Seeing certain features in a passage can identify it as belonging to a specific genre. See any numbers or data? If so, the passage isn't likely to be poetry. Similarly, florid language isn't as likely to be found in scientific reports. Before test day, read a few examples from different genres like newspaper articles, scientific journal articles, poems, novels, and literary criticism. Being able to identify a particular genre can help orient you to the rest of a passage's stylistic features.



Consider the following paired passages:

Passage 1

The general structure of the plateau region is rather simple. To great depths it is up primarily of nearly horizontal layers of rock of various sorts and colors lying above another. This structure is best shown in the walls of the Grand Canyon, but is also shown in most of the minor tributary canyons, where the absence of vegetation soil expose the plan of the structure so plainly that its simpler elements may be at a glance. Although the beds of rock appear to be horizontal, they are not absolutely so, but dip slightly to the north. Thus the beds which are about 7,800 feet above sea level at the base of the San Francisco Peaks are only 6,400 feet above sea level at the rim of the canyon forty odd miles to the north, and 4,400 feet above the same plane at the Vermilion Cliffs, nearly an equal distance farther in the same direction.

There is a second element in the structure of the plateau which appears to best advantage in the walls of the canyon. This is a series of nearly vertical fissures, by which the horizontal beds are broken up into a series of blocks. These vertical fissures run in various directions, but there are two principal sets, approximately at right angles to each other. It is the combination of horizontal lines occasioned by the bedding planes and of vertical lines occasioned by the fissures, that produces the remarkable architectural effects so common in the canyon.

Passage 2

You could not dream, then, how the Grand Canyon's labyrinth of chasms allured me—its unfathomable shadows, moving in resistless majesty; its amphitheaters swelling out, until in fancy they are peopled with form of temple and tower and town; the illusive haze that cloaks its myriad peaks; the royal purple of its shadows; its miles of color bands, and every band an age; its mile-deep walls, red with a passion sublime, reaching from this puny age of ours back to that mystic period when the waiting earth first quickened in the throbbing womb of time. The miracle of a sunrise that floods the unanswering wastes until the far reaching line of cliffs seems floating in the rising tide of a crimson sea. The carnage of a sunset that with blood-red banners marches resistless over the ruined wastes of cities until the distant watch towers flash defiance, then signal defeat, then sink back into the night, until the far-flung line of battlements goes down in the unequal struggle and this great underworld grows black in sullen wonder.

Click here to view citations for these passages. The titles of the passages convey information that may affect the difficulty of the questions that follow.



Choosing Which Passage Best Fits a Description of Style

Some style-comparison questions may present a description and ask to which of the two passages, if any, it best applies. Descriptions will likely need to focus on general, overt characteristics of the passage and not individual uses of a particular style, so try considering each passage as a whole when faced with these types of questions.

Sample Question

“But for him who wishes to build his vision on a solid ground of facts—who cares to know what story the rocks tell—the article which follows will be more interesting than a romance.”

- Adapted from *The Grand Canyon of Arizona: being a book of words from many pens, about the Grand Canyon of the Colorado river in Arizona* by Atchison, Topeka, and Santa Fe Railway Company Passenger Department (1909)

- A.** Passage 1
- B.** Passage 2
- C.** This introduction applies equally well to Passage 1 and Passage 2
- D.** This introduction likely does not describe Passage 1 or Passage 2

The quotation describes “a solid ground of facts” and the “what story the rocks tell.” Passage 1 is very objective and based on facts, whereas Passage 2 is florid and poetic, providing mostly imaginative description instead of facts. In addition, Passage 1 discusses different structural aspects of the rocks of the Grand Canyon, whereas Passage 2 doesn’t seem to focus on the meaning of the rocks any more than any other element of the canyon’s appearance. Finally, the quotation contrasts the passage it is describing with “a romance,” and Passage 2 could be described as romantic in style. From this, we can tell that the introduction applies to Passage 1 very well and Passage 2 very poorly, making “Passage 1” the correct answer.



Direct Comparisons of Style Descriptions

Some style-comparison questions may deal with adjectives describing each passage. If you jotted down an adjective or two describing each passage's style after reading it, that information could come in useful here and help you avoid mixing up the passages.

Sample Question

Whereas the author of the first passage is _____, the author of the second passage is _____.

- A.** objective . . . imaginative
- B.** poetic . . . uninterested
- C.** haughty . . . considerate
- D.** unbiased . . . prejudiced

Let's look at each of the words used to describe the first passage and see which are accurate. "Objective" and "unbiased?" Yes—the passage is focused on facts and figures and does not bring in the author's personal opinion. "Poetic?" Nope—the author does not use figurative language or imaginative, emotive description. "Haughty?" Nope—the author doesn't seem to consider himself better than his reader in any sense, and one doesn't get the impression that he is "haughty." "Trite?" No—the author doesn't use any common, overused sayings, and the information he presents is not common advice.

This leaves us with "objective . . . imaginative" and "unbiased . . . prejudiced." Between "imaginative" and "prejudiced," "imaginative" better describes Passage 2. The author of Passage 2 doesn't appear to be prejudiced against anything, and "prejudiced" has a negative connotation that doesn't mesh with the paragraph. "Imaginative," on the other hand, describes Passage 2 well, as it consists mostly of lengthy poetic description of the author's personal experience of seeing the Grand Canyon. So, the correct answer is "objective . . . imaginative."



Direct Comparisons of Textual Features

Instead of focusing on adjectives that describe each passage's style, a question might also ask about textual features found in one passage, the other, or both. Be careful not to mix up the passages!

Sample Question

Passage 1 includes _____, unlike Passage 2, but Passage 2 includes _____, unlike Passage 1.

- A.** measurements . . . figurative language
- B.** personal opinion . . . geographic data
- C.** a call to action . . . rhetorical questions
- D.** poetic language . . . false assertions

This question may be tricky because it is not only asking which features appear in a given passage, but which features are unique to that passage—that is, they only appear in that passage, and not the other one. Let's again consider the list of items attributed to Passage 1: "measurements" appear in that the author provides quantitative data; "personal opinion" is nowhere to be found, as Passage 1 is written in third-person perspective; "a call to action" does not appear in Passage 1, which makes sense, as it is presenting information, not arguing a point; "poetic language" doesn't appear in Passage 1, and "description of the appearance of the Grand Canyon" certainly appears. This leaves us with "measurements . . . figurative language" and "description of the appearance of the Grand Canyon . . . description of the canyon's effect on the author's emotions."

Considering the descriptors applied to Passage 2 in the answer choices doesn't help here, as Passage 2 contains both figurative language and description of the canyon's effect on the author's emotions. At this point, it might be easy to begin worrying, since it may seem like there are two answers that could be correct; however, it's important to consider the subtleties of the question and its use of the word "unlike." For an answer choice to be correct, whichever feature it attributes to Passage 1 cannot be found in Passage 2, and vice versa. So, our question now becomes: can you find "description of the appearance of the Grand Canyon" in Passage 2? Yes. That means that "description of the appearance of the Grand Canyon . . . description of the canyon's effect on the author's emotions" is incorrect. Just to be sure, can you find measurements in Passage 2? No. This confirms that "measurements . . . poetic language" is the correct answer.



Making Inferences About Passage Genre

A passage's stylistic features can shed light on the genre to which it belongs, and questions may test this. Considering where a passage might have been taken from, or, alternatively, where you would be surprised to find it, can help when considering these questions.

Sample Question

One of these passages comes from an article in which explanatory diagrams are included. Which passage is this most likely to be?

- A. Passage 1
- B. Passage 2
- C. Neither passage seems likely to include explanatory diagrams
- D. Both passages seem likely to include explanatory diagrams

This question is testing your knowledge of generic features. What kind of a text would you expect to include explanatory diagrams? A scientific one, perhaps, and one that provided a lot of detailed information that could be better served in an image than in writing. While Passage 2 provides a lot of visual imagery, it is clear in textual form, and diagrams are not usually used in poetic passages. Passage 1, however, provides geographical data, descriptions of angles, and a combination of horizontal and vertical lines that creates the canyon's structure. Each of these could be supported by an explanatory diagram, so "Passage 1" is the correct answer.

Making Inferences About the Authors

Author inference questions can be challenging, and the ones you may encounter with paired passages are no exception. Beware of baseless inferences—any inference you make needs to be grounded in textual support.

Sample Question

The author of one of these passages is a geologist. Which of the passages did he most likely write?

- A. Passage 1
- B. Passage 2
- C. Neither passage seems as if it were written by a geologist
- D. Both passages seem as if they were written by a geologist

What kind of writing would we expect a geologist to write, if we had to identify their profession by their writing? Well, a geologist is a scientist who studies the structure of the Earth and the various substances that make it up, like rocks. We should pick out the passage that sounds more scientific and concerns itself with rocks and the Earth's physical structure. That's Passage 1, so "Passage 1" is the correct answer.



Multi-Passage Critical Thinking: Conclusions, Inferences, and Predictions

In addition to questions [interrogating the main point of paired passages](#), and [the way in which](#) such passages arrive at those points, the ACT Reading section will ask you to think critically about paired passages in two ways: making inferences and making predictions based on the texts. As with the other tasks related to paired passages, the process for thinking critically about these passages involves analyzing not only the passages by themselves, but also understanding the passages in terms of their relationship to one another.

Drawing Inferences and Conclusions from Paired Passages

Questions that test the student's ability to make inferences or draw conclusions based on a text are considered a test of critical thinking because such questions require the student to make a judgment about something that is not explicitly stated in the text. When you make an inference, you are making a (very!) educated guess about something related to the text that is not covered directly in the text.

An example might be helpful here. So, let's say I tell you I cannot abide 400m swim races and that I specifically object to swim races of any measured distance, and that, in my opinion, competition has no place in the lanes of a swimming pool.

While I did not specifically mention competitive diving in my rant, you could infer that I would probably not be a fan of competitive diving. Although diving occurs in a different, non-laned, part of the swimming pool, it is a competitive water sport, so it makes logical and intuitive sense that I would not specifically like one competitive water sport while hating the other, given that the differences between them are fairly limited. Fundamentally, the point of what I was saying was that I hate water sports competition, and I mentioned some specific kinds of this competition, so it is reasonable to infer that I also do not enjoy some other water sports that I simply did not have time to mention.

The key to figuring out inference questions is similar to answering questions that interrogate the basic facts of the passage: have a solid base in the text. In other words, make sure you understand what is being said and how the author feels about it. From there, you will be able to make inferences about what else the author may say about a related issue, but has not explicitly included in the text provided to you.

As always with paired passages, the very pairing of the passages can work in your favor to help you figure out the inference that the question wants you to make. Passages are not just thrown together on the ACT Reading section, they are paired with thought and care, and as such there will be ample clues provided about the content and fundamental logical basis of the passages.

Drawing conclusions involves a very similar analytical process as making inferences; the difference is that questions asking you to draw a conclusion will often be less focused on the author's view, and they will rely on you to make a logical conclusion of your own.

In any case, the exact titles of these questions will not be on the test, and the analytical thinking process will remain the same, so let's press on to the fun part—the passages!



Practice Passage A

Adapted from Volume Four of *The Natural History of Animals: The Animal Life of the World in Its Various Aspects and Relations* by James Richard Ainsworth Davis (1903)

The examples of protective resemblance so far quoted are mostly permanent adaptations to one particular sort of surrounding. There are, however, numerous animals which possess the power of adjusting their color more or less rapidly so as to harmonize with a changing environment.

Some of the best known of these cases are found among those mammals and birds that inhabit countries more or less covered with snow during a part of the year. A good instance is afforded by the Irish or variable hare, which is chiefly found in Ireland and Scotland. In summer, this looks very much like an ordinary hare, though rather grayer in tint and smaller in size, but in winter it becomes white with the exception of the black tips to the ears. Investigations that have been made on the closely allied American hare seem to show that the phenomenon is due to the growth of new hairs of white hue.

The common stoat is subject to similar color change in the northern parts of its range. In summer it is of a bright reddish brown color with the exception of the under parts, which are yellowish white, and the end of the tail, which is black. But in winter, the entire coat, save only the tip of the tail, becomes white, and in that condition the animal is known as an ermine. A similar example is afforded by the weasel. The seasonal change in the vegetarian Irish hare is purely of protective character, but in such an actively carnivorous creature as a stoat or weasel, it is aggressive as well, rendering the animal inconspicuous to its prey.

Practice Passage B

Adapted from “The Colors of Animals” by Sir John Lubbock in *A Book of Natural History* (1902, ed. David Starr Jordan)

The color of animals is by no means a matter of chance; it depends on many considerations, but in the majority of cases tends to protect the animal from danger by rendering it less conspicuous. Perhaps it may be said that if coloring is mainly protective, there ought to be but few brightly colored animals. There are, however, not a few cases in which vivid colors are themselves protective. The kingfisher itself, though so brightly colored, is by no means easy to see. The blue harmonizes with the water, and the bird as it darts along the stream looks almost like a flash of sunlight.

Desert animals are generally the color of the desert. Thus, for instance, the lion, the antelope, and the wild donkey are all sand-colored. “Indeed,” says Canon Tristram, “in the desert, where neither trees, brushwood, nor even undulation of the surface afford the slightest protection to its foes, a modification of color assimilated to that of the surrounding country is absolutely necessary. Hence, without exception, the upper plumage of every bird, and also the fur of all the smaller mammals and the skin of all the snakes and lizards, is of one uniform sand color.”

The next point is the color of the mature caterpillars, some of which are brown. This probably makes the caterpillar even more conspicuous among the green leaves than would otherwise be the case. Let us see, then, whether the habits of the insect will throw any light upon the riddle. What would you do if you were a big caterpillar? Why, like most other defenseless creatures, you would feed by night, and lie concealed by day. So do these caterpillars. When the morning light comes, they creep down the stem of the food plant, and lie concealed among the thick herbage and dry sticks and leaves, near the ground, and it is obvious that under such circumstances the brown color really becomes a protection. It might indeed be argued that the caterpillars, having become brown, concealed themselves on the ground, and that we were reversing the state of things. But this is not so, because, while we may say as a general rule that large caterpillars feed by night and lie concealed by day, it is by no means always the case that they are brown; some of them still retaining the green color. We may then conclude that the habit of concealing themselves by day came first, and that the brown color is a later adaptation.



Sample Question

Based on these passages, it may be inferred that:

- A)** all animals have an appearance influenced by their desire for concealment and self-protection
- B)** only defenseless animals use their appearance to blend into their environment
- C)** the physical appearance of almost all animals is directly influenced by their physical environment
- D)** None of these answers is a reasonable inference that may be drawn from these texts.

As is so often the case in the harrowingly rich universe of critical thought, solving this question depends as heavily on your ability to navigate between errors as it does on your ability to find the correct option. The first tempting offer may be answer option A, since the two passages were both talking about the process of animal camouflage or disguise; however, the very end of Passage 1 outlines cases in which “an actively carnivorous creature as a stoat or weasel, [camouflages itself in order to be] aggressive as well, rendering the animal inconspicuous to its prey.” So, we can eliminate options A and B from consideration, leaving us with only C and D, which are, obviously, mutually exclusive choices. The inference made in answer option C is reasonable and consistent, primarily because it makes a less specific claim than did either Option A or B. The processes described in the two passages and the cumulative examples contained in the passage allow us to safely infer that almost all animals, regardless of their motivation, location, or physical structure, will have been influenced by their physical environment.

Sample Question

According to these two passages, a species that is extremely and specifically adapted to its environment is likely to:

- A)** be carnivorous
- B)** be herbivorous
- C)** be more highly evolved and higher on the food chain than an animal who is less specifically adapted to its environment
- D)** have lived and evolved in that environment for an extremely long time

It is always important to remember that these types of questions are always asking for the most reasonable answer, and that not all incorrect answer choices will be patently or obviously incorrect—there just might be another answer that is more correct. In this case, we can quickly eliminate A and B, since both herbivores and carnivores are discussed in Passage 1, and no specific mention of a species’ ability to adapt based on their diet is included in either text. That leaves us with C and D, and while both texts highlight the benefits of evolving to a particular environment, it is more reasonable to infer the more measured statement that the species would simply have “lived and evolved in that environment for an extremely long time.” Option C asks for a relative value judgment and makes specific mention of the food chain. Since rabbits, who are quite low on the food chain, are mentioned as being highly adapted to their environment, we can infer that this second statement is overreaching. Option D has a basis in the text, and is reasonable inference, especially given the description of long-term, multi-stage processes of environmental adaptation outlined toward the end of Passage 2.



Making Predictions Based on Paired Passages

In addition to questions asking you to draw conclusions or inferences from the texts, some questions on the ACT Reading section will ask you think critically by predicting future results or events based on the logic of the text. The difference between predictions and inferences is that inferences are about subjects—they are generally concerned with views or intellectual principles. Predictions are hypothetical conclusion about events.

A prediction: “I think it will rain tomorrow. I just have a feeling.”

An inference: “The guy who was just talking would agree that events can be predicted without quantitative evidence.”

At the risk (nay the certainty) of repeating past advice, when passages are paired in this manner, it is important to consider how the texts relate to one another. In the case of making predictions based on two passages, however, it is even more important to keep in mind that the questions will always be referencing both passages. Some questions asking you to make a prediction, may give you an answer that is accurate for one of the passages and not the other—this answer choice will not be correct. The predictions asked for will always cohere to the logic of both passages individually, and both passages taken together.

A Note on Objectivity

Sometimes on the ACT Reading section, a question might ask you to make a prediction that you, personally, might not agree with or think is a reasonable outcome. The key thing to remember here is that the test wants you to read objectively, to analyze the text on its own terms and to judge its claims based on the text’s internal logic. So, for example, I encounter a passage on houseplant maintenance and care, and I encounter a question asking me to decide, based on the passage, what will happen if one were to water one’s houseplants. I might think, for instance, that watering houseplants will kill them, because in my experience it so often does, but if the passage I’m being asked about makes a clear, consistent argument that houseplants need water, then I would have to choose an answer that predicted watering to help the plants grow, not kill them.

The following practice question refers back to the sample passages provided on page 271.

Sample Question

Which of the following outcomes would be most likely for a species that had evolved for many generations in a rainy climate if it was relocated to a snowy climate?

- A)** It would go on living normally, since it had already had a long process of evolution elsewhere.
- B)** It would immediately begin evolving to fit its new surroundings.
- C)** It would die immediately.
- D)** It would be vulnerable to predators or obvious to prey until it evolved to fit its new surroundings.

The key here is understanding the scope of the claims made in the two passages. The processes described, especially as they are outlined toward the end of Passage 2, are slow, evolutionary processes that take a long time. So, you can eliminate both A and B, which predict instant action, which, given the facts as they are presented in the passages, is an overreach in terms of prediction. This leaves us with options C and D, and of these options only D touches on the main points of both passages.



Practice Passages: Multi-Passage Critical Thinking**Passage A**

Adapted from *A Vindication of the Rights of Woman*
by Mary Wollstonecraft (1792)

After considering the historic page, and viewing the living world with anxious solicitude, the most melancholy emotions of sorrowful indignation have depressed my spirits, and I have sighed when obliged to confess, that either nature has made a great difference between man and man, or that the civilization, which has hitherto taken place in the world, has been very partial. I have turned over various books written on the subject of education, and patiently observed the conduct of parents and the management of schools; but what has been the result? a profound conviction, that the neglected education of my fellow creatures is the grand source of the misery I deplore; and that women in particular, are rendered weak and wretched by a variety of concurring causes, originating from one hasty conclusion. The conduct and manners of women, in fact, evidently prove, that their minds are not in a healthy state; for, like the flowers that are planted in too rich a soil, strength and usefulness are sacrificed to beauty; and the flaunting leaves, after having pleased a fastidious eye, fade, disregarded on the stalk, long before the season when they ought to have arrived at maturity. One cause of this barren blooming I attribute to a false system of education, gathered from the books written on this subject by men, who, considering females rather as women than human creatures, have been more anxious to make them alluring mistresses than rational wives; and the understanding of the sex has been so bubbled by this specious homage, that the civilized women of the present century, with a few exceptions, are only anxious to inspire love, when they ought to cherish a nobler ambition, and by their abilities and virtues exact respect.

In a treatise, therefore, on female rights and manners, the works which have been particularly written for their improvement must not be overlooked; especially when it is asserted, in direct terms, that the minds of women are enfeebled by false refinement; that the books of instruction, written by men of genius, have had the same tendency as more frivolous productions; and that they are only considered as females, and not as a part of the human species, when improvable reason is allowed to be the dignified distinction, which raises men above the brute creation, and puts a natural sceptre in a feeble hand.

Passage B

Adapted from Jon Stuart Mill's *The Subjection of Women* (1869)

The object of this Essay is to explain as clearly as I am able grounds of an opinion which I have held from the very earliest period when I had formed any opinions at all on social political matters, and which, instead of being weakened or modified, has been constantly growing stronger by the progress reflection and the experience of life. That the principle which regulates the existing social relations between the two sexes — the legal subordination of one sex to the other — is wrong itself, and now one of the chief hindrances to human improvement; and that it ought to be replaced by a principle of perfect equality, admitting no power or privilege on the one side, nor disability on the other.

The very words necessary to express the task I have undertaken, show how arduous it is. But it would be a mistake to suppose that the difficulty of the case must lie in the insufficiency or obscurity of the grounds of reason on which my convictions. The difficulty is that which exists in all cases in which there is a mass of feeling to be contended against. So long as opinion is strongly rooted in the feelings, it gains rather than loses instability by having a preponderating weight of argument against it. For if it were accepted as a result of argument, the refutation of the argument might shake the solidity of the conviction; but when it rests solely on feeling, worse it fares in argumentative contest, the more persuaded adherents are that their feeling must have some deeper ground, which the arguments do not reach; and while the feeling remains, it is always throwing up fresh entrenchments of argument to repair any breach made in the old. And there are so many causes tending to make the feelings connected with this subject the most intense and most deeply-rooted of those which gather round and protect old institutions and custom, that we need not wonder to find them as yet less undermined and loosened than any of the rest by the progress the great modern spiritual and social transition; nor suppose that the barbarisms to which men cling longest must be less barbarisms than those which they earlier shake off.



1. From these passages, it is reasonable to infer that the two authors both value:
 - A. exclusively about women's rights
 - B. social order
 - C. ethical equality in general
 - D. social decorum

2. Which of the following inferences could NOT reasonably be drawn from these passages?
 - A. The subjugation of women is restricted to the area of social mores and education
 - B. The fundamental structure of society, both on a social and a legal level, is inherently biased against women
 - C. Social customs reinforce cultural biases
 - D. There is no reasonable ethical or factual basis for treating women as inferior members of society

3. It is reasonable to assume that the positions advocated in these passages were _____ at the time they were published.
 - A. in the mainstream
 - B. popular only among academics and social elites
 - C. deeply unpopular and unconventional
 - D. None of these are reasonable assumptions to make about the contemporary popularity of these positions.



1. Correct Answer: C

Explanation: A careful reading of the two passages will reveal that both authors favor a subversion or complete overthrow of the their societies' patriarchal (male-dominated) societal norms and structures. Furthermore, the author of Passage 1 specifically condemns the system of "manners" enforced on women, so we can eliminate both answer options B and D. Both writers, far from being in favor of the status quo, are advocating for the upheaval of their culture's profoundly sexist and fundamentally unfair systems of social order and decorum.

Now, answering this question becomes a matter of understanding the nuances of the arguments made in the passage. Answer option A is an extremely limited and literal response to the arguments put forth in these passages. In the beginning of Passage 2, in explaining his stance on the subjugation of women, the author mentions that he has considered many other "social political matters," suggesting that while he is focused on women's rights, it is not his only focus. Passage 1 is focused mostly on the historical subjugation of women, and does so from a fundamental basis of arguing the inherent unfairness and "inequity" of this system, suggesting ethical equality as an important underpinning value to the author's belief system. In this case, it is more reasonable to infer that the authors both value ethical equality, rather than that they are only concerned with the specific case of inequality upon which these specific passages are focused.

2. Correct Answer: A

Explanation: In this case, you are being asked to determine which of these options is not a reasonable inference. For these types of questions, there are two equally valid approaches: eliminating options that are reasonable inferences or trying to isolate the answer that is not a reasonable inference. Since this is a practice lesson, we'll use the elimination method for the sake of being thorough. Answer option D, while never explicitly stated by either author in these particular passages, is a fairly easy inference to draw from both these passages. Since both authors are arguing for equal treatment, and arguing against unequal treatment, it makes sense that they would not think there was an acceptable reason for discriminating. Answer option C is addressed directly in both passages. In Passage 1 the author argues that a sexist system of "manners" has led to women receiving systematically poor educations, which has further reinforced negative stereotypes about their inherent capabilities. In Passage 2 the author makes numerous references to the difficulty of fighting "deeply-rooted" negative stereotypes, "especially those which gather round and protect old institutions and custom."

Answer option B is a pretty direct re-phrasing of the fundamental argument of both passages. This leaves us with answer option A, which is the correct answer. You can draw this inference in a number of ways. The first is that the general theme and style of both passages highlights the wide-reaching, deeply-entrenched nature of the discrimination discussed, so an inference that specifically limits the nature of this discrimination to two areas of life is not reasonable, especially when compared to the other answer options. Also, the general discourse surrounding "rights" in both passages suggests that legal, as well as social, discrimination is a part of the system described.

3. Correct Answer: C

Explanation: Both passages make it fairly clear that the positions advocated in both these passages fly in the face of ancient, "deeply-rooted" conventions and opinions (this is especially highlighted at the beginning of Passage 2), so we can easily eliminate option A. Passage 1 specifically highlights the inherent presence of the subjugation discussed in the education system, so the supposition that this position would be popular among academics and cultural elites (largely responsible for enforcing and perpetuating the entrenched social mores under discussion) is not reasonable. So, all we have left to do is evaluate answer option C, and given the amount of time the authors both spend discussing the popularity and widespread cultural fixity of the positions against which the authors are arguing, it is reasonable to assume that these tracts would have deeply unpopular, in that they are directly anti-conventional. They specifically critique behaviors and social norms that they assert are widespread. Answer C is thus correct.



Practice Passage: Comparing What's Said**Practice Passage A**

Adapted from *On War* by James Boswell
(1777)

When I saw workingmen engaged with grave assiduity in fashioning weapons of death, I was struck with wonder at the shortsightedness of human beings, who were soberly preparing the instruments of destruction of their own species. I have since found upon a closer study of man, that my wonder might have been spared. The views of most individuals are limited to their own happiness; and the workmen whom I beheld so busy in the arsenal of Venice saw nothing but what was good in the labor for which they received such wages as procured them the comforts of life. That their immediate satisfaction was not hindered by a view of the remote consequential and contingent evils for which they were responsible, would not surprise one who has had seen too much of the world. We must have the telescope of philosophy to make us perceive distant ills; further, we know that there are individuals of our species to whom the immediate misery of others is nothing in comparison with their own advantage—for we know that in every age there have been found men very willing to perform the office of executioner.

Practice Passage B

Adapted from *What is Patriotism?* by Max Eastman (1915)

With proper recognition of the possible variation of individuals, we can say that patriotism is one of these unalterable facts of man's nature. A talent for fighting solidarity with a group is a part of the instinct of human beings. It is composed of two tendencies that are laid down in his nervous system when he is born called pugnacity and gregariousness, or group-loyalty. All men and most animals are pugnacious. They love to fight. Everybody loves to fight. Some people get all the fighting they want at the breakfast table, and other people have to carry it out in the law courts or the battlefield, where it makes more noise. [Theodore] Roosevelt loves to charge up San Juan Hill, and then he loves to prosecute for libel anybody that says he didn't charge up San Juan Hill. War people fight for war and peace people fight for peace. When Roosevelt calls the peace people mollicoddles and college sissies, I only want to walk up and smash him.

It is far better though that we should conquer our instinct to fight and put faith in reason. It may seem gigantic; but it is by no means a utopian undertaking to unite the whole world of nations in such a federation. For all the organic interests of men, except their sheer love of patriotic fighting itself, are against the perpetual recurrence of international war. War and the mere joy of existence are incompatible. War makes it impossible to live, and it makes it impossible even to die for a noble purpose. Let men but understand themselves, and the mechanism of their emotions by which they are brought into this perennial catastrophe, and they will be ready enough to take gigantic measures to prevent it.



1. How would the author of Passage 2 likely respond to the statement made by the author of Passage 1 that “the views of most individuals are limited to their own happiness?”
 - A. Individual happiness is nonetheless often derived from group association.
 - B. Happiness is not relevant to the understanding of war.
 - C. Happiness can never come from patriotic fever.
 - D. Most individuals lack the understanding to work for anything other than their own happiness.
2. What is the “telescope of philosophy” that the author of the first passage describes?
 - A. The devotion that the workers of the arsenal of Venice have to their labor
 - B. The ability to understand the experience of distant suffering
 - C. The wonder felt by an observer of human tendencies
 - D. The love of patriotic fighting
3. How would the author of Passage 2 likely respond to the statement made by the author of Passage 1 that “the views of most individuals are limited to their own happiness?”
 - A. Happiness is not relevant to the understanding of war.
 - B. Happiness can never come from patriotic fever.
 - C. It is man’s nature always to avoid war.
 - D. Individual happiness is nonetheless often derived from group association.
4. In what way does Passage 1 differ from Passage 2?
 - A. Passage 1 offers a personal account.
 - B. Passage 1 draws on a reference to authority.
 - C. Passage 1 is more argumentative in tone.
 - D. Passage 1 references a historical figure.
5. As the author of Passage 2 uses it in the passage’s last line, the word “perennial” most nearly means _____.
 - A. occasional
 - B. nostalgic
 - C. recurrent
 - D. intrepid
6. Passage 2 is more _____ in tone and wording than Passage 1.
 - A. moderate
 - B. inflexible
 - C. informal
 - D. nostalgic
7. The author of Passage 2 mentions Theodore Roosevelt to _____.
 - A. provide an example supporting the preceding argument
 - B. reference the wisdom of a known authority
 - C. create a contrast to an earlier statement
 - D. dramatize the subsequent statement
8. Which statement made by the author of Passage 1 would support the concluding argument made by the author of Passage 2 that if “men but understand themselves . . . they will be ready enough to take gigantic measures to prevent [war]?”
 - A. “Their immediate satisfaction was not hindered by a view of the remote consequential and contingent evils for which they were responsible.”
 - B. “I have since found upon a closer study of man, that my wonder might have been spared.”
 - C. “For we know that in every age there have been found men very willing to perform the office of executioner.”
 - D. “We must have the telescope of philosophy to perceive distant ills.”



Practice Passage: Comparing What's Said: Answer Key

1. What is the “telescope of philosophy” that the author of the first passage describes?

- A. The devotion that the workers of the arsenal of Venice have to their labor
- B. The ability to understand the experience of distant suffering
- C. The wonder felt by an observer of human tendencies
- D. The love of patriotic fighting

2. How would the author of Passage 2 likely respond to the statement made by the author of Passage 1 that “the views of most individuals are limited to their own happiness?”

- A. Individual happiness is nonetheless often derived from group association
- B. Happiness is not relevant to the understanding of war
- C. Happiness can never come from patriotic fever
- D. Most individuals lack the understanding to work for anything other than their own happiness

3. How would the author of Passage 2 likely respond to the statement made by the author of Passage 1 that “the views of most individuals are limited to their own happiness?”

- A. Happiness is not relevant to the understanding of war
- B. Happiness can never come from patriotic fever
- C. It is man’s nature always to avoid war
- D. Individual happiness is nonetheless often derived from group association

The “telescope of philosophy” is meant to describe the ability to perceive the suffering of people who live far away. You can solve this problem by reading the sentence that precedes the one that mentions the “telescope of philosophy.” In that preceding sentence, the author describes individuals who are not considerate of the long-distance suffering caused by their work fashioning weapons. The author contrasts these individuals, who represent the majority of mankind, with those individuals who are able to understand the experience of distant suffering. These enlightened people are referred to by the author as possessing the “telescope of philosophy.”

The author of Passage 2 makes numerous references to the construction of human identity and satisfaction through group association. He states that all human beings are born with a natural tendency toward group-loyalty, and it is clear that the author believes human beings derive happiness from patriotic association. The author of Passage 2 would never agree that happiness is irrelevant to war or that it cannot be derived from patriotic fever. Likewise, the author of Passage 2 clearly believes it is man's nature to go to war and, as evidenced in the conclusion, that man can work to avoid war and consider the greater good. The author of Passage 2 would view individual happiness and apathy as less significant to the cause of war and would place greater emphasis on group association.

Passage 1 differs from Passage 2 in that the first passage offers a personal account. The author of Passage 1 uses evidence gathered on his visit to an armory in Venice to help make his argument. Specifically, he states, “The views of most individuals are limited to their own happiness, and the workmen whom I beheld so busy in the arsenal of Venice saw nothing but what was good in the labor for which they received such wages as procured them the comforts of life.” Contrast this with the second passage, where the author makes no reference to a personal account.



4. In what way does Passage 1 differ from Passage 2?

- A. Passage 1 offers a personal account
B. Passage 1 draws on a reference to authority
C. Passage 1 is more argumentative in tone
D. Passage 1 references a historical figure

5. As the author of Passage 2 uses it in the passage's last line, the word "perennial" most nearly means _____.

- A. occasional
B. nostalgic
→ C. recurrent
D. intrepid

6. Passage 2 is more _____ in tone and wording than Passage 1.

- A. moderate
B. inflexible
→ C. informal
D. nostalgic

Passage 1 differs from Passage 2 in that the first passage offers a personal account. The author of Passage 1 uses evidence gathered on his visit to an armory in Venice to help make his argument. Specifically, he states, "The views of most individuals are limited to their own happiness, and the workmen whom I beheld so busy in the arsenal of Venice saw nothing but what was good in the labor for which they received such wages as procured them the comforts of life." Contrast this with the second passage, where the author makes no reference to a personal account.

The word "perennial" is most frequently used to describe something that is either *recurring* or *enduring*. If you did not know this, it is difficult to solve this question from the context of the sentence, unless you go about it by means of eliminating incorrect answers. "Occasional" loosely means *infrequent* and goes against the author's intention when plugged in to the sentence. "Nostalgic" means *looking back on past events fondly*, and "intrepid" means *adventurous*. Neither of these answers adequately describes a "catastrophe" like war. "Miserly" means *not generous*, and this also does not fit. If you were able to eliminate all four of the other answers, you would arrive at "recurrent" as the solution.

The primary difference between Passage 2 and Passage 1 in terms of tone is that Passage 2 is more informal than Passage 1. Neither passage could rightly be called "restrained" or "moderate" given the strength of feeling each author displays. Similarly, both passages look back on the past as a part of an ongoing problem, and therefore would not be "nostalgic" in tone. Both passages are similarly "inflexible" because the authors clearly feel that flexibility on the topics that they are discussing is wrong. Passage 2 differentiates itself as more informal in the conclusion to its first paragraph when the author states "When Roosevelt calls the peace people mollicoddles and college sissies, I only want to walk up and smash him." This momentary shift to the first-person perspective, as well as the actual content of the sentence, is a good deal less formal than the language of the first passage.



7. The author of Passage 2 mentions Theodore Roosevelt to _____.

- A. provide an example supporting the preceding argument
B. reference the wisdom of a known authority
C. create a contrast to an earlier statement
D. dramatize the subsequent statement

8. Which statement made by the author of Passage 1 would support the concluding argument made by the author of Passage 2 that if “men but understand themselves . . . they will be ready enough to take gigantic measures to prevent [war]?”

- A. “Their immediate satisfaction was not hindered by a view of the remote consequential and contingent evils for which they were responsible.”
B. “I have since found upon a closer study of man, that my wonder might have been spared.”
C. “For we know that in every age there have been found men very willing to perform the office of executioner.”
→ D. “We must have the telescope of philosophy to perceive distant ills.”

The author of Passage 2 makes reference to the attitude of Theodore Roosevelt to provide an example of the type of war-loving people he mentions in his preceding argument. The author states, “They love to fight. Everybody loves to fight. Some people get all the fighting they want at the breakfast table, and other people have to carry it out in the law courts or the battlefield, where it makes more noise.” Following these statements, the author uses the behavior of Theodore Roosevelt to provide an example to support his argument.

When the author of Passage 2 states that if “men but understand themselves . . . they will be ready enough to take gigantic measures to prevent [war]” he is expressing a belief in the ability of wisdom to overcome mankind’s tendency toward warfare. This is very similar in intent and belief to the statement made by the author of Passage 1 that “We must have the telescope of philosophy to perceive distant ills.” Both authors believe that the key to conquering our urge to ignore the suffering of others and make war is consideration and thoughtfulness.



The ACT Science Section

Introduction

The ACT Science section is 35 minutes long and contains 40 multiple-choice questions based on given passages, many of which refer to figures, charts, graphs, equations, and other data representation tools. Despite the section title, the ACT Science section emphasizes critical thinking and analytic skills, rather than the ability to recall scientific content. Nevertheless, basic knowledge in science is required to be successful in this section of the ACT.

Content

While the ACT Science section does not primarily test your ability to recall scientific content, certain technical skills and knowledge of the scientific method will be emphasized. The ACT Science section assumes you are in the process of taking at least three years of core science courses such as biology, chemistry, physics, and Earth/space science. Based on this requisite knowledge, you will be expected to interpret graphs and tables, establish relationships between variables, and analyze the efficacy of experimental setups. Again, most of the questions/passages will provide you with the core information you need to answer the questions. Your main requirement in this section is to use critical thinking, analysis, problem-solving, interpretation, and evaluation skills to apply knowledge obtained from past courses or from the passage/question in a new or specific way. Rather than separating the content areas into scientific disciplines, we will look at the various formats in which questions may be asked.

At a Glance

Tests: critical thinking, data analysis, ability to read and interpret charts and graphs

Timing: $\frac{35 \text{ Minutes}}{40 \text{ Questions}} = \frac{52.5 \text{ Seconds}}{1 \text{ Question}}$

Section Breakdown:

- Five or six questions per passage
- Seven passages total
- Three types of passages: Data Interpretation, Experimental Setups and Results, and Evaluating Conflicting Theories

Common Passage Topics

Biology

Cell biology, microbiology (bacteria and viruses), plant biology, genetics, ecology, evolution, systems biology and physiology

Chemistry

Atomic theory, subatomic particles, chemical bonds, chemical reactions, thermodynamics, states of matter, periodic table

Physics

Mechanics, fluids, electricity and magnetism, waves, conservation of energy

Earth and Space Sciences

Astronomy, geology, meteorology, environmental science, oceanography



ACT Science Passage Types

Data Interpretation (30-40%)

These questions involve the presentation of data in tables and/or graphs, mimicking those found in scientific texts. Be prepared to interpret, analyze, and translate the information depicted in graphs, charts, and/or other figures. To answer these questions, you may be required to apply basic mathematical functions to data (average, median, range, reading a graph, slope of graph, etc.).

Experimental Setups and Results (45-55%)

This format involves descriptions of one or more related scientific experiments. Questions focus on experimental design and interpretation of experimental results. Be familiar with dependent and independent variables, confounding variables, control groups, experimental groups, measurements, and overall experimental design. You may also be required to interpret graphs and establish relationships between variables based on textual or visual descriptions.

Evaluating Conflicting Theories (15-20%)

The passages for these types of questions include two or more hypotheses that are inconsistent with each other. These inconsistencies usually arise from incomplete data, similar experiments with differing results, or different experiments with similar results. Questions do not necessarily ask which hypothesis is more commonly accepted; rather, they ask the reader to interpret, analyze, compare, and contrast aspects of the conflicting theories. It is important to read these passages particularly carefully because the two theories/hypotheses often closely resemble one another. This format combines the other two question formats and requires exceptional reading comprehension skills, as well as the ability to infer the effect of new information on the validity of each hypothesis.



How to Use This Book

Each section in this chapter investigates one ACT Science topic in-depth and walks you step-by-step through relevant practice questions. You can test your skills before trying some of the full ACT diagnostic tests at the end of the book.

Chapter Outline

[Review](#)

[Experiments and Procedures](#)

[Control Groups](#)

[Independent Variable](#)

[Dependent Variable](#)

[Graphs](#)

[Axes and Labels](#)

[Dual Axes](#)

[Linear Relationships](#)

[Exponential Relationships](#)

[Graphing an Equation](#)

[Determining an Equation for a Graph](#)

[Practice Questions: Graphs](#)

[Math, Science, and Equations](#)

[Reading an Equation](#)

[Applying an Equation](#)

[Units](#)

[Terminology and Jargon](#)

[Data Interpretation](#)

[Identifying a Point from a Table](#)

[Identifying a Trend from a Table](#)

[Identifying a Point from a Graph](#)

[Identifying a Trend from a Graph](#)

[Anticipating Placement of New Data Points](#)

[Further Applications](#)

[Practice Passage: Data Interpretation](#)

[Experimental Setups and Results](#)

[Distinguishing Experimental Purpose](#)

[Interpreting Textual Setups](#)

[Interpreting Diagrammed Setups](#)

[Describing Results](#)

[Visualizing Results](#)

[Applying Results to New Scenarios](#)

[Experimental Integrity and Flaws](#)

[Further Applications](#)

[Evaluating Conflicting Theories](#)

[Given Background](#)

[Identifying Theses and Arguments](#)

[Identifying Agreement](#)

[Comparing Arguments](#)

[Applying Arguments](#)

[Integrating New Information](#)

[Further Applications](#)



Review

Each section of the ACT requires certain skills that are applied to the questions presented, but are not directly assessed. This Review section seeks to reinforce and supplement these foundational skills before introducing the content that is directly tested in the ACT Science section. Mastering these topics and skills is essential for providing the framework and context for analyzing the questions in this section. To review question types and begin test-accurate review, you can skip this section; however, it is highly recommended that you take a moment to refresh your fundamentals before attempting more advanced review.

Section Summary

[Experiments and Procedures](#)

[Control Groups](#)

[Independent Variable](#)

[Dependent Variable](#)

[Graphs](#)

[Axes and Labels](#)

[Dual Axes](#)

[Linear Relationships](#)

[Exponential Relationships](#)

[Graphing an Equation](#)

[Determining an Equation for a Graph](#)

[Practice Questions: Graphs](#)

[Math, Science, and Equations](#)

[Reading an Equation](#)

[Applying an Equation](#)

[Chemical Reactions and Notations](#)

[Units](#)

[Terminology and Jargon](#)



Review: Experiments & Procedures

Experimental designs and procedures are important components of the ACT Science section. Identifying variables and their interactions with one another is key to proper scientific investigations. The scientific method relies on the use and understanding of control groups as well as that of independent and dependent variables. Control groups are not given a treatment and act as a standard to test for change in treatments. The independent variable represents the treatment that will cause change in the dependent variable. Scientists make changes to dependent variables and dependent variables test whether or not treatments cause or correlate with an observable effect. A hypothesis suggests a testable explanation for a particular phenomenon. Scientists manipulate and choose their variables carefully in order to discern causality between several forces.

Let's look at an example scenario to practice applying these principles.

A scientist wants to test the effects of nitrogen on plant development. The scientist takes 15 of the same plants and organizes them into three groups. The scientist makes sure that all of the plants exist in identical conditions with the exception of nitrogen soil content. Group A receives no extra nitrogen and is left unaltered. Plants in group B are given an additional 5 grams of nitrogen each week and plants in group C are given an additional 10 grams of nitrogen each week. The plants are monitored for 10 weeks and their development is recorded to test the effects of nitrogen on development.

Control Groups

Control groups are used to establish baseline measurements in experimental studies. They are identical to the experimental groups in every possible way, with the exception of treatment with regard to the independent variable. To identify the control group, look for the sample that corresponds with no change in the independent variable. Note that the control will still (usually) register change in the dependent variable. In the given example study, the plants are expected to grow some amount (dependent variable), regardless of whether they are exposed to additional nitrogen or not (independent variable).

Group A represents the control group. It receives no treatment, but is identical to groups B and C. The growth of plants in group A can then be compared to the growth of plants in groups B and C to determine how the exposure to nitrogen changed the experimental plants. Without a control group, there is no reference point for comparison and the experimental results will be inconclusive.

Independent Variable

The independent variable is the factor that is intentionally altered in a deliberate manner during experimentation to measure some effect. The experimenter is in complete control of the independent variable, and changes are usually made in distinct increments. Common independent variables are time between samples, dosage size, temperature, or strength of an applied change. These factors can all be carefully controlled in their administration so that each subject in a group receives exactly the same treatment.

In the given example, the added nitrogen is the independent variable. It is carefully controlled and administered by the experimenter in order to observe the effect it has on plant growth. Each plant in group B will receive exactly 5 grams per week, and each plant in group C will receive exactly 10 grams per week. The growth of plants in each group can be compared to the growth of plants in the control group to



draw conclusions about the effects of the added nitrogen. When graphing these results, the independent variable will be displayed on the horizontal axis (x-axis).

Dependent Variable

The dependent variable is measured as a result of the experiment. It is not administered, controlled, or altered by the experimenter, but arises naturally as an effect of other changes to the independent variables in the experimental groups. The trends observed in the dependent variable should directly correlate to the purpose and hypothesis of the experiment, answering the question that the procedure was designed to answer.

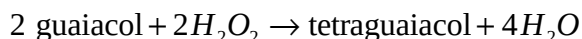
In the example, plant development (or growth) is the dependent variable. The experimenter does nothing to control the plant growth except for alter the amount of nitrogen available. Variance in plant growth and development directly corresponds with the purpose of the experiment (does nitrogen affect plant development?). The conclusions of the experiment can be drawn by looking at the changes made in the independent variable and the effects these changes produced in the dependent variable. When graphing these results, the dependent variable will be displayed on the vertical axis (y-axis).

Now that we have been able to explore the basics of experimental design and procedure, we can explore the mechanics of experimental design and procedure using questions and walkthroughs.



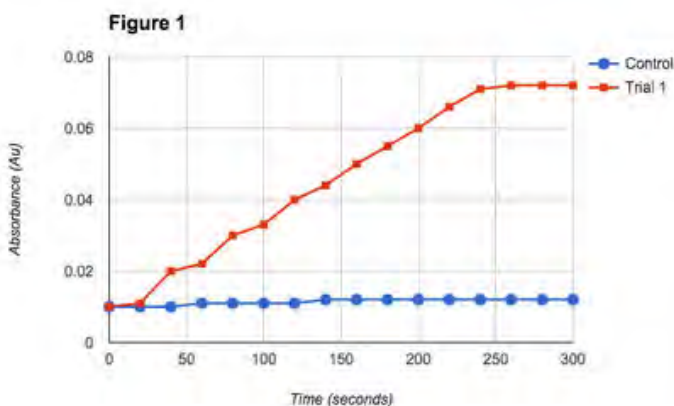
Sample Passage: Experiments and Procedures

Chemical reactions involve two main components, reactants and products. The reactants, often referred to as substrates, interact with each other and rearrange in order to be converted into products. The speeds of these reactions are often defined by substrate concentration and the presence of enzymes. Enzymes are referred to as catalysts. Peroxidase is traditionally derived from turnips; however, it is commonly found in many plant and animal cells. This enzyme helps plant cells by removing hydrogen peroxide from cells in the form of tetraguaiacol.



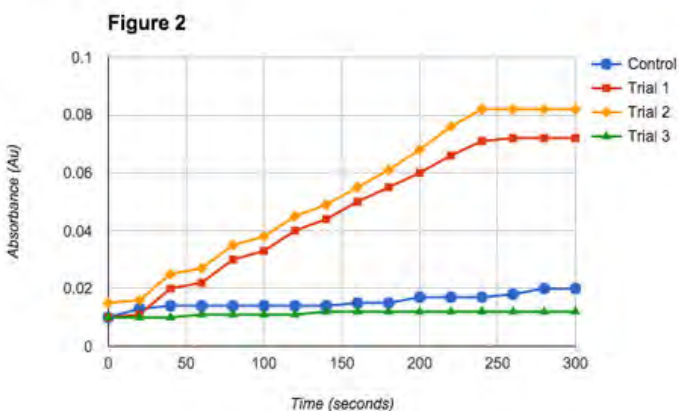
Study 1

A scientist wants to observe the production of tetraguaiacol by observing a reaction between hydrogen peroxide and guaiacol. The product of this reaction is orange-brown in color. The scientist measures the intensity of color in each sample using a spectrophotometer. In the control experiment, the scientist mixed the substrates together and measured the reaction rate. In the test experiment, a peroxidase enzyme was added to a new set of substrates and rate of reaction was measured. The results of these reactions are plotted in Figure 1.



Study 2

A research team decides to study the effects of the peroxidase facilitated reaction in the presence of heat. Reaction rates are known to speed up when heat is applied; however, at a certain point enzymes, such as peroxidase, denature and the reaction slows. The scientists perform a control trial at room temperature—20 degrees Celsius—and test trials at 40, 60, and 100 degrees Celsius. The results are plotted in Figure 2.



Sample Question

Which of the following best characterizes the role of tetraguaiacol in Study 1?

- A) Independent variable
- B) Control
- C) Dependent variable
- D) Extraneous variable

Choice C is the correct choice.



In the given equation, tetraguaiacol is the product of the reaction. The researcher in Study 1 wants to observe the orange-brown color of the product tetraguaiacol using a spectrophotometer. Tetraguaiacol is the dependent variable because its production was hypothesized to be changed by the presence or absence of the enzyme peroxidase.

Sample Question

Which of the following best describes the difference between the control and Trial 1 in Study 1?

- A)** The control contained the enzyme treatment and Trial 1 did not
- B)** Trial 1 contained the enzyme treatment and the control did not
- C)** Trial 1 contained hydrogen peroxide and the control did not
- D)** The control contained hydrogen peroxide and Trial 1 did not

Choice B is the correct choice.

The only difference between the control and Trial 1 is the presence of the independent variable: the peroxidase enzyme. The control represents a reaction standard that is not assisted by the enzyme. Trial 1 possesses all of the same components as the control but was treated with the enzyme to observe its effects. According to the given chemical equation, both the control and trial would contain hydrogen peroxide because it is a required substrate for the reaction; therefore, answers including this chemical are incorrect.

Sample Question

Which of the following best explains the difference in reaction rates presented in Figure 1?

- A)** The control had a higher rate of reaction than Trial 1
- B)** The control and Trial 1 had the same rate of reaction
- C)** Trial 1 had a higher rate of reaction than the control
- D)** Cannot be determined

Choice C is the correct choice.

The Trial 1 data series in figure 1 has a higher rate of reaction than the control. At no point in the graph is the absorbance of the control greater than that of Trial 1. Absorbance directly correlates with the amount of tetraguaiacol produced. In other words a darker sample will possess both a greater absorbance on the spectrophotometer and greater concentration of tetraguaiacol.



Sample Question

Which of the following choices best represents the dependent variable in Study 2?

- A)** Turnips
- B)** Heat
- C)** Peroxidase enzyme
- D)** Tetraguaiacol production

Choice D is the correct choice.

The dependent variable in Study 2 was tetraguaiacol production. The reaction rate is directly correlated with the concentration of tetraguaiacol produced in an enzyme facilitated reaction in the presence of different temperatures. Temperature was the independent variable because it effected the rate of reaction as seen in Figure 1. Peroxidase enzyme was a component within all of the trials. Last, the choice “turnips” would be incorrect because they are refined to produce peroxidase and are otherwise absent from this study.

Sample Question

Which of the following choices best represents the independent variable in Study 2?

- A)** Peroxidase enzyme
- B)** Heat
- C)** Tetraguaiacol production
- D)** Hydrogen peroxide

Choice B is the correct choice.

The independent variable causes change in the dependent variable. In this study, tetraguaiacol production is dependent upon environmental temperatures. Peroxidase enzyme and hydrogen peroxide are both substrates needed to initiate the reaction; therefore, they are incorrect.

Sample Question

The control in Study 2 was performed at which of the following temperatures?

- A)** 20 degrees Celsius
- B)** 40 degrees Celsius
- C)** 60 degrees Celsius
- D)** 100 degrees Celsius

Choice A is the correct choice.

The control in Study 2 was performed at room temperature. 20 degrees Celsius is a scientific constant for room temperature. The other temperature choices represented the temperatures of the varying trials.

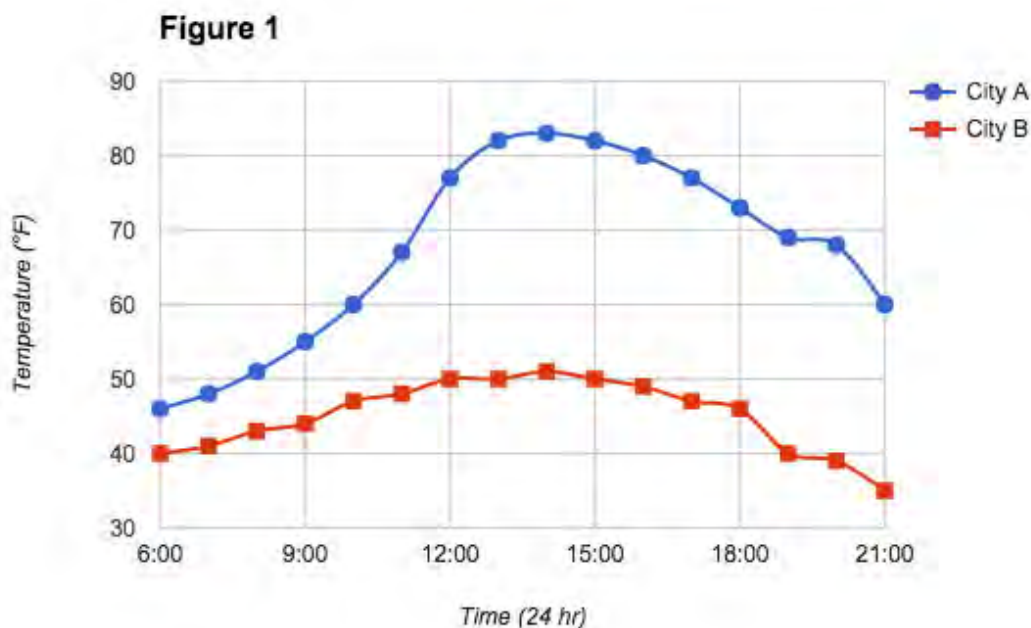


Review: Graphs

Passages belonging to the “Data Interpretation” and “Experimental Setups and Results” categories often include graphical representation of information. Various types of graphs may be present in the ACT Science section, including bar graphs, line graphs, scatter plots, and pie charts. Typically, these graphs will depict data from experiments mentioned in the passage.

Axes and Labels**Sample Question**

The average temperature of two cities is taken every hour for 15 hours. The data is shown in the figure below.



The first thing to do when presented with a graph is to get oriented by identifying the axes. Time is on the horizontal axis (also known as the x -axis) and it is measured in hours on the 24-hour clock. Temperature is on the vertical axis (also known as the y -axis) and it is measured in degrees Fahrenheit. Generally, the independent variable is on the x -axis and the dependent variable is on the y -axis. Here, it can be said that the temperature depends on the time of day. This graph, like most on the ACT Science section, lacks a descriptive title. However, we could name it “Temperature Vs. Time”—note that the name follows the general format: “ y -axis vs. x -axis” or “dependent variable vs. independent variable.” Some graphs on the ACT will provide figure legends, like the one in the top right corner of this one. We see that City A is represented by the blue circles, connected by the blue line, and City B is represented by the red squares, connected by the red line. Now we know exactly what information is presented in this graph and can begin using it to answer pertinent questions.

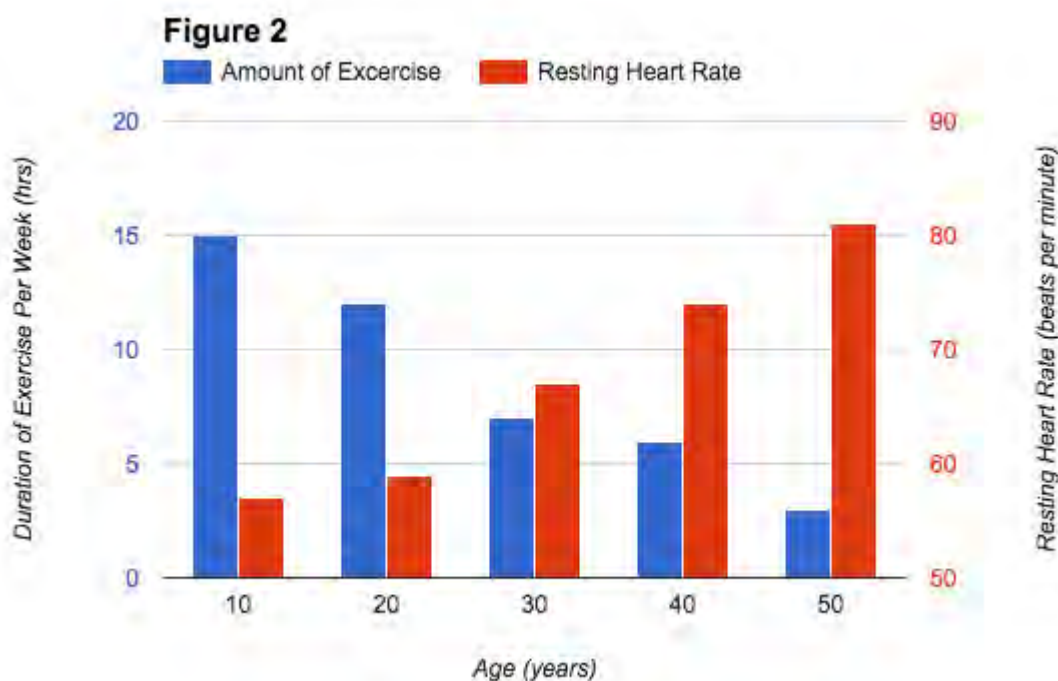


Dual Axes

Some graphs on the ACT may include dual axes. This means that two or more sets of data are being represented simultaneously. The basics of graphs with dual axes and those with single axes are the same, however, remember to recognize that each axis represents its own set of data and uses its own scale and units.

Sample Question

A scientist is interested in studying the relationships between age, amount of exercise, and resting heart rate. He performs a six-month experiment on fifty subjects of ages 10, 20, 30, 40 and 50, split evenly to include ten subjects per age group. The subjects record their exercise habits, and have their resting heart rates measured weekly. The averages of the results of the experiment are shown in Figure 2.



Which age group had the highest average resting heart rate?

To answer this question, we must use the right vertical axis, which corresponds with the red bars in the graph. Thus we see that the group with the highest resting heart rate is the 50-year-old group, who have an average resting heart rate of about 81 beats per minute.

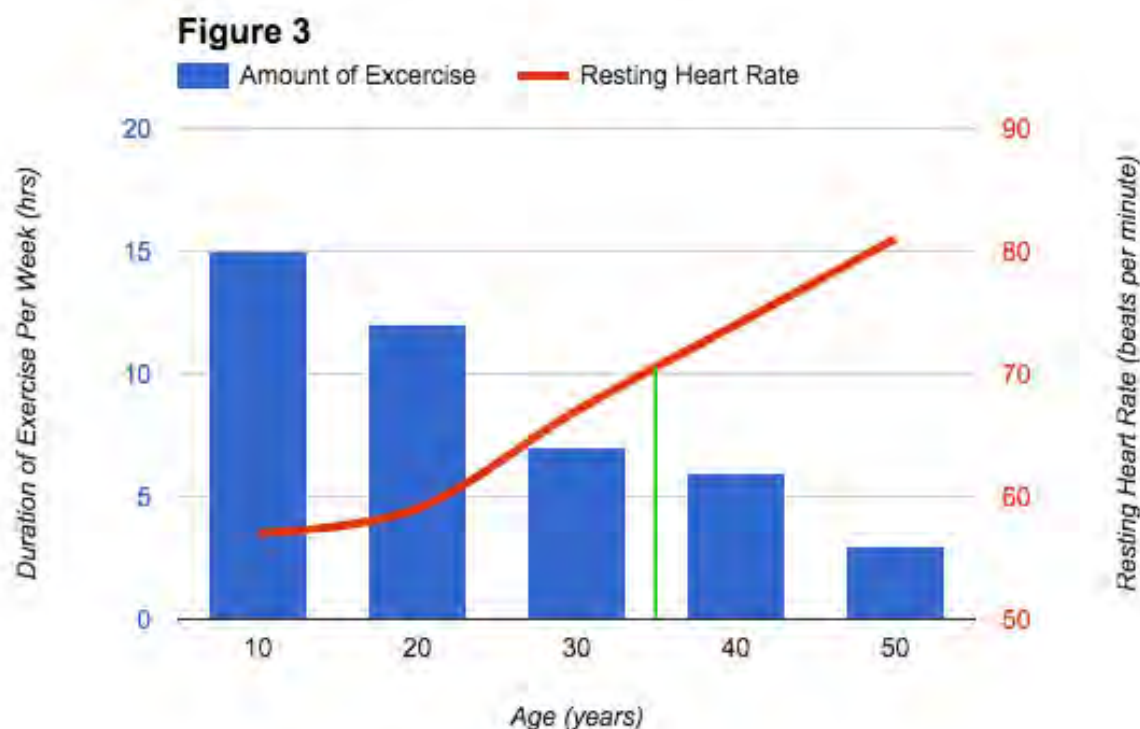
Which age group(s) exercise at least 8 hours each week?

Since we are asked about exercise, focus on the blue bars and use the scale on the left vertical axis to determine our desired values. It is clear that the 10- and 20-year-old groups both exercise more than 10 hours per week. As for the 30-year-olds, notice that the bar does not pass the halfway point between 10 and 15 hours on the left axis scale. Although subtle, we need to use this information to determine that the 30-year-old group, on average, does not exercise at least 8 hours per week. Thus our answer is: The 10- and 20-year-old groups both exercise at least 8 hours per week.



Linear Relationships**Sample Question**

Based on this data, what would be the approximate resting heart rate for a 35-year-old individual?

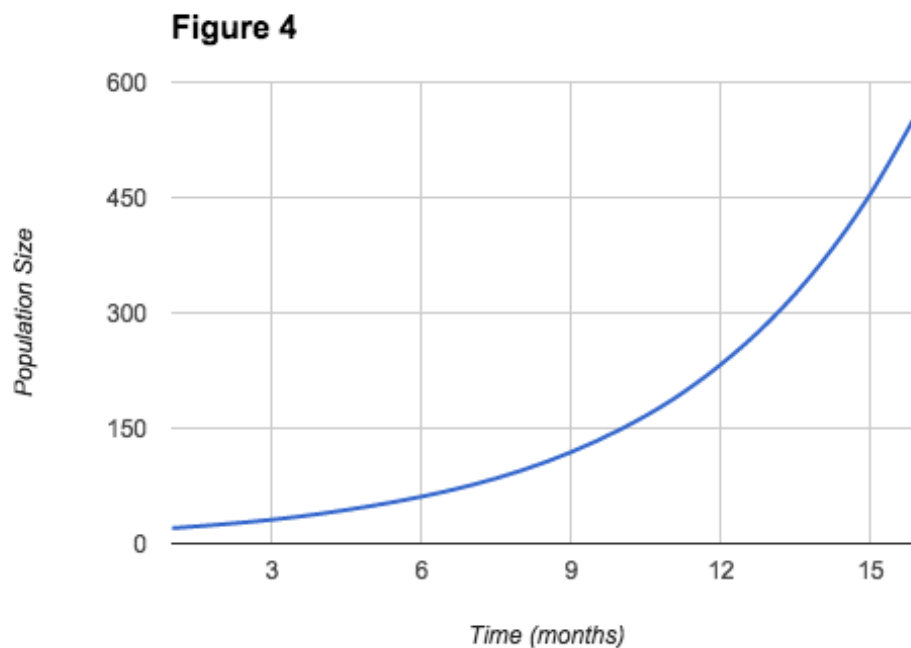


Even though there is no data for 35-year-olds explicitly given in this graph, we can interpolate, or approximate, the desired value since it falls within the range of ages studied. In order to get a good estimate, we must first recognize the relationship between age and resting heart rate: as age increases, resting heart rate increases. This is referred to as a direct relationship. Furthermore, it is possible to draw a relatively straight line connecting the average heart rates for each age group. Now draw a perpendicular line (shown in green in Figure 3) from approximately 35 on the x-axis up until the line intersects with the line we drew connecting the average heart rates. Based on this, we can estimate the average heart rate for 35-year-old individuals to be about 70 beats per minute.



Exponential Relationships**Sample Question**

A population of 20 rabbits was introduced to an environment with surplus food, water, shelter, and space, and an absence of predators. The population size grew by 25% each month over a period of 16 months. This data is shown in Figure 4.



If the population of rabbits continues to follow this growth pattern, what will be the approximate population of rabbits after 18 months?

First, we must recognize this as an exponential growth curve, which we do by observing that the slope of this curve is constantly increasing. The relationship between time and population is not linear; rather, it is exponential. Since the question asks for an estimate, we need not apply any formulas. Since the slope is constantly increasing, to make the most accurate estimate, we will start by looking at the last (16th) month. We see that there are approximately 450 rabbits in the beginning of month 16, and approximately 575 rabbits at the end of the month. Thus the population increased by about 125 during the last month. Since the slope is constantly increasing, the population must increase by more than 125 during the next month—let's say it increases by 200. Similarly, the increase in population during the final month must be more than 200—let's say it increases by 300. Thus the final population size after 18 months is about $575 + 200 + 300 = 1075$.



Graphing an Equation

Sample Question

Joe has committed to reading a long series of books. He has already read the first two books of the series, which were 280 pages each, when he sets his goal of reading 40 pages of the new books each day for two weeks. Construct a graph that represents Joe's reading progress from the time he sets his goal.

Since Joe is reading at a constant rate of 40 pages per day, we know that his reading rate, or slope, is constant. A constant slope corresponds to a linear relationship between number of pages read and time. In order to construct a graph, we must first write an equation that describes the scenario. Since we are dealing with a linear relationship, we use the general equation for a line:

$$y = mx + b$$

Here, y is the value on the y -axis (number of pages), m is the slope of the line, x is the value on the x -axis, and b is the y -intercept (the y -coordinate of the point at which our line intersects the y -axis).

Our graph will be set up with pages on the y -axis, and time on the x -axis (number of pages vs. time). Using the information given in the passage, we can determine the slope of this line:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\triangle \text{number of pages}}{\triangle \text{time}} = \frac{40 \text{ pages}}{1 \text{ day}} = 40 \frac{\text{pages}}{\text{day}}$$

Next, recognize that the y -intercept is given in the passage. The y -axis intercepts the x -axis at time zero. Since Joe has read two books, for a combined total of 560 pages before he sets his goal, we use this as the y -intercept. Thus, the equation for the line we will be graphing is:

$$y = 40x + 560$$

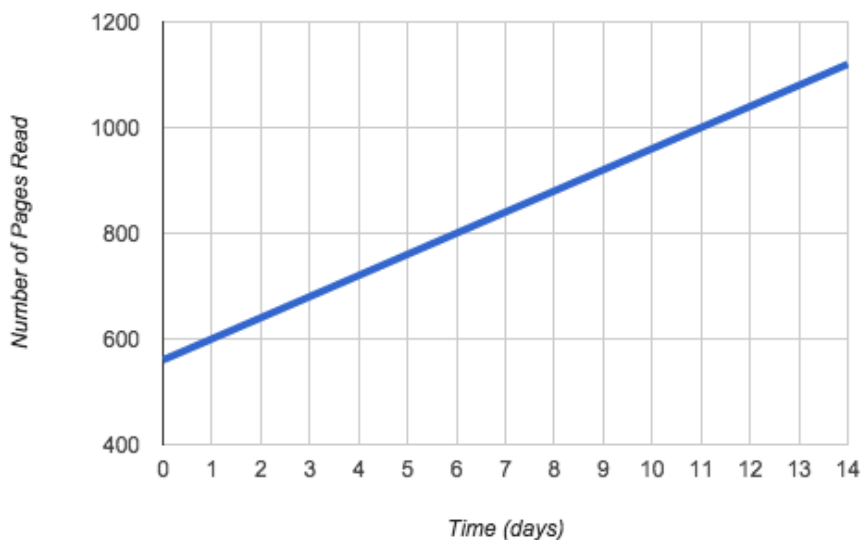
To make the graph, begin by labeling the y -axis (number of pages), and the x -axis (time). Be sure to use a scale that makes it easy to visualize the data. Now we may begin plotting points on our line. Begin by plotting the y -intercept (0,560). At this point we can approach plotting points in two ways. One way is to count 40 units in the positive y -direction (up), and one unit in the positive x -direction (right), plot a point, then repeat this process until we have reached two weeks. Another way we can get our line is to plug in the final x -value in our equation, and solve for the corresponding y -value.

$$y = 40(14) + 560$$

$$y = 1120$$

Either method of plotting the points is correct. Now draw a line connecting the points to obtain the graph of the equation, shown in Figure 5.

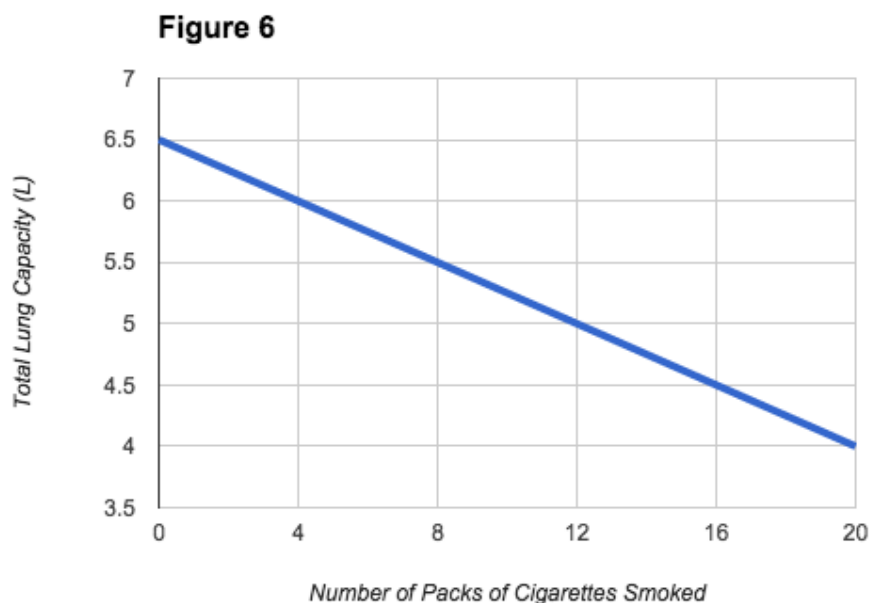
Figure 5



Determining an Equation for a Graph

Sample Question

To determine the effects of cigarettes on total lung capacity, a doctor has evaluated the lungs of a smoker who has just begun smoking. From the data collected over some time, the doctor has constructed the graph shown in Figure 6. What is the equation of the graph?



First, recognize that the relationship between the total lung capacity of a smoker and the number of packs of cigarettes smoked is linear. Therefore, we know the equation follows the general form:

$$y = mx + b$$

To find the equation of this graph, begin by determining b , the y -intercept of the line. In other words, the point at which the line intersects the y -axis. By looking at the graph, we determine this point to be $(0, 6.5)$. Note that the y -intercept will always have an x -value of zero. To find the slope of the graph, we need to choose two points. For example, use the first point on the graph, $(0, 6.5)$, and the point $(4, 6)$. From here, we can use the slope formula to find m in our equation. The slope formula is:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

It does not matter which point we use as (x_1, y_1) and which point we use as (x_2, y_2) , as long as we are consistent.

$$\text{slope} = \frac{6 - 6.5}{4 - 0} = -\frac{0.5}{4} = -\frac{1}{8} = -0.125$$

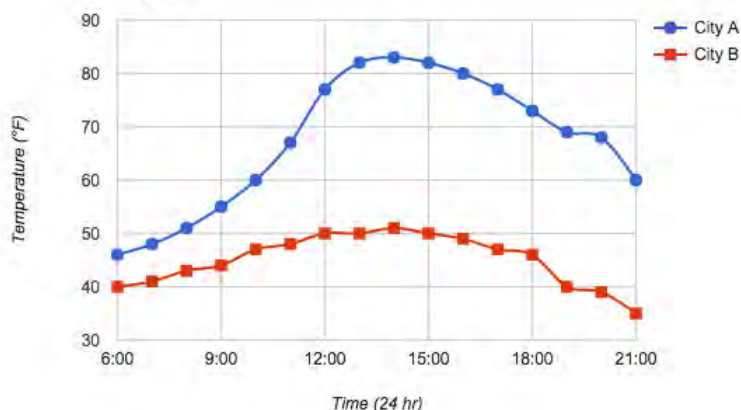
We may use either the fraction or the decimal form of the slope of the line, m . Notice that the slope of this graph is negative (it points down and to the right). Each pack of cigarettes decreases the smoker's total lung capacity by 0.125L . Now substitute our calculated values into the general equation for a line.

$$y = -\frac{1}{8}x + 6.5$$



Practice Questions: Graphs

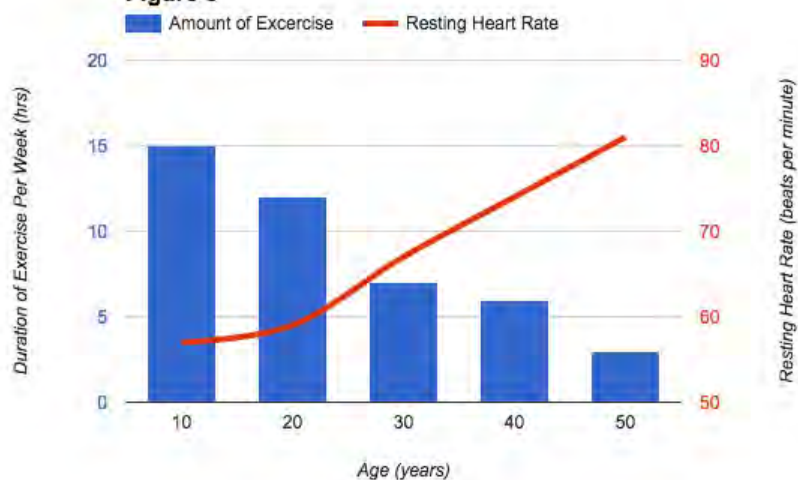
Figure 1



1. Refer to Figure 1. During which three-hour time interval does each city experience the greatest change in temperature?

- A. City A = 9:00 to 12:00 and City B = 18:00 to 21:00
- B. City A = 18:00 to 21:00 and City B = 9:00 to 12:00
- C. City A = 9:00 to 12:00 and City B = 12:00 to 15:00
- D. City A = 18:00 to 21:00 and City B = 12:00 to 15:00

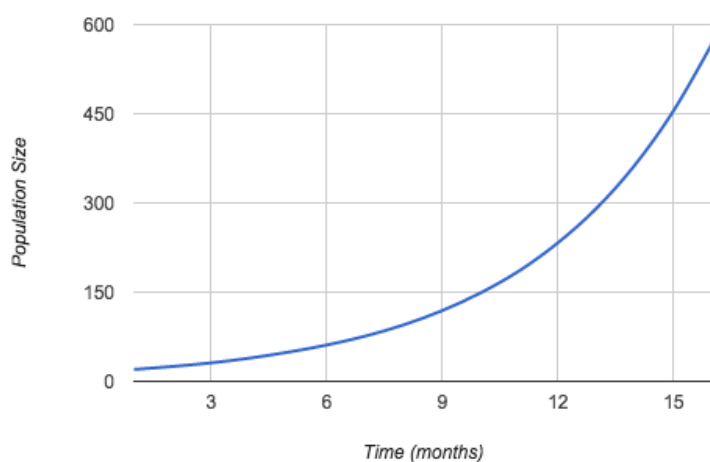
Figure 3



2. Refer to Figure 3. Based on this data, what is the approximate resting heart rate for a 65-year-old individual?

- A. 91.5 beats per minute
- B. 81.0 beats per minute
- C. 55.0 beats per minute
- D. 100.5 beats per minute

Figure 4



3. Refer to Figure 4. Use the following exponential growth formula to calculate the population size after 18 months:

$$y = n(1 \pm r)^x$$

Here, y is the population size, r is the growth rate and x is the number of time intervals.

- A. 1110 rabbits
- B. 760 rabbits
- C. 1325 rabbits
- D. 2460 rabbits



Practice Questions: Graphs: Answer Key**Question #1**

Refer to Figure 1. During which three-hour time interval does each city experience the greatest change in temperature?

- A.** City A = 9:00 to 12:00 and City B = 18:00 to 21:00
- B.** City A = 18:00 to 21:00 and City B = 9:00 to 12:00
- C.** City A = 9:00 to 12:00 and City B = 12:00 to 15:00
- D.** City A = 18:00 to 21:00 and City B = 12:00 to 15:00

Since this question asks us for a time interval, we cannot look at only one point on the graph. Rather, we must find the interval during which the change in temperature occurs the fastest. In other words, we need to find the interval on the graph that has the slope with the largest magnitude. To do this, use the slope formula to find the three-hour interval for each city during which the graph is the steepest.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{temperature}}{\text{time}}$$

Upon first glance, it appears that the steepest part of the curve for City A occurs during the interval from 9:00 to 12:00.

$$\text{slope} = \frac{77 - 55}{12 - 9} = \frac{22^\circ\text{F}}{3\text{hr}} \approx 7.33 \frac{^\circ\text{F}}{\text{hr}}$$

To ensure this is the correct answer, compare this to another three-hour interval that you think may have the slope with the second largest magnitude. Upon computation of the slopes of other intervals, we find that the greatest change in temperature does indeed occur between 9:00 and 12:00.

City B has a significant decrease in temperature, and thus a large magnitude of slope, during the last three-hour time interval. We will measure the slope of the graph during the interval from 18:00 to 21:00.

$$\text{slope} = \frac{35 - 46}{21 - 18} = -\frac{11^\circ\text{F}}{3\text{hr}} \approx -3.67 \frac{^\circ\text{F}}{\text{hr}}$$

Note that even though the slope is negative during this time period, the magnitude (absolute value) of the change in temperature of this number is positive. If we calculate the slopes of the other intervals, we will find that City B experiences the most rapid change in temperature from 18:00 to 21:00.

Thus the correct answer is: City A = 9:00 to 12:00; City B = 18:00 to 21:00.



Question #2

Refer to Figure 1. During which three-hour time interval does each city experience the greatest change in temperature?

- A. 91.5 beats per minute
- B. 81.0 beats per minute
- C. 55.0 beats per minute
- D. 100.5 beats per minute

Since the graph only includes groups of individuals up to age 50, we must use the given data to extrapolate the resting heart rate for a 65-year-old individual. Notice the relationship between average resting heart rate and age is a direct relationship. The opposite is true for the relationship between age and amount of exercise: as age increases, the amount of exercise decreases. This type of relationship is referred to as an indirect relationship. The data follows a roughly linear distribution, that is, we can find an approximate equation for the line that is best fit for resting heart rate and age. To do this, we must first start by getting the average slope of the line. Again, since we do not have exact coordinates for the points, we must do our best to estimate. Any two points on the line will do, but since we are looking for information on 65-year-olds, let's choose the two points closest to that: 40 and 50-year-olds.

$$\text{slope} = \frac{81 - 74}{50 - 40} = \frac{7}{10} = \frac{7 \text{ beats per minute}}{10 \text{ years}}$$

Thus we see that for every 10 additional years of age, the average resting heart rate increases by 7 beats per minute. Since we need to add 15 years to 50 in order to get to 65-years-old, we can set up an equation as follows:

$$\frac{7 \text{ beats per minute}}{10 \text{ years}} = \frac{x \text{ beats per minute}}{15 \text{ years}}$$

Cross multiply and simplify:

$$x = 10.5$$

Thus, the total average resting heart rate for a 65-year-old individual is:

resting heart rate at 50 years old + x

$$81 + 10.5 = 91.5 \text{ beats per minute}$$



Question #3

Refer to Figure 4. Use the following exponential growth formula to calculate the population size after 18 months:

$$y = n(1 \pm r)^x$$

Here, y is the population size, r is the growth rate and x is the number of time intervals.

- A. 1110 rabbits
- B. 760 rabbits
- C. 1325 rabbits
- D. 2460 rabbits

First, write the equation for the exponential growth curve shown.

$$y = 20(1 + 0.25)^x$$

We use 18 as the number of time intervals because there is a 25% increase in population each month, which is also why we add our growth rate, not subtract it as we would if the population were experiencing exponential decay.

$$y = 20(1 + 0.25)^{18}$$

$$y = 1110$$

Note that when we plug this into the calculator, the actual answer is 1110.22, but since rabbits are discrete entities, we must round to the nearest whole number.



Review: Math, Science, and Equations

Although success on the ACT Science section does require memorization of a few formulas and relationships, such as the area of a circle and the slope formula, the exam's focus is to test critical thinking and analytical skills. It is likely that a passage will include formulas that are unfamiliar to you. Neither memorization nor prior familiarity of these formulas is required to be able to apply them to the passage; however, you should be comfortable manipulating and working with such equations and/or formulas.

Reading an Equation

Coulomb's law describes electric forces between two charged particles. It can be written mathematically as:

$$F_E = k \frac{q_1 q_2}{r^2}$$

Here, F_E is the electric force, in Newtons, experienced by two particles of charge q_1 and q_2 , in Coulombs, r is the distance between the two particles in meters, and k is Coulomb's constant, which is

equal to $9 \cdot 10^9 \frac{N \cdot m^2}{C^2}$ and will be provided on the ACT if it is required for a question. When we look at this equation, first recognize what it equates: the electric force between two charged particles to the product of Coulomb's constant and the charges of said particles, divided by the square of the distance between them. Be prepared to rearrange equations such as this one to find the value of any of the variables. Also, note the units in which each parameter is given; some ACT passages require unit conversions of given values before they can be used in equations (more on units later).

Applying an Equation

Kinematics is a branch of mechanical physics that describes the motion of objects. A student wants to predict the motion of projectiles using the following kinematics formulas:

$$d = \frac{1}{2}(v_i + v_f)t$$

$$d = v_i + \frac{1}{2}at^2$$

Here, d is the total distance travelled by the object in question in meters, v_i is the initial velocity of the object in meters per second, v_f is the final velocity of the object in meters per second, a is the acceleration of the object in meters per second squared, and t is the time (duration of the motion) in seconds.

Sample Question

A 10kg bowling ball, initially at rest, is pushed horizontally by some force, such that its acceleration is due west. How long will it take the bowling ball to travel west a distance of 250m? Ignore the effects of friction.



First, recognize what the question is asking us to find; here we are looking for the time it takes for the bowling ball to travel 250m west. Thus, we are looking for time, t . Next, gather all the relevant information:

$$d = 250m$$

$$a = 5 \frac{m}{s^2}$$

$$v_i = 0 \frac{m}{s}$$

Note that the initial velocity of the bowling ball is zero because we are told that it starts at rest. Also, notice that the mass of the object is extraneous to this question. Now that we clearly have all relevant information, we must choose one of the two given equations to use to find the time. The first equation cannot be used for this question since we do not have the final velocity of the bowling ball. We must use the second equation because we have three of the four variables. Thus we plug in to equation two and solve for time (remember that the initial velocity of the bowling ball is zero). It may be simpler to solve for t before plugging in any numbers as such:

$$t = \sqrt{\frac{2d}{a}}$$

Finally, plug in the appropriate values and solve.

$$t = \sqrt{\frac{2 \cdot 250m}{5 \frac{m}{s^2}}}$$

Simplify.

$$t = 10s$$

Sample Question

What is the final velocity of the bowling ball after ten seconds?

Since we are looking for final velocity, v_f , we must use the first equation. Again, rearrange the equation to solve for v_f , noting that : $v_i = 0 \frac{m}{s}$.

$$v_f = \frac{2d}{t}$$

Now, plug in the appropriate values and solve.

$$v_f = \frac{2 \cdot 250m}{10s} = 50 \frac{m}{s}$$

Units are incredibly important for all questions involving numerical values. Answer choices on the ACT may have the same numerical value but different units, (e.g. 10km, 10m, 10cm, 10mm), so it is vital to keep them organized throughout calculations. A basic understanding of the relative magnitude of units will be helpful. Also, keep in mind which values can be measured by which units and prepare to convert between units.



Units**Commonly used Prefixes and Values**

"milli-" = 10^{-3} (Example: $1\text{mm} = 10^{-3}\text{m} = \frac{1}{1000}\text{m}$)

"centi-" = 10^{-2} (Example: $1\text{cm} = 10^{-2}\text{m} = \frac{1}{100}\text{m}$)

"deci-" = 10^{-1} (Example: $1\text{dm} = 10^{-1}\text{m} = \frac{1}{10}\text{m}$)

"kilo-" = 10^3 (Example: $1\text{km} = 10^3\text{m} = 1000\text{m}$)

"mega-" = 10^6 (Example: $1\text{Mm} = 10^6\text{m} = 1000000\text{m}$)

Length:

1 millimeter (1mm) is about the thickness of a guitar string

1centimeter (1cm) $\approx$ 0.4 inches (in) $\approx$ length of a sugar cube

1 meter (1m) $\approx$ 3.3 feet (ft) $\approx$ length of a guitar (think of a yard stick or meter stick)

1 kilometer (1km) $\approx$ 0.62 miles (mi) $\approx$ three laps around an athletic track

Volume:

1 milliliter (1mL) $\approx$ 0.2 teaspoon (tsp) $\approx$ 20 drops of liquid

1 liter (1L) $\approx$ 34 fluid ounces (oz) $\approx$ two standard plastic water bottles

Weight:

1 gram (1g) $\approx$ 0.033 ounces (oz) $\approx$ weight of a medium paper clip

1 kilogram (1kg) $\approx$ 2.2 pounds (lb) $\approx$ weight of an average textbook



Sample Question

How many ounces are in 20g?

For unit conversions, start by writing the given value, then multiply it by the appropriate conversion factor such that the units in the given value cancel with the units in the denominator of the conversion factor:

$$20g \cdot \frac{0.033oz}{1g} = 0.66oz$$

Sample Question

How many seconds are there in a year?

Take the same approach, starting with the given value and multiply it by the appropriate conversion factor such that we cancel each unit, until we reach seconds.

$$1 \text{ year} \cdot \frac{365 \text{ days}}{1 \text{ year}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} = 31536000 \text{ seconds}$$

Sample Question

Approximately how many meters long is a big rig truck?

- A. 2.5m
- B. 25m
- C. 250m
- D. 2500m

When we imagine a big rig truck, we know that it is much longer than a standard family car, which is about 2.5m. If we put 250 one-meter-long sticks end to end, it would be about the length of two and a half football fields, which is too long for a big rig. Similarly, 2500m is over a mile long. Thus, by process of elimination, the best answer is B, 25m.



Terminology and Jargon

The ACT Science passages will likely include unfamiliar terms, which are often italicized. Don't get thrown off by terms you don't know, since they are often defined in the passage. If they are not defined in the passage, chances are that the exact definition is not necessary to correctly answer the questions. For example, there may be an unfamiliar parameter that is being measured and is part of a table or graph. Remember that the ACT is a critical thinking and problem-solving test, not a test on whether you know what a novel term means.

Practice Passage

Species competition is driven by a variety of factors. Resources such as water, food, sunlight, and suitable habitat are among the top contributors that influence interspecific (between different species) competition and intraspecific (between members of the same species) competition.

One interesting example of interspecific competition is that of two barnacle species that inhabit intertidal zones. *Balanus balanoides* inhabits the lower intertidal zone and *Chthamalus stellatus* inhabits the upper intertidal zone. A researcher attempts to study this phenomenon.

The researcher removes the *Balanus* species from the lower intertidal zone and observes that the *Chthamalus* species expands its range to inhabit the lower intertidal zone and the upper intertidal zone. The researcher then removes the *Chthamalus* species from the upper tidal zone of a different area and observes that the *Balanus* species does not extend its range. The researcher concludes that competition has allowed each species to exist simultaneously by forming specialized niches that promote survivorship for each species.

Sample Question

A new study investigated the tide's effect on water levels in relation to species diversity. It suggests that the *Balanus* species is absent from the upper intertidal zone due to issues involved with desiccation (the process of removing moisture from something). How would the researcher react to this claim?

- A. The researcher would disagree because these results do not correspond with collected data.
- B. The researcher would agree because data on tidal effects on ocean water levels was absent in the barnacle study.
- C. The researcher would disagree because the two species would not inhabit the same space despite tidal influences.
- D. The researcher would agree with the claim because it corresponds with the data that was collected.

The new study contains information that aids in the explanation of the observed results of the researcher's initial investigation. The *Balanus* species does not inhabit the upper intertidal zone. The data in the new study supports this information with a possible explanation for the niche-forming abilities of the new species. Thus the correct answer is D; none of the other choices are supported by the passage.



Data Interpretation

Data Interpretation involves the deciphering of graphs and tables. Questions on the ACT Science section will frequently present test-takers with data from a given experimental set-up or trial, with the expectation that students can interpret the graphs, figures, and tables given. For these questions, students will generally need a strong knowledge of graph construction and a basic understanding of mathematical principles. Any scientific relationships tested will be given within the context of the passage, and can generally be identified within the first 2-3 sentences of the passage.

Data Interpretation questions strictly address the data presented in the passage. Common question types require students to identify increasing and decreasing trends, observe direct and indirect proportionalities, anticipate new data points based on the given trends, and generate graphical representations of data. Data Interpretation questions will not ask how or why these relationships occur; these questions fall within the domain of Experimental Setups and Results.

Of the three primary question types found on the ACT Science section, Data Interpretation questions generally require the smallest time investment due to their direct correlation with the given material in the passage. These questions are designed to test students' abilities to recognize and interpret relationships; essentially, these questions are more oriented toward skill, as opposed to critical thinking and reasoning.

Section Outline

- [Identifying a Point from a Table](#)
- [Identifying a Trend from a Table](#)
- [Identifying a Point from a Graph](#)
- [Identifying a Trend from a Graph](#)
- [Anticipating Placement of New Data Points](#)



Identifying a Point from a Table**Sample Question**

Chemical compounds have different boiling points based on several molecular characteristics.

Table 1

Solid carbon dioxide is known as dry ice, and sublimates at a temperature of -78.5°C . According to Table 1, which compound has a boiling point closest to the sublimation point of dry ice?

Compound Name	Molecular Formula	Molecular Weight	Boiling Point
Methane	CH_4	$16.04 \frac{\text{g}}{\text{mol}}$	-161.49°C
Ethane	C_2H_6	$30.07 \frac{\text{g}}{\text{mol}}$	-88.5°C
Propane	C_3H_8	$44.10 \frac{\text{g}}{\text{mol}}$	-42.2°C
Butane	C_4H_{10}	$58.12 \frac{\text{g}}{\text{mol}}$	-1.0°C

We can tell that this question requires us to identify a point because it asks for a specific piece of data that we can extract from Table 1. Keep in mind that we do not actually need any scientific background to reason through these types of questions. We do not need to know the definition of sublimation, which is the phase change when a solid transitions directly into a gas—in this case the reason why dry ice looks like it is “smoking.” Instead, we only need to be able to analyze context.

The question tells us the sublimation point of dry ice (-78.5°C) and asks that we identify the compound in the table with a boiling point closest to this value. Looking in the fourth column of the table, we can find the heading “Boiling Point.” This means that we are looking for the value closest to -78.5°C in this column. We can see that the second row has the closest value: -88.5°C .

To see which compound has this boiling point, follow the second row (the row with our target value) to the left until you find the compound name. In this case, the compound with a boiling point of -88.5°C is ethane. This means that ethane will be the correct answer.



Identifying a Trend from a Table

Sample Question

A student has several resistors, each with a resistance of 2 ohms. He arranges different numbers of the resistors into two different configurations, and records the total equivalent resistance of each circuit.

Table 1

Number of Resistors Used	Equivalent Resistance	
	Configuration A	Configuration B
1	2Ω	2Ω
2	1Ω	4Ω
3	0.5Ω	8Ω
4	0.25Ω	16Ω

According to Table 1, how does the equivalent resistance change as fewer resistors are used in Configuration A?

Questions requiring us to identify trends will generally ask for the change in one variable with relation to another. We can identify this question type by looking for relationships in the question stem (“As *variable A* increases, *variable B* _____” or “How does *variable A* change as *variable B* decreases?”). We can also examine the answer options. When identifying trends, our answers will indicate relationships, such as “increases” or “decreases.”

Our first step in this question will be to identify the variables. The first variable is equivalent resistance, based on the question stem, “*how does the equivalent resistance change.*” This also tells us that the final answer we are looking for will be a trend in equivalent resistance.

Our second variable will be the number of resistors; we are looking at the trend for equivalent resistance “*as fewer resistors are used in Configuration A.*” This indicates that we need to observe the second column of the table (Configuration A), viewing the change in the upwards direction (as fewer resistors are used). We know that we are looking in the upward direction because the number of resistors used in the first column of the table decreases as we move upward.

Finally, identify the trend in the second column as we move upward. 0.25Ω is the smallest value, while 2Ω is the largest value. This means that the value *increases* upward within the column. This means that the correct trend is that equivalent resistance increases as fewer resistors are used in Configuration A.

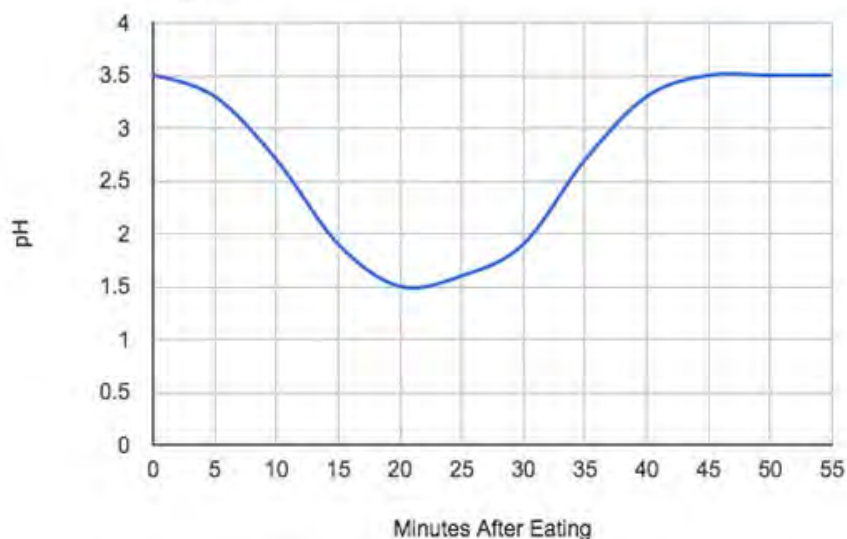


Identifying a Point from a Graph

Sample Question

After consuming a meal, cells in the stomach are activated to release hydrochloric acid. The acid lowers the pH of the stomach to aid in digestion. More acidic conditions will result in a lower pH value.

Figure 1



How many minutes after eating will the stomach be most acidic?

This question is asking us for a specific detail based on the information presented in the graph. We are being asked to identify a single point at which a given variable (in this case, acidity) reaches a maximum or minimum. To answer this question, we will need to decipher the information presented in the graph.

It should be immediately apparent that “acidity” is not directly represented on the graph. Our two axes are labeled “pH” and “Minutes After Eating.” Which of these can be used to find pH?

The passage text tells us that “*more acidic conditions will result in a lower pH value.*” We know we are looking for the most acidic point, meaning we must be looking for the lowest pH.

pH is given on the vertical axis of our graph, meaning that smaller pH values are represented by points closer to the bottom, and larger pH values are represented by points closer to the top. To find the most acidic point, we need to find the smallest pH value. We can see that the smallest pH value that the graph reaches is 1.5.

Now we need to find the time value that corresponds to this point on the graph. Time is displayed on the horizontal axis of the graph. Starting at our point on the graph where the pH is 1.5, we can draw a line straight downward. This line will tell us the time value for the point. Tracing straight downward from the graph at 1.5, we can see that the correct time for this point is 20 minutes after eating.



Identifying a Trend from a Graph

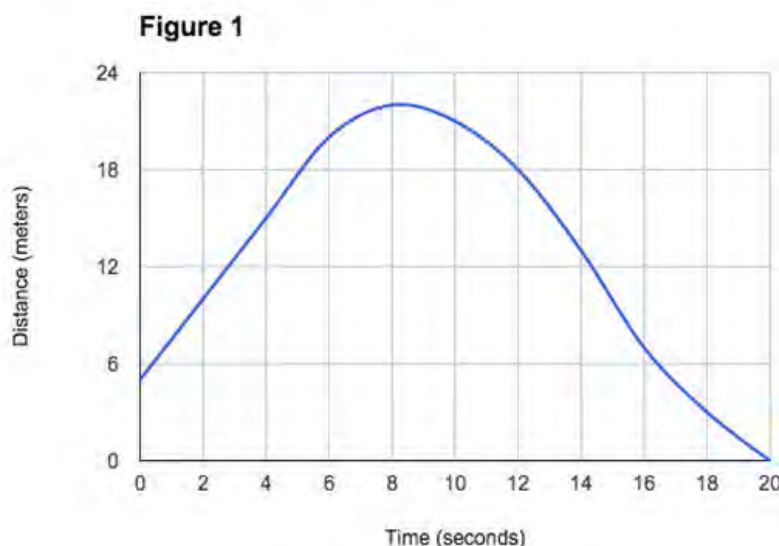
Sample Question

Velocity is given by the equation

$$v = \frac{\text{distance}}{\text{time}}$$

A student has measured the distance a ball rolls over a period of time and recorded her results in Figure 1.

Over what time interval is the ball's velocity decreasing?



Though this question may at first look similar to the identification question format, there is a key difference: the variable in question (the velocity) is not actually displayed on this graph. The question does, however, give us the tools to find it.

We have the given equation for velocity, as well as the graph of our data. At this point, we see how certain analytical skills are incorporated into the ACT Science section. We know that the velocity will be equal to the distance divided by the time, and we have a graph with distance on the vertical axis and time on the horizontal axis. Recalling the properties of linear equations, you should be able to recognize that the velocity will be equal to the slope of this graph.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{distance}}{\text{time}} = \text{velocity}$$

Since we know that the question is asking for the interval where the velocity is decreasing, we can interpret this to be the interval where the slope is decreasing.

What does it mean to have a decreasing slope? A slope of zero will be a horizontal line; a decreasing slope will be approaching horizontal. In this case, we are looking for the region of the graph that approaches horizontal, or gets more “flat.”

Our graph can be broken up into four basic segments. The trend is a straight line between 0 and 5. It then begins to turn (becoming more flat) between 5 and 8 until it completes the turn and begins to follow a path downward. Between 8 and 11, the graph becomes steeper, until it once more becomes a straight line between 11 and 20.

Based on these trends, we can see that the velocity is decreasing between 5 seconds and 8 seconds.



Anticipating Placement of New Data Points

Sample Question

A titration is a chemical procedure used to determine the concentration of an unknown sample of acid or base. An indicator compound is added to the unknown sample, and a different base or acid of known concentration is added to the unknown sample in small amounts. The indicator is designed to change color at a known pH value, allowing us to determine how much of the unknown sample has reacted with the known additive.

A student is performing a titration of an unknown weak acid with a strong base. Data from the experiment is collected in Table 1.

Based on the results in Table 1, how many milliliters (mL) of base must be added for the solution to have a pH of 4.5?

Table 1	
Volume of Base Added (mL)	pH
0	1.0
2.5	1.2
5.0	1.3
7.5	1.5
10.0	1.7
12.5	2.0
15.0	7.0
17.5	11.9
20.0	12.1
22.5	12.3
25.0	12.4
27.5	12.5
30.0	12.5

Though it may seem like we are simply being asked to identify a point in the table, this question actually requires a bit more critical thinking. We can see in the second column that pH values jump abruptly from 2.0 to 7.0, yet the question asks us to find the amount of base added for a pH of 4.5.

It is not possible to tell from this passage the exact value that will result in a pH of 4.5, but we can determine a range. We know that a pH of 4.5 is between a pH of 2.0 and a pH of 7.0. Looking at our table, we can find the volume of base added to arrive at each of these pH values. A pH of 2.0 corresponds to 12.5mL of base, while a pH of 7.0 corresponds to 15.0mL of base. Since we know that our pH must fall within this range, we know that our volume of added base must also fall in this range.

Now, we can compare our range to the given answer options for the question. We know that the correct answer will be between 12.5mL and 15.0mL.

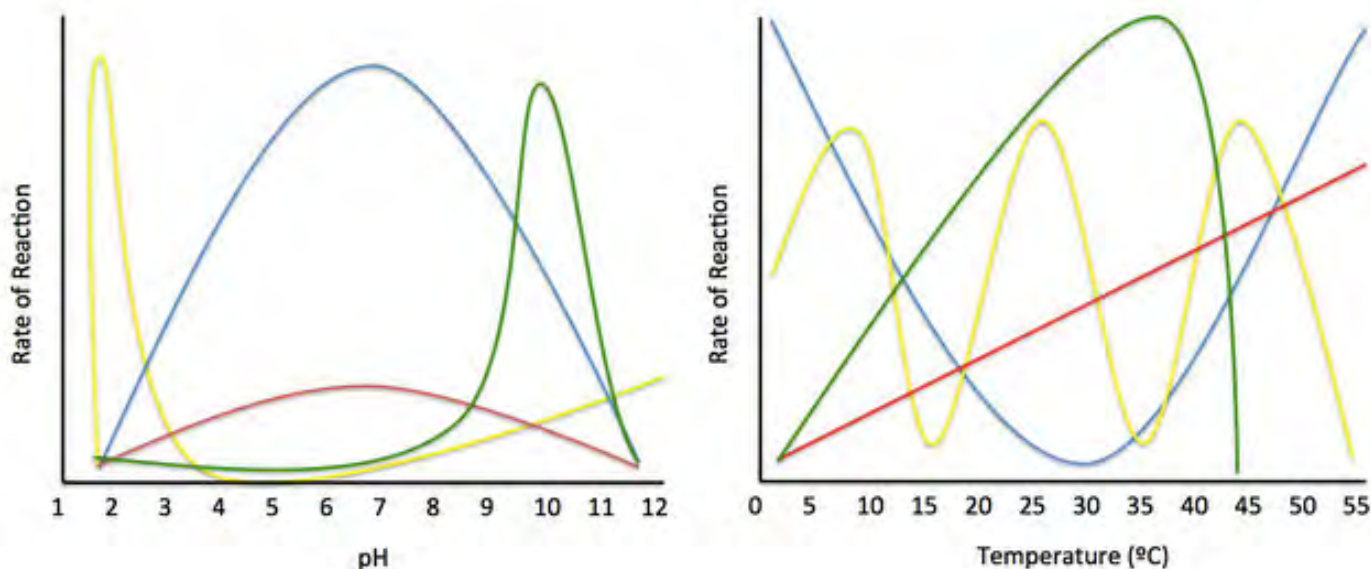


Practice Passage: Data Interpretation

Amino acids, when strung together in extensive chains, serve as the building blocks of muscles and proteins. At around 37°C, these amino acid chains allow the body to carry out both macroscopic processes, like moving arms and legs, and microscopic processes, like increasing the rate of chemical reactions. A special class of proteins called enzymes assists in combining reactants to produce products by speeding up the rate of a reaction in one of three ways.

The first way enzymes increase reaction rate is by lowering the activation energy of a reaction. This is done by balancing positively charged amino acids with negatively charged amino acids, creating an electrically neutral environment. This process is called electrostatic interaction. Another way enzymes increase reaction rate is through the use of non-charged amino acids, such as valine and isoleucine, in a process called Van der Waals interactions. In Van der Waals interactions, the non-charged amino acids become temporarily polarized, similar to the permanent polarity of positively and negatively charged amino acids. This interaction brings non-charged amino acids together to stabilize the reactants. The final way enzymes increase reaction rates is by sharing the electrons in their hydrogen atoms with nitrogen, oxygen, or fluorine on the reactant molecules to trap them at the active site. The active site is the part of an enzyme where molecules bind and undergo a chemical reaction.

Enzymes are designed to work in specific parts of the body depending on their functions. For example, an enzyme in the stomach responsible for breaking down food would work most effectively at low pH, while an enzyme in the small intestine responsible for absorbing food would work most effectively at high pH. Some enzymes, such as those that function in the blood, work best at intermediate pH. Some enzymes function better at lower temperatures, while others require higher temperatures. All enzymes have exponential relationships between their rates of reaction and both pH and temperatures, meaning that they function best in narrow pH and temperature windows. Graphs of four enzymes and their rates of reaction at various pH levels and temperature are presented below.



1. Based on the figures, which enzyme functions optimally at body temperature (about 37°C)?
 - A. The enzyme represented by the yellow curve
 - B. The enzyme represented by the blue curve
 - C. The enzyme represented by the green curve
 - D. The enzyme represented by the red curve
2. Blood has an intermediate pH of roughly 7.5. Based on the figures, which enzyme is most likely to function in blood?
 - A. The enzyme represented by the yellow curve
 - B. The enzyme represented by the blue curve
 - C. The enzyme represented by the green curve
 - D. The enzyme represented by the red curve
3. Based on the figures, which enzyme functions best at temperature extremes?
 - A. The enzyme represented by the yellow curve
 - B. The enzyme represented by the blue curve
 - C. The enzyme represented by the green curve
 - D. The enzyme represented by the red curve
4. A certain enzyme in plants allows for improved conversion of sunlight and carbon dioxide into glucose and oxygen during photosynthesis. As temperature increases, the enzyme is able to help produce more oxygen. According to the figures, which enzyme is most likely to exist in plants?
 - A. The enzyme represented by the yellow curve
 - B. The enzyme represented by the blue curve
 - C. The enzyme represented by the green curve
 - D. The enzyme represented by the red curve
5. Based on the figures, which enzyme is most likely to function in the stomach?
 - A. The enzyme represented by the yellow curve
 - B. The enzyme represented by the blue curve
 - C. The enzyme represented by the green curve
 - D. The enzyme represented by the red curve



Data Interpretation Sample Passage: Answers

1. Based on the figures, which enzyme functions optimally at body temperature (about 37°C)?

A. The enzyme represented by the yellow curve

B. The enzyme represented by the blue curve

→ C. The enzyme represented by the green curve

D. The enzyme represented by the red curve

2. Blood has an intermediate pH of roughly 7.5. Based on the figures, which enzyme is most likely to function in blood?

A. The enzyme represented by the yellow curve

→ B. The enzyme represented by the blue curve

C. The enzyme represented by the green curve

D. The enzyme represented by the red curve

3. Based on the figures, which enzyme functions best at temperature extremes?

A. The enzyme represented by the yellow curve

→ B. The enzyme represented by the blue curve

C. The enzyme represented by the green curve

D. The enzyme represented by the red curve

Based on the figure comparing rate of reaction to temperature, we see that only the green curve has a peak rate of reaction at around 37°C.

The passage states that enzymes that function in the blood work best at intermediate pH. Based on the figure comparing rate of reaction to pH, we see that only the blue curve has a high rate of reaction at intermediate pH, indicating that it would function best in the neutral pH environment of the blood.

Based on the figure comparing rate of reaction to temperature, we see that the blue curve has the highest rates of reaction at temperature extremes.



4. A certain enzyme in plants allows for improved conversion of sunlight and carbon dioxide into glucose and oxygen during photosynthesis. As temperature increases, the enzyme is able to help produce more oxygen. According to the figures, which enzyme is most likely to exist in plants?

A. The enzyme represented by the yellow curve
B. The enzyme represented by the blue curve
C. The enzyme represented by the green curve

→ D. The enzyme represented by the red curve

5. Based on the figures, which enzyme is most likely to function in the stomach?

→ A. The enzyme represented by the yellow curve
B. The enzyme represented by the blue curve
C. The enzyme represented by the green curve
D. The enzyme represented by the red curve

The question asks us to determine the relationship between temperature and rate of reaction. We know from the question that as temperature increases, the enzyme has a faster rate of reaction. We are looking for the curve that increases in rate of reaction as temperature increases. The only curve that does this across the entire temperature spectrum is the red curve.

The passage states that enzymes that work in the stomach function best at low pH. Based on the figure comparing rate of reaction to pH, we see that only the yellow curve has a high rate of reaction at low pH, indicating that it would function best in the low pH environment of the stomach.



Data Interpretation: Further Applications

The skills required for Data Interpretation questions on the ACT Science section are some of the most practical abilities tested on the exam. Being able to rapidly decipher data and interpret relationships can help in a multitude of practical scenarios, whether you simply need to check if the weather in your city is finally getting warmer and signaling an end of winter, or if you are working in a laboratory and trying to determine if a new drug is actually showing results toward reducing cancer incidence.

Perhaps even more valuable than basic interpretational skills is the ability to anticipate results based on trends. If you can determine how given data behaves—for example, that your phone takes 10 minutes to charge when it has 90% battery and 50 minutes to charge when it has 50% battery—then you can predict future data points. Based on this trend, you know that your phone will probably take about 100 minutes to charge from 0%, letting you plan ahead to avoid the travesty of having a phone with no battery.

On a more advanced level, being able to understand empirical relationships can help you better decipher things like interest rates on student loans, credit card bills, and mortgages. Understanding the implications of these relationships can be invaluable in calculating your finances and avoiding debt.



Experimental Setups and Results

Experimental design is an important component of both the scientific method and ACT Science passages that you may encounter on test day. Scientific theories are wrought from practical laboratory procedures. The purpose or hypotheses present in a passage are not always obvious. Practice and experience in reading comprehension can enable you to become familiar with scientific language. They can also help you identify the purpose of a study. Likewise, results are often illustrated in figures or tables; however, comprehension of the passage helps to decipher their implications for the study. Understanding how these procedures are carried out is crucial to understanding the purpose of scientific experiments illustrated on the ACT. Identifying the subtle mechanics of these exercises will assist you come test day.

Section Summary

[Distinguishing Experimental Purpose](#)

[Interpreting Textual Setups](#)

[Interpreting Diagrammed Setups](#)

[Describing Results](#)

[Visualizing Results](#)

[Applying Results to New Scenarios](#)

[Experimental Integrity and Flaws](#)

[Further Applications](#)



Distinguishing Experimental Purpose

Studies are often performed in order to solve a particular problem, identify causality, or identify correlations for future research. The purpose of a passage can be synonymous with its hypothesis—an if-then statement that presents a possible explanation for a particular phenomenon. Identifying the purpose embedded with scientific jargon, descriptions, and results may seem difficult to understand at first, but familiarizing yourself with the format and language commonly associated with the Science section of the ACT test can help you identify experimental purpose with ease.

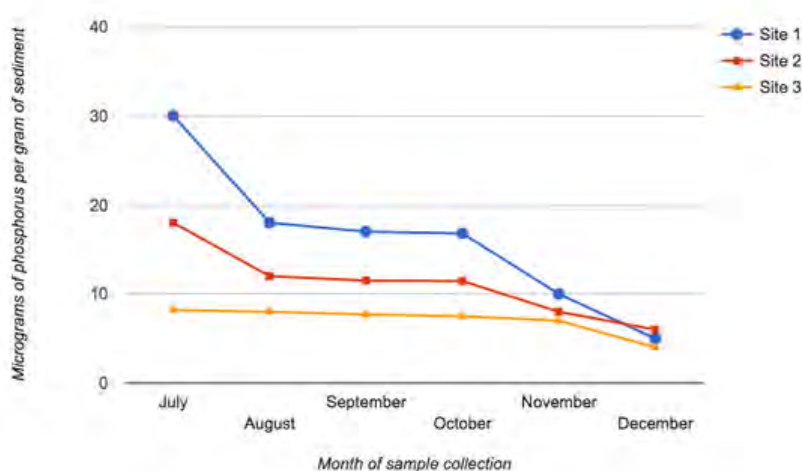
Sample Passage

A series of studies is performed in order to find a relationship between plant enzyme activity and phosphorus levels in aquatic biomes. Researchers studying the invasive Eurasian milfoil, *Myriophyllum Spicatum*, believe that they can control the spread of the plant into neighboring waters if they can limit the nutrients in its environment. Plants need nutrients such as phosphorus to grow and proliferate; therefore, understanding how this species uses phosphorus could lead to conservation practices to limit this invasive species' impact on the environment.

Study 1

Three standing ponds were sampled at six different time periods in the same year from July to December. The phosphorus content of each sample was determined through an ascorbic acid assay. Sediment collections were divided into samples weighing one sixteenth of a gram using coning and quartering techniques. These samples underwent a persulfate digestion and were vacuum filtered to remove excess sediment. Last, the samples were diluted and analyzed for phosphorus content using the ascorbic acid procedure. The solutions were measured spectrophotometrically at an absorbance of 880 nanometers using a reagent blank as a reference solution. Concentrations were calculated through the use of a standard curve of known values of phosphorus. Results are represented in Figure 1.

Figure 1



Study 2

In the second study, the soil collections underwent an alkaline phosphatase assay. One to two milliliters of collected sediment was centrifuged until the sediment formed a pellet. The excess fluid was removed.

A buffer 0.25 ml of artificial substrate was added to each sample. The samples incubated for 15 minutes. The reaction between the phosphatase enzymes present in the soil and the artificial substrate was halted through the addition of 0.25 ml of 1N NaOH. The samples were centrifuged and their absorbance was measured with a spectrophotometer set at 420 nanometers to measure levels of p-nitrophenol produced by the enzyme reaction. A standard curve of known quantities of p-nitrophenol was used to calculate the enzyme activity per unit time. Results are represented in Figure 2.

Figure 2

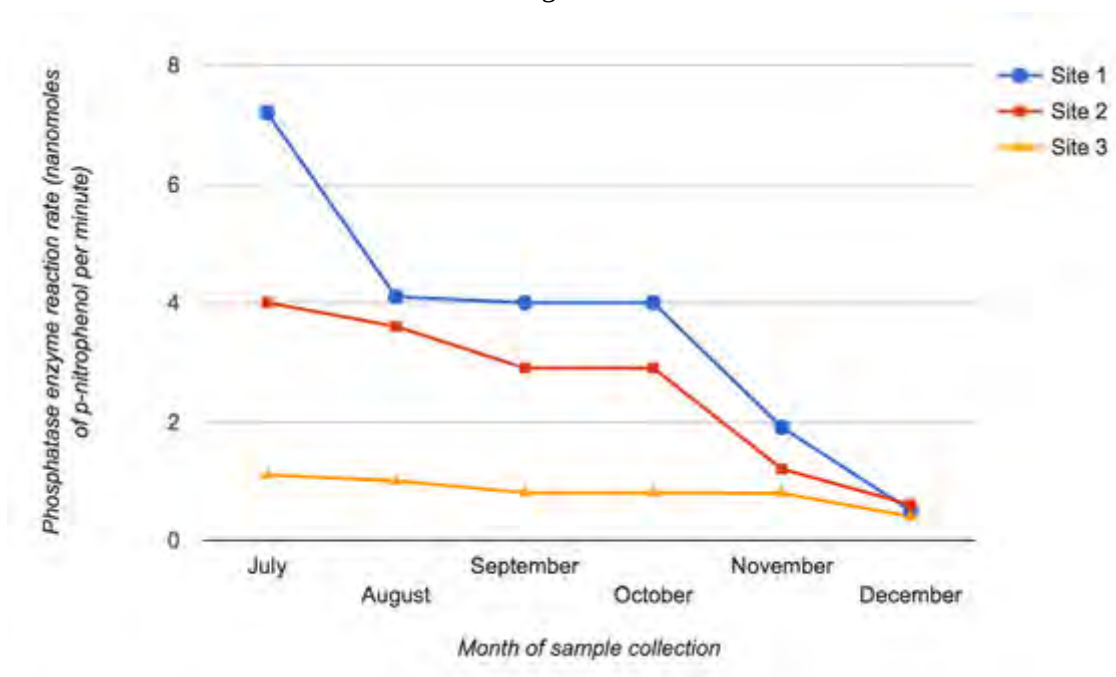
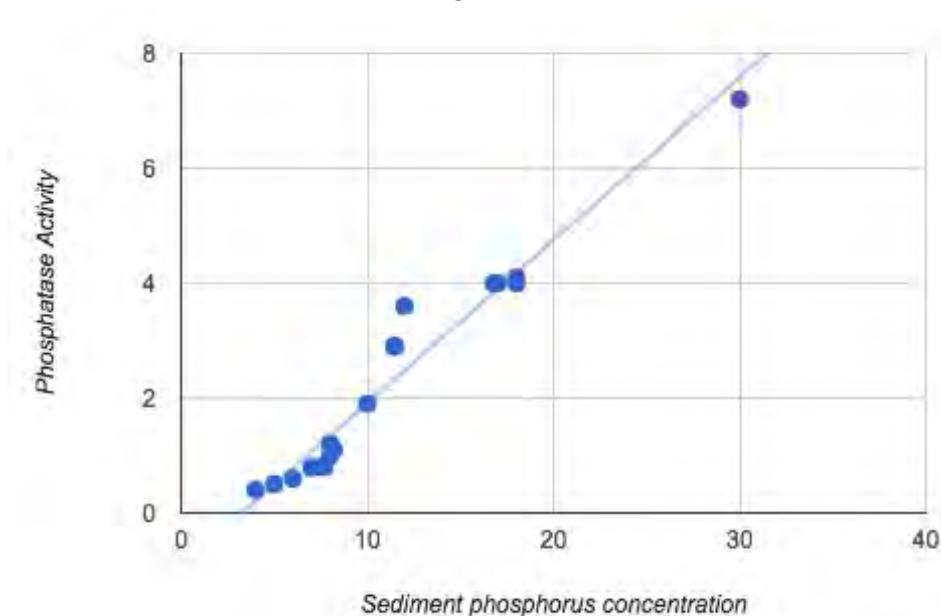


Figure 3 represents a correlation between phosphorus concentration and phosphatase activity in all the sites across all time periods.

Figure 3



Sample Question

What is the purpose of this experiment?

- A.** Identify a connection between enzyme activity and phosphorus concentration
- B.** Measure levels of phosphorus in the environment
- C.** Measure levels of plant enzyme activity
- D.** None of these

Choice A is the correct choice.

The purpose of this passage is to develop a relationship between enzyme activity and phosphorus concentrations. In Study 1, the researchers measured the levels of phosphorus present within the environment. In Study 2, the researchers measured the soil's enzyme activity. The main purpose of this study was to develop a relationship or correlation between these two studies.

Sample Question

What was the purpose of Study 1?

- A.** Identify a connection between enzyme activity and phosphorus concentration
- B.** Measure levels of phosphorus in the environment
- C.** Measure levels of plant enzyme activity
- D.** None of these

Choice B is the correct choice.

Study 1 measured the concentration of phosphorus per gram of sediment in the environment. Figure 1 corroborates this information and visualizes the collected data. Study 1 is a part of the overall purpose to identify a relationship between enzyme activity and phosphorus concentration.

Sample Question

What is the Purpose of Study 2?

- A.** Identify a connection between enzyme activity and phosphorus concentration
- B.** Measure levels of phosphorus in the environment
- C.** Measure levels of plant enzyme activity
- D.** None of these

Choice C is the correct choice.

Study 2 measured the enzyme activity by way of reaction rates present in sediment samples. Figure 2 corroborates this information and visualizes the collected data. Study 2 is a part of the overall purpose to identify a relationship between enzyme activity and phosphorus concentration.



Interpreting Textual Setups

Sometimes experimental setups are described in visual or textual forms. Textually described setups possess an inherent difficulty because they lack diagrams to illustrate the experiment. In this section, we will introduce a text-based setup passage that will provide an example of how to navigate questions commonly associated with these items on the ACT.

Sample Passage

Naturally occurring water in lakes and reservoirs used as sources for drinking water feature a variety of dissolved minerals such as magnesium, sodium, and calcium. Water treatment plants must closely monitor the levels of these minerals to ensure they do reach unsafe levels. An experiment carried out by a scientist at a water treatment plant is described below.

Experiment 1

A common way to determine the concentration of a particular chemical is by titration. In this titration, 10 milliliters of the treated water sample was placed in a flask. A buret was placed above the flask. A buret is a specialized funnel with volume markings on the side and a knob on the bottom to control the volume of substance dispensed from the device. The buret was filled with 50 milliliters of a 20 parts per million (ppm) solution of EDTA, a chemical that can react with magnesium to chemically remove it from the water. An indicator—a substance that changes color to indicate a chemical change—was also placed in the flask. This indicator appears purple in solutions containing magnesium and blue in solutions without magnesium. The buret was used to dispense the EDTA solution until enough EDTA had been added to the magnesium-containing solutions to remove all the magnesium. Once the EDTA removed the magnesium from the samples, the solutions changed color from purple to blue. The volume, in milliliters, of EDTA solution added to each of five water samples is recorded in Table 1.

Table 1

Trials	Volume of EDTA Added
Trial 1	31.5 mL
Trial 2	32.2 mL
Trial 3	29.6 mL
Trial 4	27.9 mL
Trial 5	30.4 mL



Sample Question

Which of the following best describes the initial contents of the sample flasks?

- A. 10 milliliters of treated water, indicator solution, and 50 milliliters of EDTA
- B. 10 milliliters of treated water and indicator solution
- C. 10 milliliters of treated water, indicator solution, and 20 milliliters of EDTA
- D. 10 milliliters of treated water, indicator solution, and 30.3 milliliters of EDTA

Choice B is the correct choice.

Initially, the flasks contained the treated water sample and indicator solution. EDTA was added to bind with the magnesium and turn the solutions from purple to blue. No EDTA was present in the sample flasks initially.

Sample Question

Which of the following substances was contained in the buret?

- A. EDTA
- B. Calcium
- C. Indicator solution
- D. Magnesium

Choice A is the correct choice.

The buret contained the EDTA that was added to the samples in order to remove magnesium. The indicator solution was contained in each of the samples and changed their color from purple to blue depending on the magnesium content. Last, calcium is a mineral that may naturally occur in freshwater sources.

Interpreting Diagrammed Setups

Diagrams are often used in ACT Science passages to describe experimental setups. These diagrams can be useful when trying to identify or clarify concepts that may not be easily visualized by textual means. In this section, we will revisit the sample passage used in the previous text-based example and include a figure that visualizes the titration. Afterwards, we will examine a new sample passage and diagram. Mastering the hints provided in figures may help you better understand passages and make the most of your time on test day.



Sample Passage 1

Naturally occurring water in lakes and reservoirs used as sources for drinking water feature a variety of dissolved minerals such as magnesium, sodium, and calcium. Water treatment plants must closely monitor the levels of these minerals to ensure they do not reach unsafe levels. An experiment carried out by a scientist at a water treatment plant is described below.

Experiment 1

A common way to determine the concentration of a particular chemical is by titration. In this titration, 10 milliliters of the treated water sample was placed in a flask. A buret was placed above the flask (see Figure 1). A buret is a specialized funnel with volume markings on the side and a knob on the bottom to control the volume of substance dispensed from the device (see Figure 1). The buret was filled with 50 milliliters of a 20 parts per million (ppm) solution of EDTA, a chemical that can react with magnesium to chemically remove it from the water. An indicator—a substance that changes color to indicate a chemical change—was also placed in the flask. This indicator appears purple in solutions containing magnesium and blue in solutions without magnesium. The buret was used to dispense the EDTA solution until enough EDTA had been added to the magnesium-containing solutions to remove all the magnesium. Once the EDTA removed the magnesium from the samples, the solutions changed color from purple to blue.



Figure 1

Sample Passage 2

Bacteria is often grown on plates and used for testing. In a certain experiment, a researcher attempts to test the efficacy of three antibiotics on a particular strain of *Escherichia coli* bacteria. The scientist takes four plates containing swabs of the bacteria culture and places four squares of parchment paper soaked in one of the following: saline solution, ampicillin, penicillin, and amoxicillin. The control plate contained saline, Trial I contained ampicillin, Trial II contained penicillin, and Trial III contained amoxicillin (see Figure 1). The plates are left to incubate for 48 hours. After incubation, the scientist observes the bacterial growth on each of the plates (see Figure 2). In Figure 2, the grey coloring represents bacterial colonies, while the white indicates that no bacteria is present.

Figure 1

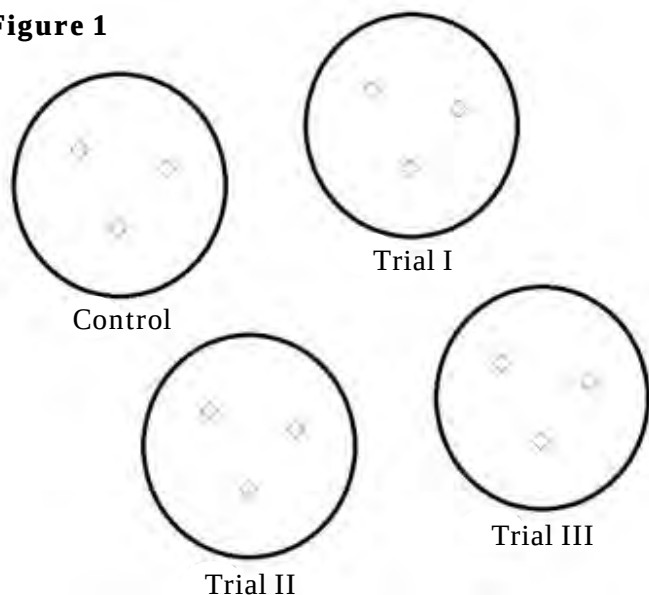
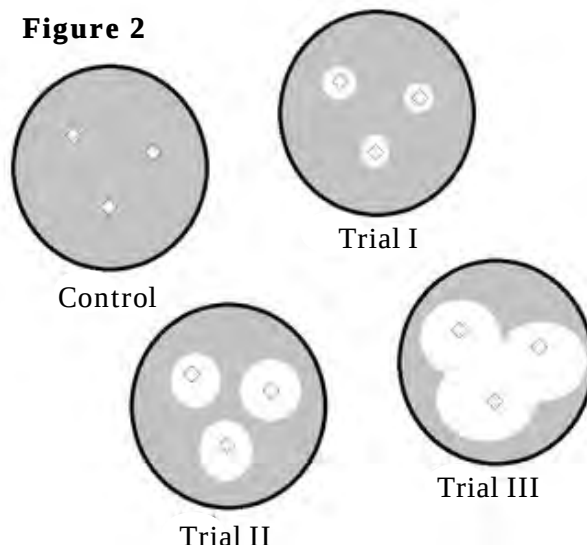


Figure 2



Sample Question

Which of the following controls the flow of EDTA out of the buret and into the flask?

- A. Cylinder
- B. Funnel
- C. Volumetric pressure
- D. Knob

Choice D is the correct choice.

According to the passage's description of a buret, the knob controls the flow of EDTA from the cylinder into the flask. Alternatively, one could observe Figure 1 and see that the knob on the side of the cylinder would be the most likely choice to control EDTA flow.

Sample Question

The buret contained which of the following chemicals?

- A. 10 milliliters of treated water
- B. 50 milliliters of 20 ppm EDTA solution
- C. Indicator solution
- D. All of the other answers are correct

Choice B is the correct choice.

The buret contains the EDTA solution. The flask contains the indicator solution and the 10 milliliters of treated water.

Sample Question

Which antibiotic was most effective at treating the bacteria?

- A. Ampicillin
- B. Amoxicillin
- C. Penicillin
- D. Control

Choice B is the correct choice.

The best way to answer this question is to look at Figure 2. Figure 2 illustrates the bacteria plates after they incubated for 48 hours. The Trial III plate has the most amount of white space; therefore, amoxicillin was best at treating the bacteria. No other plate treated the bacteria as well as Trial III.



Sample Question

Which of the following is true regarding the control plate?

- A.** It was incubated for 48 hours
- B.** It was swabbed with bacteria
- C.** It contained papers soaked in saline
- D.** All of the other answer choices are correct

Choice D is the correct choice.

The control plate represents a trial standardized for “normal” conditions. As a result, the control plate is treated similar to each of the trials. The control was swabbed with bacteria, contained papers soaked in saline, and incubated for 48 hours. The control contained saline in order to remain standardized with the other plates containing papers. The saline would not have any effect on bacterial growth. This is supported by Figure 2.



Describing Results

Results can be described in a number of ways. The ACT may reveal results in a passage outlining different studies or viewpoints, or it may illustrate them using tables and figures. This section will help you decipher scientific language and results embedded in the text of a passage. Mastering this skill will assist you in understanding and identifying scientific results on test day. This process essentially rests upon reading comprehension skills, and this section will refine your ability to pay attention to detail.

Sample Passage

Scientists attempt to study bacterial immunity to antibiotics.

Study 1

Bacteria is often grown on plates and used for testing. In a certain experiment, a researcher attempts to test the efficacy of three antibiotics on a particular strain of *Escherichia coli* bacteria. The scientist takes four plates containing swabs of the bacteria culture and places four squares of parchment paper soaked in one of the following: saline solution, ampicillin, penicillin, and amoxicillin. The control plate contained saline, Trial I contained ampicillin, Trial II contained penicillin, and Trial III contained amoxicillin. The plates are left to incubate for 48 hours. After incubation, the scientist observes the bacterial growth on each of the plates. The scientists observed the following: bacteria covered the control plate completely, 90 percent of the Trial I plate, 75 percent of the Trial II plate, and 60 percent of the Trial III plate.

Study 2

The scientists conclude that *Escherichia coli* can be effectively treated with amoxicillin. They create a culture using the surviving bacteria on the Trial III plate. They create four new plates containing the same control and trial components as Study 1 and swab them with the bacteria cultured from the surviving bacteria on Study 1's Trial III plate. They allow the plates to incubate for 48 hours. After incubating the plates, the scientists observe the bacterial growth on each of the plates and note the following: bacteria covered the control plate completely, 95 percent of the Trial I plate, 85 percent of the Trial II plate, and 98 percent of the Trial III plate.

Sample Question

Which of the following treatments was most effective at eradicating the bacteria in Study 1?

- A. Trial I
- B. Trial II
- C. Trial III
- D. Control

Choice C is the correct choice.

The passage states that the plate in Trial III only contained 60 percent bacterial coverage. This plate had the lowest percentage of bacterial coverage; therefore, amoxicillin was most successful at killing the bacteria in Study 1.



Sample Question

Which of the following treatments was most effective at eradicating the bacteria in Study 2?

- A.** Trial I
- B.** Trial II
- C.** Trial III
- D.** Control

Choice B is the correct choice.

The passage states that the plate in Trial II only contained 85 percent bacterial coverage. This plate had the lowest percentage of bacterial coverage; therefore, penicillin was most successful at killing the bacteria in Study 2.

Sample Question

The bacteria in the study developed resistance to which of the following treatments?

- A.** Trial I
- B.** Trial II
- C.** Trial III
- D.** All of the answer choices are correct

Choice D is correct.

The surviving bacteria of Trial III in Study 1 were cultured and placed on the plates in Study 2. These bacteria developed resistance to all of the antibiotics. This makes sense because all of the antibiotics are similarly derived from fungi.



Visualizing Results

We have covered passages that simply list the experiment's results. In this section, we will look at passages that use tables and figures to visualize the results. Tables and figures neatly pack data into accessible illustrations, and knowing how to use both the text and data in conjunction with one another is key to understanding ACT Science passages. In this section, we will work on developing reading comprehension and data analysis skills.

Sample Passage

A series of studies is performed in order to find a relationship between plant enzyme activity and phosphorus levels in aquatic biomes. Researchers studying the invasive Eurasian milfoil, *Myriophyllum spicatum*, believe that they can control the spread of the plant into neighboring waters if they can limit the nutrients in its environment. Plants need nutrients such as phosphorus to grow and proliferate; therefore, understanding how this species uses phosphorus could lead to conservation practices to limit this invasive species' impact on the environment.

Study 1

Three standing ponds were sampled at six different time periods in the same year from July to December. The phosphorus content of each sample was determined through an ascorbic acid assay. Sediment collections were divided into samples weighing one sixteenth of a gram using coning and quartering techniques. These samples underwent a persulfate digestion and were vacuum filtered to remove excess sediment. Last, the samples were diluted and analyzed for phosphorus content using the ascorbic acid procedure. The solutions were measured spectrophotometrically at an absorbance of 880 nanometers using a reagent blank as a reference solution. Concentrations were calculated through the use of a standard curve of known values of phosphorus. Results are represented in Figure 1.

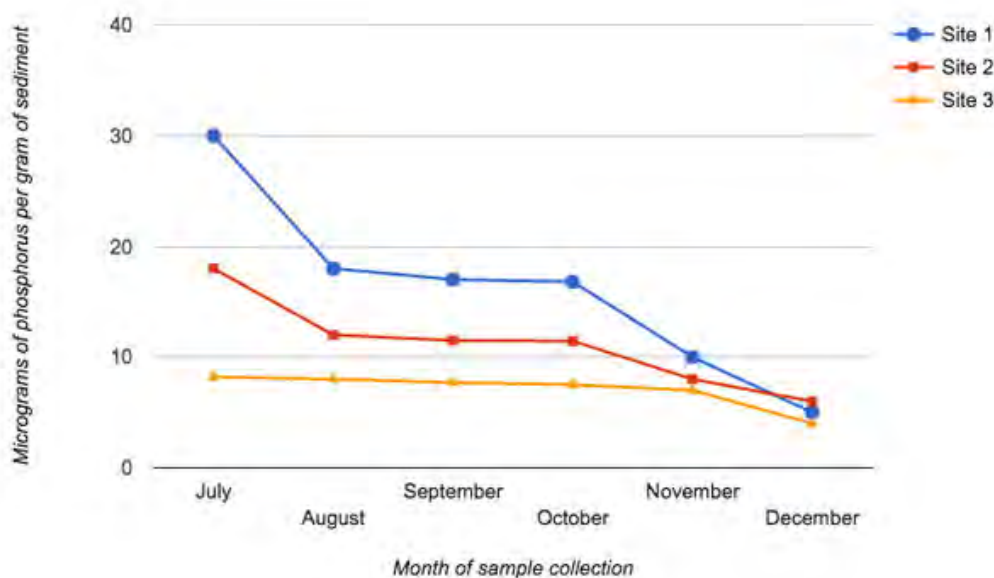


Figure 1



Study 2

In the second study, the soil collections underwent an alkaline phosphatase assay. One to two milliliters of collected sediment was centrifuged until the sediment formed a pellet. The excess fluid was removed. A buffer 0.25 ml of artificial substrate was added to each sample. The samples incubated for 15 minutes. The reaction between the phosphatase enzymes present in the soil and the artificial substrate was halted through the addition of 0.25 ml of 1N NaOH. The samples were centrifuged and their absorbance was measured with a spectrophotometer set at 420 nanometers to measure levels of p-nitrophenol produced by the enzyme reaction. A standard curve of known quantities of p-nitrophenol was used to calculate the enzyme activity per unit time. Results are represented in Figure 2.

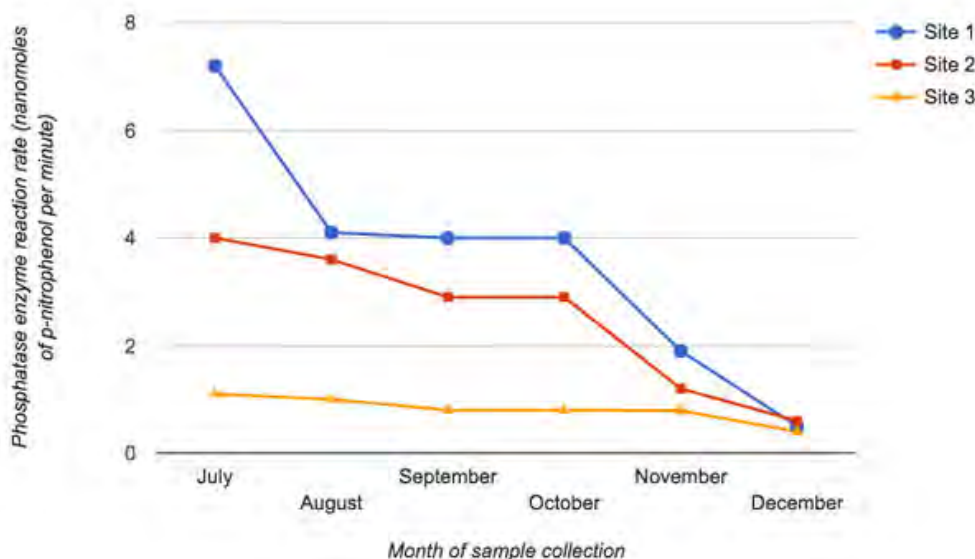


Figure 2

Figure 3 represents a correlation between phosphorus concentration and phosphatase activity in all the sites across all time periods.

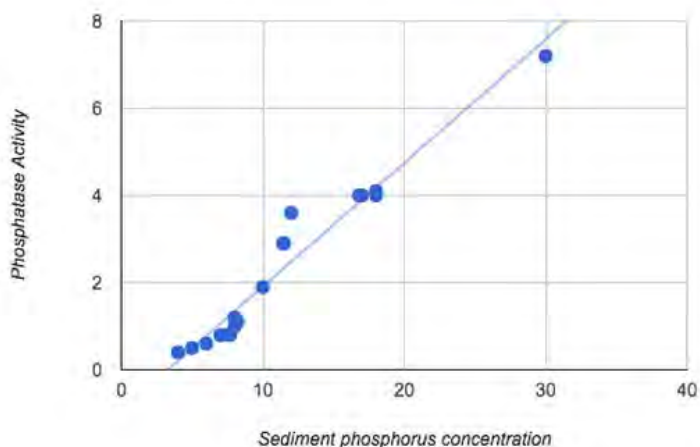


Figure 3



Sample Question

Which of the following choices best describes the pattern of phosphorus concentration over time?

- A.** Phosphorus concentration increased over time
- B.** Phosphorus concentration decreased over time
- C.** Phosphorus concentration increased and then decreased over time
- D.** Phosphorus concentration decreased and then increased over time

Choice B is the correct choice.

Figure 1 shows that over the six-month period the phosphorus concentration decreased over time in all cases. You can trace each one by hand and see that all of them decrease over time.

Sample Question

Which of the following choices best describes the pattern of enzyme activity over time?

- A.** Enzyme activity increased over time
- B.** Enzyme activity decreased over time
- C.** Enzyme activity increased and then decreased over time
- D.** Enzyme activity decreased and then increased over time

Choice B is the correct choice.

Figure 2 shows that over the six-month period, the enzyme activity decreased over time in all cases. You can trace each one by hand and see that all of them decrease over time.

Sample Question

Which of the following best describes the relationship between phosphorus concentration and enzyme activity?

- A.** They are inducible—as one increase or decreases so does the other
- B.** They are opposing—as one increases the other decreases
- C.** They are inversely correlated to one another
- D.** None of these

Choice A is the correct choice.

The effects are inducible. The two variables follow a near-linear relationship with one another according to Figure 3. As one factor increases or decreases, so does the other. In other words, when phosphorus is present, phosphatase enzymes are released by plants to obtain it. In many biomes, phosphorus is bound to soil molecules. Phosphatase enzymes are released by plants to free phosphorus for the plant's use; therefore, if phosphorus is present in the soil, then plants will release enzymes to use it.



Applying Results to New Scenarios

Results obtained through experimental methods are often used to inform future studies and provide context for additional information. In this section, we will cover how the ACT may use evidence obtained in one study to inform questions related to a new scenario.

By determining the outcome of a study and drawing particular conclusions about the hypotheses and claims tested, experimentation can lead to new stores of knowledge. The ACT demands that you not only decipher the conclusions presented by a passage about experimental setups and results, but also apply the newfound information in new contexts. These types of questions generally have a longer question stem, presenting some new, tangentially related scenario for students to assess. To address the new scenario, you will need to filter through the information presented in the passage to find the most applicable conclusions before determining an inferred conclusion to the new situation.

Consider an experimental setup in which soil samples are taken from varying depths in the Earth's surface and measured for saturation percentage of various gases. The results determine the saturation of oxygen, nitrogen, carbon dioxide, hydrogen, and ozone gases at seven different soil depths: 5m, 10m, 15m, 20m, 25m, 35m, and 45m.

	Sample Depth (Earth)						
	5m	10m	15m	20m	25m	35m	45m
Oxygen	30%	27%	21%	15%	10%	3%	1%
Nitrogen	70%	66%	52%	31%	12%	2%	2%
Carbon Dioxide	40%	20%	20%	5%	2%	1%	1%
Hydrogen	7%	7%	6%	6%	6%	5%	4%
Ozone	2%	2%	2%	2%	2%	1%	1%

Sample Question

Several soil samples from the Moon are taken at a depth of 40m and are found to be very similar in oxygen, carbon dioxide, and hydrogen composition to comparable samples on Earth. Soil samples from the Moon at a depth of 30m indicate that carbon dioxide and hydrogen levels are similar to those on Earth, but oxygen levels are significantly lessened. What conclusions can be drawn about the oxygen soil saturation of the Moon at 20m?

- A. It is less than 15%
- B. It is greater than 15%
- C. It is equal to 15%
- D. It is between 15% and 10%

This experiment garners a wide range of applicable data—35 data points in total, corresponding to each combination of soil depth and type of gas, as well as various trends for which gases penetrate farthest into the soil. With such a variety of applicable conclusions available from the data collected, answering a question about a new scenario first requires you to determine what conclusion is being addressed by the new information. Otherwise, these questions can quickly become overwhelming.



Let's start sorting through the new information presented. First, we are told that Moon samples of oxygen, carbon dioxide, and hydrogen at 40m are comparable to samples on Earth. Next, we know that Moon samples of carbon dioxide and hydrogen at 30m are also comparable to Earth samples; however, oxygen levels are lower. Finally, we are asked about the levels of oxygen in Moon samples at 20m. This information is summarized in an improvised table below:

	Sample Depth (Moon)		
	20m	30m	40m
Oxygen	?	<Earth	=Earth
Carbon Dioxide		=Earth	=Earth
Hydrogen		=Earth	=Earth

Based on the trend for the oxygen sample taken at 30m on the Moon, we can infer that the sample taken at 20m will also be **less** than the comparable sample on Earth. This trend is the key to finding the correct answer: for depths less than 40m, oxygen saturation on the Moon will be less than oxygen saturation on Earth. Equipped with our new conclusion, we can quickly deduce the correct answer to the question.

We can cross-reference the Moon sample trends with the information presented in our table of Earth samples. The most important conclusion, of course, will come from the value of the oxygen Earth sample at 20m. Note that we can still fill in values for every cell of the table by comparing ranges of values; samples taken at 30m, for example, will fall between measurements at 25m and 35m.

	Sample Depth (Moon)		
	20m	30m	40m
Oxygen	LESS THAN 15%	LESS THAN 3%	1-3%
Carbon Dioxide		1-2%	1%
Hydrogen		5-6%	4-5%

Using our new table as a reference, we can return to the question at hand. Answer option A states that oxygen soil saturation of the Moon at 20m is less than 15%, corresponding to the conclusions we found. Thus, answer option A is correct.



Experimental Integrity and Flaws

Every experiment has its limitations, and some are flawed in their methodology and/or conclusions. It is important to know the limitations of a particular study. Being able to accurately gauge a study's validity and reliability is an important skill necessary for the ACT. Validity pertains to a study's ability to accurately correspond to the "real world" phenomenon under study. Reliability, on the other hand, refers to a study's ability to reproduce similar results. If a study is lacking either of these, then it may be flawed and not be factually sound. This section will help you analyze a passage, test its integrity, and tease out its possible flaws.

Sample Passage

Species competition is driven by a variety of factors. Resources such as water, food, sunlight, and suitable habitat are among the top contributors that influence interspecific and intraspecific competition. Interspecific competition is competition between different species, while intraspecific competition is between members of the same species.

Study 1

One interesting example of interspecific competition is that of two barnacle species that inhabit intertidal zones. *Balanus balanoides* inhabits the lower intertidal zone and *Chthamalus stellatus* inhabits the upper intertidal zone. A researcher attempts to study this phenomenon.

The researcher removes the *Balanus* species from the lower intertidal zone and observes that the *Chthamalus* species expands its range to inhabit the lower intertidal zone and the upper intertidal zone.

Study 2

The researcher then removes the *Chthamalus* species from the upper intertidal zone of a different area and observes that the *Balanus* species does not extend its range. The researcher concludes that competition has allowed each species to exist simultaneously by forming specialized niches that promote survivorship for each species.

Sample Question

How could this study be improved?

- A. The scientist could test each barnacle's ability to survive in different conditions using a laboratory setting
- B. The scientist could study the interspecific competition of each species
- C. The scientist could test the barnacle's survival rate in fresh and saltwater environments
- D. None of the other answer choices is correct

Choice A is the correct choice.

The main improvement to this study would be to test the different environmental conditions that affect each barnacle's environmental niche. Competition between species may play a role; however, environmental conditions that make a habitat unfavorable to one species may keep that species out of a particular area (i.e. *Balanus* barnacles desiccate in the upper intertidal zone, and *Chthamalus* barnacles



cannot escape predation in the lower intertidal zone). Observing different environmental conditions in a laboratory setting would tease out factors besides competition that affect where each species is found; thus, this study limited its scope to competition. It could be improved by looking at environmental conditions as well.

Sample Question

Which of the following choices represents an alternative explanation for why the *Balanus* species does not extend its range in Study 2?

- A.** Competition between the two barnacle species creates specialized niches for each
- B.** The species is preyed upon by local predators
- C.** The species desiccates in the upper zone
- D.** All of the other answers are correct

Choice D is the correct choice.

Competition does play a role in the respective habitat for each species; however, environmental factors such as predation and desiccation from a drier environment also affect the location of each species. It is improper to make a single conclusion when other factors may be at play. Competition does play a role, but it is not the only factor.



Experimental Setups and Results: Further Applications

Experimental setups and results are crucial components of the scientific method. Setups dictate how an experiment is conducted, and results illustrate the data collected. Understanding how to understand, interpret, and identify the limitations of these processes is crucial for the ACT. Moreover, these skills are necessary for any work in the natural sciences. Studies can never prove anything with complete certainty; however, when conducted professionally, each one brings the scientific community closer to an understanding of particular phenomena of the natural world.

Experiments and the application of their results depends upon several factors: validity, reliability, and the extent to which findings may be applied to other areas of research. Validity indicates that the metrics used in a study are accurate measures of “real world” indicators. Reliability indicates that a study obtains similar results during multiple replications. Last, if these measures are strong, then the study may be applied to a particular community; however, we cannot overextend our theoretical findings.

Beyond the ACT, the skills inherent in this lesson will help you to become familiar with simplified experimental setups and illustrations of results using text, figures, and tables. Any field related to medicine, medical research, ecology, and the like requires you to understand the nuances in experimental setups as well as the information obtained from research. These skills will introduce you to methodologies that are complicated in upper level research. With this foundation, you can grasp the core concepts required for scientific research and build upon them as you grow in complexity and study focus.

The skills in this chapter hinge upon reading comprehension. The ACT will provide all of the evidence necessary to solve particular problems. It is up to you to familiarize yourself with scientific literatures that explain methodologies, setups, and results. Practice will help you to engage with the text, figures, tables, and diagrams to understand the experiments and results collected in ACT passages. These skills will enable you to master these questions and be prepared for test day.



Evaluating Conflicting Theories

In contrast to the Data Interpretation and Experimental Setups and Results passages in ACT Science, Evaluating Conflicting Theories questions involve a great deal of reading comprehension, logical reasoning, and literary analysis. Unlike the tables and graphs that frequently appear in other sections, Conflicting Theories passages generally appear as a wall of text. This can be intimidating to test-takers, unless you know how to approach the structure of these passages.

Passages will be broken into segments of given information, followed by two or more differing theories that describe the background topic. Questions will address the primary arguments of each theory, as well as comparisons between the theories. Various questions will require students to evaluate the strengths and weaknesses of each theory in light of new information, interpret similarities and key differences between theories, and apply the given theories to certain scenarios. Note that both theories will be scientifically plausible; you will never be asked which theory is “correct” or “right.”

For those more comfortable with diagrams and figures, Conflicting Theories passages can be time-consuming and strenuous. In order to move quickly through these types of passages, it is essential to learn how to identify and apply the defining features of each argument quickly and accurately.

Section Outline

- [Given Background](#)
- [Identifying Theses and Arguments](#)
- [Identifying Agreement](#)
- [Comparing Arguments](#)
- [Applying Arguments](#)
- [Integrating New Information](#)
- [Further Applications](#)



Given Background

Conflicting Theories passages have two primary components: given information and the theories themselves. While most questions about the passage will require analysis and application of the theories at hand, there will frequently be a question addressing the given information upon which the theories are based.

Identifying these questions is relatively easy: they will not reference the theories being discussed. In other words, they will not contain any reference to “Scientist A,” “Theory 1,” or any other specific designations. Instead, they will frequently reference relatively obscure, pinpoint details about the topic. You are not expected to know these details off the top of your head! Use the introductory text of the passage to find the keywords or topics presented in the question. Most often, this will lead you directly to the correct answer.

Sample Passage

Sleep plays a vital role in defining the daily activities of virtually all animals. During periods of sleep, the parasympathetic nervous system becomes active and induces a relaxed state in response to increased levels of the hormone melatonin. Yet, despite its ubiquity in the animal kingdom, the purpose of sleep and its role in our daily lives has been disputed by scientists. Two scientists discuss their theories about the purpose of sleep.

Scientist 1

During periods of sleep, animals are able to conserve energy that they would otherwise be spending on unnecessary activity. If an animal's primary food source is most abundant during daylight, it is a waste of precious energy to be moving about at night. For example, many herbivores, such as squirrels, are diurnal (asleep during the night) because their food source is available during the day, while many insectivores, such as bats, are nocturnal (asleep during the day) because their food source is available during the night. Food sources, as an animal's most valuable resource, dictate their sleep cycles. Many animal traits observable today evolved as a result of the supply and demand of food in their natural habitat.

Scientist 2

During waking hours, it is true that the body utilizes large amounts of energy; however, the role of sleep is to restore biological products that were utilized during periods of wakefulness, rather than simply to avoid utilizing energy in the first place. Many types of biological molecules, such as hormones, are released throughout the body while an animal is active. Sleep serves as a period of inactivity during which the body can manufacture and store a supply of these molecules, for future use during the next period of activity; furthermore, sleep allows the body to repair cellular damage that has accumulated during waking hours. Experimental evidence shows that when animals are deprived of sleep, their immune system quickly weakens and death rates increase. Sleep is necessary for animals to prevent accumulation of damage and to regenerate crucial biomolecules for daily life.



The following is a question on the given background of this passage. Note that it does not make direct reference to either scientist or their theories.

Sample Question

Which of the following is likely true of melatonin?

- A.** Diurnal animals will express high levels of melatonin at night
- B.** Melatonin causes animals to seek food sources
- C.** Nocturnal animals will express high levels of melatonin at night
- D.** Melatonin causes animals to use more energy

To answer this question, we need to find the part of the passage that relates to melatonin. Since we have identified the question as addressing the background information, based on the omission of direct reference to the scientist, we should look for “melatonin” in the introductory information. When we do so, we find this phrase:

During periods of sleep, the parasympathetic nervous system becomes active and induces a relaxed state in response to increased levels of the hormone melatonin.

This tells us that melatonin is found at high levels when the animal is asleep. Now we simply need to select an answer option that reflects this information.

We can immediately eliminate answers B and D. We know that melatonin induces sleep; it does not cause energy expenditure or hunger. This leaves answers A and C. To determine which of these is correct, you must understand the definitions of “diurnal” and “nocturnal.” These definitions are given in the theory described by Scientist 1. Diurnal animals are asleep during the night, while nocturnal animals are asleep during the day.

Now, we simply need to put together the given information. Melatonin is expressed when an animal is asleep—diurnal animals sleep at night, and nocturnal animals sleep during the day. Based on these facts, we can infer that melatonin will be high in diurnal animals at night, and melatonin will be high in nocturnal animals during the day (corresponding to when each type of animal is asleep). Based on this reasoning, the correct answer must be A, “Diurnal animals will express high levels of melatonin at night.”

Notice how we did not need to analyze the thesis of either scientist’s argument or establish in-depth comprehension of the arguments presented. The information to address a question on the given background will always be readily available, and presented directly in the passage.



Identifying Theses and Arguments

Identifying the theses of the arguments presented in a passage concerning conflicting theories is one of the most critical steps in approaching the majority of the questions about the passage. Not only will certain questions require you to summarize the defining features of each argument, but you will also be asked to apply these defining elements to various scenarios, compare them between arguments, and identify their strengths and weaknesses. As such, being able to quickly and accurately determine the theses of the presented theories is arguably the most important skill for this type of passage.

Begin by reading each theory once to gain a basic understanding of the arguments being made. This will give you the background necessary to establish exactly what defines each argument. Next, look for distinct information in each theory; that is, look for any information or key terminology that is mentioned in one theory, but not the other. Finally, look for information that appears in both theories. This final step will reveal two types of information: agreement (information that is presented as true in both theories) and disagreement (information that is presented as true in one theory, but false in the other).

The thesis of each theory will be heavily determined by the distinct information that appears only in that particular portion of the passage. Let's apply this approach to the theories from the passage given in the previous section:

Sample Passage

Sleep plays a vital role in defining the daily activities of virtually all animals. During periods of sleep, the parasympathetic nervous system becomes active and induces a relaxed state in response to increased levels of the hormone melatonin. Yet, despite its ubiquity in the animal kingdom, the purpose of sleep and its role in our daily lives has been disputed by scientists. Two scientists discuss their theories about the purpose of sleep.

Scientist 1

During periods of sleep, animals are able to conserve energy that they would otherwise be spending on unnecessary activity. If an animal's primary food source is most abundant during daylight, it is a waste of precious energy to be moving about at night. For example, many herbivores, such as squirrels, are diurnal (asleep during the night) because their food source is available during the day, while many insectivores, such as bats, are nocturnal (asleep during the day) because their food source is available during the night. Food sources, as an animal's most valuable resource, dictate their sleep cycles. Many animal traits observable today evolved as a result of the supply and demand of food in their natural habitat.

Scientist 2

During waking hours, it is true that the body utilizes large amounts of energy; however, the role of sleep is to restore biological products that were utilized during periods of wakefulness, rather than simply to avoid utilizing energy in the first place. Many types of biological molecules, such as hormones, are released throughout the body while an animal is active. Sleep serves as a period of inactivity during which the body can manufacture and store a supply of these molecules for future use during the next period of activity. Furthermore, sleep allows the body to repair cellular damage that has accumulated during waking hours. Experimental evidence shows that when animals are deprived of sleep, their immune systems quickly weaken and death rates increase. Sleep is necessary for animals to prevent accumulation of damage and to regenerate crucial biomolecules for daily life.



Sample Question

Which of the following best describes how the scientists view the role of sleep?

- A. Scientist 1: conserve energy; Scientist 2: resource availability
- B. Scientist 1: regenerate biomolecules; Scientist 2: restore the body
- C. Scientist 1: conserve energy; Scientist 2: restore the body
- D. Scientist 1: restore the body; Scientist 2: regenerate biomolecules

Scientist 1's theory immediately introduces a tight connection with food. The first two sentences outline the necessity of conserving energy and the expenditure of energy in the search for food. The next two sentences give examples. The final two sentences give a summary of the argument that the availability of food sources determines sleep patterns. The reason for this correlation goes back to the first sentence of the theory, giving us the full picture: animals sleep in order to conserve energy that would otherwise be wasted by searching for food when none is available. Based on this information, we can eliminate choices B and D.

Scientist 2's theory also begins by discussing energy expenditure, but immediately proposes that sleep is used for restoration rather than simple energy retention. The next two sentences describe the restoration of biological molecules and the repair of cellular damage. The sentence on experimental evidence provides further depth to the argument that sleep helps keep animals healthy. The final sentence of the theory provides the best summary: sleep is used to restore the body's resources and keep it healthy. Answer options B, C, or D correlate with this thesis; however, we have already eliminated B and D based on the thesis for theory 1. Thus, the correct answer must be choice C.

When a question addresses only one theory, it can become even easier to eliminate answer choices. By looking for distinct information in each theory, we can make logical assumptions about the questions and answer options presented. If a question about the passage explicitly addresses *Scientist 1*, but an answer option contains reference to biological molecules, the immune system, cellular damage, or restoration, we immediately know that that answer option is incorrect; none of these concepts appear in *Scientist 1's* theory. Similarly, if answer options about *Scientist 2's* theory address food sources or energy conservation, we can immediately narrow down potential options.



Identifying Agreement

When approaching a Conflicting Theories passage, it can be easy to become singly focused on the discrepancies between arguments—you are asked to identify the defining factors of each theory and to apply them to various analyses. This mindset can make identifying agreement a challenge.

Typically, agreement between conflicting theories can be found in two places: the background information presented at the beginning of the passage, or in the theories themselves. The given background, as previously discussed, is generally meant to be universally applicable. This is the one portion of the passage that is presented as fact, rather than argument. The given background, by nature of its universality, can be assumed to be a point of agreement between the two theories presented, as it provides the foundational elements upon which the theories are based.

Agreement can also be found in the theories themselves. To find points of agreement, look for terminology or keywords that appear in both theories. One strategy to accomplish this is to read one theory in-depth, and then skim the second theory for instances of matching terms. Finding a matching term, however, does not guarantee agreement! After you identify a term that appears in both passages, carefully compare what is being said in the sentences containing the term. Terms like “but,” “yet,” “however,” and “not” may indicate that the term is actually being used in a contrasting manner. Be careful to actually analyze the material presented, rather than jumping to conclusions once a matching term is identified.

Sample Passage

Sleep plays a vital role in defining the daily activities of virtually all animals. During periods of sleep, the parasympathetic nervous system becomes active and induces a relaxed state in response to increased levels of the hormone melatonin. Yet, despite its ubiquity in the animal kingdom, the purpose of sleep and its role in our daily lives has been disputed by scientists. Two scientists discuss their theories about the purpose of sleep.

Scientist 1

During periods of sleep, animals are able to conserve energy that they would otherwise be spending on unnecessary activity. If an animal's primary food source is most abundant during daylight, it is a waste of precious energy to be moving about at night. For example, many herbivores, such as squirrels, are diurnal (asleep during the night) because their food source is available during the day, while many insectivores, such as bats, are nocturnal (asleep during the day) because their food source is available during the night. Food sources, as an animal's most valuable resource, dictate their sleep cycles. Many animal traits observable today evolved as a result of the supply and demand of food in their natural habitat.

Scientist 2

During waking hours, it is true that the body utilizes large amounts of energy; however, the role of sleep is to restore biological products that were utilized during periods of wakefulness, rather than simply to avoid utilizing energy in the first place. Many types of biological molecules, such as hormones, are released throughout the body while an animal is active. Sleep serves as a period of inactivity during which the body can manufacture and store a supply of these molecules, for future use during the next period of activity. Furthermore, sleep allows the body to repair cellular damage that has accumulated during waking hours. Experimental evidence shows that when animals are deprived of sleep, their immune systems quickly weaken and death rates increase. Sleep is necessary for animals to prevent accumulation of damage and to regenerate crucial biomolecules for daily life.



Sample Question

The scientists agree on which of the following principles?

- A.** Animals accumulate biological damage while awake.
- B.** Animals use large amounts of energy while awake.
- C.** Animals spend the most time searching for food while awake.
- D.** Animals have evolved the need for sleep based on their diet.

A quick review of the given background for this passage makes it evident that none of the background appears in the answer options presented. To answer this question correctly, we must delve into the theories themselves.

Reading through *Scientist 1's* theory gives us background about energy expenditure and activity, sleep, wakefulness, food availability and food sources, and the eating habits of nocturnal and diurnal animals. Skimming over *Scientist 2's* theory for these same terms, we encounter “energy,” “sleep,” and “wakefulness” in the first sentence. The second and third sentences contain the terms “activity” and “inactivity,” respectively. These are all possible areas of agreement.

Closely reading the first three sentences of *Scientist 2's* theory tells us that animals “utilize large amounts of energy” during waking hours, that biological molecules are released when animals are active, and that animals are inactive during sleep. Comparing these statements to *Scientist 1's* theory (as well as the given answer options) shows a clear correct answer.

Scientist 1's theory never discusses biological molecules, and the release of biological molecules is not a viable answer option. The claim that animals are inactive during sleep appears to be a point of agreement, echoed in the first and second sentences of *Scientist 1's* theory and in the third sentence of *Scientist 2's* theory; however, none of the given answer choices present this option. Finally, the claim made in the first sentence of *Scientist 2's* theory (that animals “utilize large amounts of energy” during waking hours) matches with the claim in the first sentence of *Scientist 1's* theory: “During periods of sleep, animals are able to conserve energy that they would otherwise be spending on unnecessary activity.” Both scientists agree that animals expend large amounts of energy while awake. This corresponds to answer option B, giving us our final answer.



Comparing Arguments

Questions requiring argument comparison will generally have answer options that correspond to the two theories directly. Test-takers will be required to establish a comprehensive understanding of the theories presented (identifying the theses), and then to select the theory that best agrees with the presented statements. These questions follow a rather formulaic format, generally asking “which” theory/scientist agrees or disagrees with a given statement of scenario.

When approaching a Comparing Arguments question, it is important to already have a concrete understanding of the arguments presented. Ambiguity in initial comprehension can easily lead to confusion and frustration when addressing these questions. To review how to analyze the arguments presented, return to the Identifying Theses and Arguments section.

Sample Passage

Sleep plays a vital role in defining the daily activities of virtually all animals. During periods of sleep, the parasympathetic nervous system becomes active and induces a relaxed state in response to increased levels of the hormone melatonin. Yet, despite its ubiquity in the animal kingdom, the purpose of sleep and its role in our daily lives has been disputed by scientists. Two scientists discuss their theories about the purpose of sleep.

Scientist 1

During periods of sleep, animals are able to conserve energy that they would otherwise be spending on unnecessary activity. If an animal's primary food source is most abundant during daylight, it is a waste of precious energy to be moving about at night. For example, many herbivores, such as squirrels, are diurnal (asleep during the night) because their food source is available during the day, while many insectivores, such as bats, are nocturnal (asleep during the day) because their food source is available during the night. Food sources, as an animal's most valuable resource, dictate their sleep cycles. Many animal traits observable today evolved as a result of the supply and demand of food in their natural habitat.

Scientist 2

During waking hours, it is true that the body utilizes large amounts of energy; however, the role of sleep is to restore biological products that were utilized during periods of wakefulness, rather than simply to avoid utilizing energy in the first place. Many types of biological molecules, such as hormones, are released throughout the body while an animal is active. Sleep serves as a period of inactivity during which the body can manufacture and store a supply of these molecules, for future use during the next period of activity. Furthermore, sleep allows the body to repair cellular damage that has accumulated during waking hours. Experimental evidence shows that when animals are deprived of sleep, their immune systems quickly weaken and death rates increase. Sleep is necessary for animals to prevent accumulation of damage and to regenerate crucial biomolecules for daily life.



Sample Question

Which scientist would most likely agree with the statement, “Sleep evolved as a means of resting the body, allowing the animal to store energy for future activity?”

- A. Scientist 1 because molecular damage accrues during wakefulness
- B. Scientist 2 because sleep is a period for biomolecular regeneration
- C. Scientist 2 because animals search for food while awake
- D. Scientist 1 because animals conserve energy during periods of sleep

The answer choices presented in Comparing Arguments questions can be true to the theories presented, or they may intentionally misattribute information. Before even addressing the statement presented, we can eliminate certain answer options simply by comparing them to the known arguments of the given theories. We know that *Scientist 1* argues that sleep is a means of conserving energy, whereas *Scientist 2* argues that sleep is a period of biomolecular regeneration. Based on this knowledge, we can see that answer options B and D are in accordance with the presented theories. In contrast, option A pairs *Scientist 1* with an element of *Scientist 2*'s theory, and option C pairs *Scientist 2* with an element of *Scientist 1*'s theory; we can eliminate these two options, as they are inconsistent with the information presented.

Our next step is to determine whether answer option B or answer option D pairs better with the question stem. The given statement relates to rest and energy storage, as opposed to regeneration or damage. Based on these elements, we can determine that the statement is most in alignment with *Scientist 1*'s theory. Looking at answer option D, it becomes evident that the conservation of energy during sleep is in direct agreement with the statement regarding rest and energy storage. Thus, we can determine that D is the correct answer.



Applying Arguments

Questions requiring argument application generally relate only to one specific theory. The question stem will present a specific situation and ask test-takers to apply their knowledge of the presented theory, attributing a new conclusion to the theory in question. These questions are relatively abundant in the ACT Science section, as they require a great deal of critical reasoning. In order to correctly answer the question, you must recognize the established premise of the theory, integrate the information presented in the question, and draw a new conclusion that relates the two. The result will be a novel, implied element of the theory that was not directly discussed in the passage.

Our example will look at a proposed conclusion related to Scientist 2's theory.

Sample Passage

Sleep plays a vital role in defining the daily activities of virtually all animals. During periods of sleep, the parasympathetic nervous system becomes active and induces a relaxed state in response to increased levels of the hormone melatonin. Yet, despite its ubiquity in the animal kingdom, the purpose of sleep and its role in our daily lives has been disputed by scientists. Two scientists discuss their theories about the purpose of sleep.

Scientist 1

During periods of sleep, animals are able to conserve energy that they would otherwise be spending on unnecessary activity. If an animal's primary food source is most abundant during daylight, it is a waste of precious energy to be moving about at night. For example, many herbivores, such as squirrels, are diurnal (asleep during the night) because their food source is available during the day, while many insectivores, such as bats, are nocturnal (asleep during the day) because their food source is available during the night. Food sources, as an animal's most valuable resource, dictate their sleep cycles. Many animal traits observable today evolved as a result of the supply and demand of food in their natural habitat.

Scientist 2

During waking hours, it is true that the body utilizes large amounts of energy; however, the role of sleep is to restore biological products that were utilized during periods of wakefulness, rather than simply to avoid utilizing energy in the first place. Many types of biological molecules, such as hormones, are released throughout the body while an animal is active. Sleep serves as a period of inactivity during which the body can manufacture and store a supply of these molecules, for future use during the next period of activity. Furthermore, sleep allows the body to repair cellular damage that has accumulated during waking hours. Experimental evidence shows that when animals are deprived of sleep, their immune systems quickly weaken and death rates increase. Sleep is necessary for animals to prevent accumulation of damage and to regenerate crucial biomolecules for daily life.



Sample Question

Coordinate compound X is a damaging by-product of the brain that is produced during waking hours. If Scientist 2's theory is correct, which of the following is likely true?

- A. Coordinate compound X levels remain constant while an animal is sleeping.
- B. In nocturnal animals, coordinate compound X levels are high in the morning.
- C. If an animal has enough food, coordinate compound X levels will decrease.
- D. An animal that is more active will produce more coordinate compound X.

This question requires in-depth analysis and application of the basic principles presented in *Scientist 2's* argument. In order to answer correctly, we must establish the basic premise of the theory, and then apply it to the given information.

We know that *Scientist 2* argues that sleep is used to repair biomolecular damage and regenerate molecular stores. We know that *Scientist 2* does **not** address food or activity levels; these concepts are only discussed in *Scientist 1's* theory, making them irrelevant to this question. As such, we can eliminate answer options C and D.

The question tells us that coordinate compound X is a damaging molecule, and we know that *Scientist 2* argues for the elimination of similar damaging molecules during sleep. Based on these tenets, we can logically conclude that—according to *Scientist 2*—coordinate compound X will accumulate while the animal is awake and decrease while it is asleep, as it is eliminated during biomolecular repair processes. This conclusion fits well with answer option B: nocturnal animals are awake during the night, meaning that they will have the highest accumulation of damaging compounds (such as coordinate compound X) in the morning, soon before going to sleep. Answer option A is less logical; we would expect levels of coordinate compound X to decrease during periods of sleep, as opposed to remaining constant. The correct answer, in this case, would be option B.



Integrating New Information

To this point, we have discussed methods for establishing the key elements of conflicting arguments, comparing those elements, and applying those elements to determine a conclusion attributed to one of the theories in question. Now, we will take the final step in analyzing conflicting theories: integrating new information. These questions will frequently be formatted in such a way as to require assessment of the new information in the context of strengthening or weakening one of the given theories, but they may also ask for a given scientist's opinion or reaction to the presentation of new information. Integrating New Information will require you to incorporate multiple degrees of analysis and critical reasoning, and these questions can be some of the most difficult questions in the ACT Science section.

To approach these questions, start by reestablishing the core argument of each theory presented, paying particular attention to any theory specifically referenced in the question stem. Next, you will need to look at the new information and work to interpret it in the context of the presented theories. The new information may be presented in the question stem or given as answer options, and will often be completely novel and relatively disconnected from the theories at hand. Regardless of how the information is presented, it is critical to decode the intended meaning in context. Try analyzing the given question's answer choices in the context of the presented theory:

Sample Passage*Scientist 1*

During periods of sleep, animals are able to conserve energy that they would otherwise be spending on unnecessary activity. If an animal's primary food source is most abundant during daylight, it is a waste of precious energy to be moving about at night. For example, many herbivores, such as squirrels, are diurnal (asleep during the night) because their food source is available during the day, while many insectivores, such as bats, are nocturnal (asleep during the day) because their food source is available during the night. Food sources, as an animal's most valuable resource, dictate their sleep cycles. Many animal traits observable today evolved as a result of the supply and demand of food in their natural habitat.

Sample Question

Which of the following, if true, would most support Scientist 1?

- A.** Many rodents are nocturnal, but squirrels are diurnal
- B.** Bats that are given infinite food availability sleep more
- C.** Bears hibernate in the winter because their food source is scarce
- D.** Human ancestors evolved from being nocturnal to being diurnal

Answer option A - Though the passage notes that squirrels (a type of rodent) are diurnal, we do not have any information about other rodents. This answer option suggests that squirrels are an exception to the rodent population ("Many rodents are nocturnal, but squirrels are diurnal"); however, the meaning of this exceptionalism is lost due to the ambiguity of information about the general rodent population. Answer A offers no meaning in context and must be incorrect.

Answer option B - We know that *Scientist 1* argues that sleep is a means of energy conservation, and that animals are less active when their food is not readily available, as a means of conserving energy stores.



The essential summary is that more food availability leads to more activity (less sleep), while less food availability leads to less activity (more sleep). Answer option B states that more food availability will lead to more sleep. This is indirect conflict with *Scientist 1's* argument; if true, this statement would weaken *Scientist 1's* theory. Since we are looking for an answer option that strengthens *Scientist 1's* theory, option B must be incorrect.

Answer option C - We can summarize similarly to our approach in answer option B: The essential summary of *Scientist 1's* theory is that more food availability leads to more activity (less sleep), while less food availability leads to less activity (more sleep). Answer option C states that more sleep (hibernation) is caused by less food availability. This is in accordance with *Scientist 1's* theory, making it the strongest choice for the correct answer.

Answer option D - The passage does not directly mention humans at all. This answer option seems to suggest that human food sources were once more readily available at night than during the day, causing the change from nocturnal to diurnal habits. However, this does not directly relate to *Scientist 1's* claims and is not useful in strengthening (or weakening) the presented theory, making it incorrect.



Evaluating Conflicting Theories: Further Applications

Reading comprehension, critical reasoning, and prose analysis are essential skills when approaching literary works and English coursework, but may seem out of place in a science section. In reality, interpreting scientific findings and articles can be even more difficult than applying analytic skills to typical prose. It takes a very different mindset to dissect the material presented in a research paper than it does to interpret the symbolism in classical literature. Yet, both skills sets are essential to gaining a better appreciation for the intellectual environment in which many occupations function.

Anyone entering a field of science, medicine, technology, engineering, social science, or even English will eventually find the need to research technical articles. Filled with jargon and a complicated matrix of proposed theory and proven fact, these articles pose a puzzle unlike any contemporary literature. Developing the skill to sift through this complex information to find specific kernels of knowledge, and subsequently learning to apply this newly acquired knowledge to given problems or projects at hand, can take extensive study and experience. The Conflicting Theories passages on the ACT Science section give students a taste of technical analysis, testing their ability to approach dense scientific study with a critical reasoning mindset.



The ACT Mathematics Section

The Mathematics section of the ACT is 60 minutes long and contains 60 multiple-choice questions. Like all sections of the ACT, there is no penalty for guessing, so answer every question. All of the questions on this section of the exam may be solved without the use of a calculator; however, a permitted calculator is allowed on this section only. In general, any 4-function (addition, subtraction, multiplication, division), scientific, or graphing calculator will be permitted. Note that not all graphing calculators are permitted, and some must be modified before use. It is your responsibility as the test-taker to make sure your calculator is permitted, and in working condition. Scratch paper, if needed, will be provided.

At a Glance

Tests: mathematical problem-solving skills relevant to pre-algebra, algebra, geometry, and trigonometry

Timing: $\frac{60 \text{ Minutes}}{60 \text{ Questions}} = \frac{1 \text{ Minute}}{1 \text{ Question}}$

Section Breakdown:

- Some problems are presented as word problems, others in equation format
- Certain calculators are permitted
- Scratch paper is available for you to use

Preparation

Practice on the ACT Mathematics section is key. Take as many practice sections and/or full-length exams as possible before you take the ACT. Make the most of your practice—work in a quiet environment that mimics the testing center and time yourself. When reviewing the results of your practice work, try to understand why you missed certain questions, and then address those issues.

General Test-Taking Tips

Read each question carefully to identify exactly what is being asked. Solve the problem, make sure your answer is reasonable, and then find your answer among the answer choices. Be sure to bubble in the correct answer choice on the answer sheet. Each question is worth the same amount of points, so do not spend too much time on any one question. If you have extra time, check your work. Some other items to be aware of: unless otherwise stated, assume figures are *not* drawn to scale, geometric figures lie in a plane, “line” denotes a straight line, and “average” denotes the arithmetic mean.



Content

The ACT Mathematics section tests concepts taught up to the beginning of twelfth grade, and includes pre-algebra, elementary algebra, intermediate algebra, coordinate geometry, plane geometry, and trigonometry.

Pre-Algebra (20-25%)

Simple probability and statistics; ordering numbers in a series; data collection, representation, and interpretation; basic operations involving whole numbers, fractions, decimals, and integers; least common denominator; factors; single-variable linear equations; understanding place values; approximations; absolute value; ratios; percentages; square roots; exponents; scientific notation; unit analysis.

Elementary Algebra (15-20%)

Solving equations using algebraic functions and operations; factoring to solve quadratic functions; understanding exponential and square root operations; using substitution to solve algebraic expressions; solving functions; representing functions in terms of variables; understanding data represented in charts and figures.

Intermediate Algebra (15-20%)

Using the quadratic formula to solve equations and inequalities; solving systems of equations using substitution and linear combination; polynomials; recognizing and understanding sequences and patterns; equations, expressions, inequalities, and functions that contain exponential and square root functions; matrices; complex numbers; using and developing algebraic models.

Coordinate Geometry (15-20%)

Finding the slope, distance, and midpoint of graphically represented data; using and slope-intercept and point-slope form; understanding relationships between graphs and equations; parallel and perpendicular lines; functions of a line; graphing curves, inequalities, and conic sections.

Plane Geometry (20-25%)

Understanding properties and relations of plane figures including circles, triangles, rectangles, parallelograms, and trapezoids; surface area and volume; transformations; geometric proofs.

Trigonometry (5-10%)

Right triangle trigonometric values and relationships; properties of trigonometric functions; trigonometric identities; graphing trigonometric functions; solving trigonometric equations.



How to Use This Book

Each section in this chapter investigates one ACT Math topic in-depth and walks you step-by-step through relevant practice questions.

Each section concludes with a practice quiz so that you can test your skills before trying some of the full ACT diagnostic tests at the end of the book.

Chapter Outline

[Arithmetic](#)

[Number Theory and Basic Operations](#)

[Classifying Numbers](#)

[Order of Operations](#)

[Absolute Value](#)

[Sets and Sequences](#)

[Factors, Greatest Common Factor, and Least Common Multiple](#)

[Fractions and Decimals](#)

[Simplifying Fractions](#)

[Mixed Fractions](#)

[Operations with Fractions](#)

[Compound and Complex Fractions](#)

[Ratios and Proportions](#)

[Decimals](#)

[Combinations and Permutations](#)

[Combinations](#)

[Permutations](#)

[Squaring, Exponents, and Square Roots](#)

[Perfect Squares](#)

[Simplifying Square Roots](#)

[Equations with Square Roots](#)

[Squaring and Square Roots with Fractions](#)

[Squaring and Square Roots with Decimals](#)

[Data Analysis](#)

[Probability](#)

[Probability of Single Events](#)

[Probability of Multiple Events: “And” and “Or”](#)

[Dependent vs. Independent Events](#)

[Basic Statistics](#)

[Mean](#)

[Median](#)

[Mode](#)

[Range](#)

[Basic Statistics Review](#)

[Geometry](#)

[Plane Geometry](#)

[Squares](#)

[Area of a Square](#)

[Perimeter of a Square](#)

[Diagonal of a Square](#)

[Rectangles](#)

[Rectangular Area](#)

[Rectangular Diagonals](#)

[Triangles](#)

[Area of a Triangle](#)

[Equilateral Triangles](#)

[Isosceles Triangles](#)

[Circles](#)

[Bringing It All Together](#)

[Complex Shapes](#)



[Solid Geometry](#)[Cubes](#)[Cylinders](#)[Spheres](#)[Angles](#)[Triangular Angles](#)[Circular Angles](#)[Clock Math](#)[Algebra](#)[Rules of Exponents](#)[Variables and Algebraic Notation](#)[Monomials](#)[Adding and Subtracting Monomials](#)[Multiplying and Dividing Monomials](#)[Factoring Monomials](#)[Practice Problems: Monomials](#)[Binomials](#)[Adding and Subtracting Binomials](#)[Multiplying and Dividing Binomials](#)[Factoring Binomials](#)[FOIL](#)[Practice Problems: Binomials](#)[Polynomials](#)[Adding Polynomials](#)[Subtracting Polynomials](#)[Multiplying Polynomials](#)[Dividing Polynomials](#)[Factoring Polynomials](#)[Practice Problems: Polynomials](#)[Equations and Inequalities](#)[Linear Equations](#)[Linear Inequalities](#)[Absolute Value Equations](#)[Direct and Inverse Variation](#)[Rational Equations](#)[Radical Equations](#)[Quadratic Equations](#)[Function Notation](#)[Inequalities](#)[Rules of Inequalities](#)[Linear Inequalities](#)[Non-Linear Inequalities](#)[Solving Systems of Equations](#)[Solving Using Substitution](#)[Solving Using Elimination](#)[Coordinate Geometry](#)[X and Y Intercept](#)[Lines](#)[Distance Formula](#)[Midpoint Formula](#)[Parallel, Perpendicular, and Tangent Lines](#)

[Graphing](#)[Graphing Points](#)[Graphing Lines](#)[Graphing Quadratic Functions](#)[Graphing Exponential Functions](#)[Graphing Inequalities](#)[Trigonometry](#)[SOHCAHTOA](#)[Trigonometric Identities](#)[Reciprocal Identities \(Trigonometric Ratios\)](#)[Quotient Identities](#)[Pythagorean Identities](#)[Conversions Between Radians and Degrees](#)[Unit Circle](#)[Trigonometric Function Graphs](#)[Sine Graphs](#)[Cosine Graphs](#)[Tangent Graphs](#)[Amplitude, Period, Frequency, and Phase Shifts](#)[Solutions for Triangles](#)[Practice Problems: Trigonometry](#)

Arithmetic

Arithmetic involves the mechanics of how numbers relate to one another on the most fundamental level. While most arithmetic subjects are only tested indirectly on the ACT, a comprehensive understanding of these concepts is fundamental to applying more complex concepts in algebra and geometry. Our first segment of ACT Math review will focus on revisiting the foundational principles of mathematics and on reinforcing the basic knowledge upon which our other segments will build.

Section Outline

[Number Theory and Basic Operations](#)

[Classifying Numbers](#)

[Order of Operations](#)

[Absolute Value](#)

[Sets and Sequences](#)

[Factors, Greatest Common Factor, and Least Common Multiple](#)

[Fractions and Decimals](#)

[Simplifying Fractions](#)

[Mixed Fractions](#)

[Operations with Fractions](#)

[Compound and Complex Fractions](#)

[Ratios and Proportions](#)

[Decimals](#)

[Combinations and Permutations](#)

[Combinations](#)

[Permutations](#)

[Squaring, Exponents, and Square Roots](#)

[Perfect Squares](#)

[Simplifying Square Roots](#)

[Equations with Square Roots](#)

[Squaring and Square Roots with Fractions](#)

[Squaring and Square Roots with Decimals](#)



Number Theory and Basic Operations

Any math review must start with a few basic concepts and definitions. In this section, we will address the various classifications and applications of number theory, as well as a few simple operations. Number theory is used to help define numeric relationships, while operations are designed as a shorthand to apply relationships between numbers. Together, these two concepts form the core of arithmetic.

Classifying Numbers

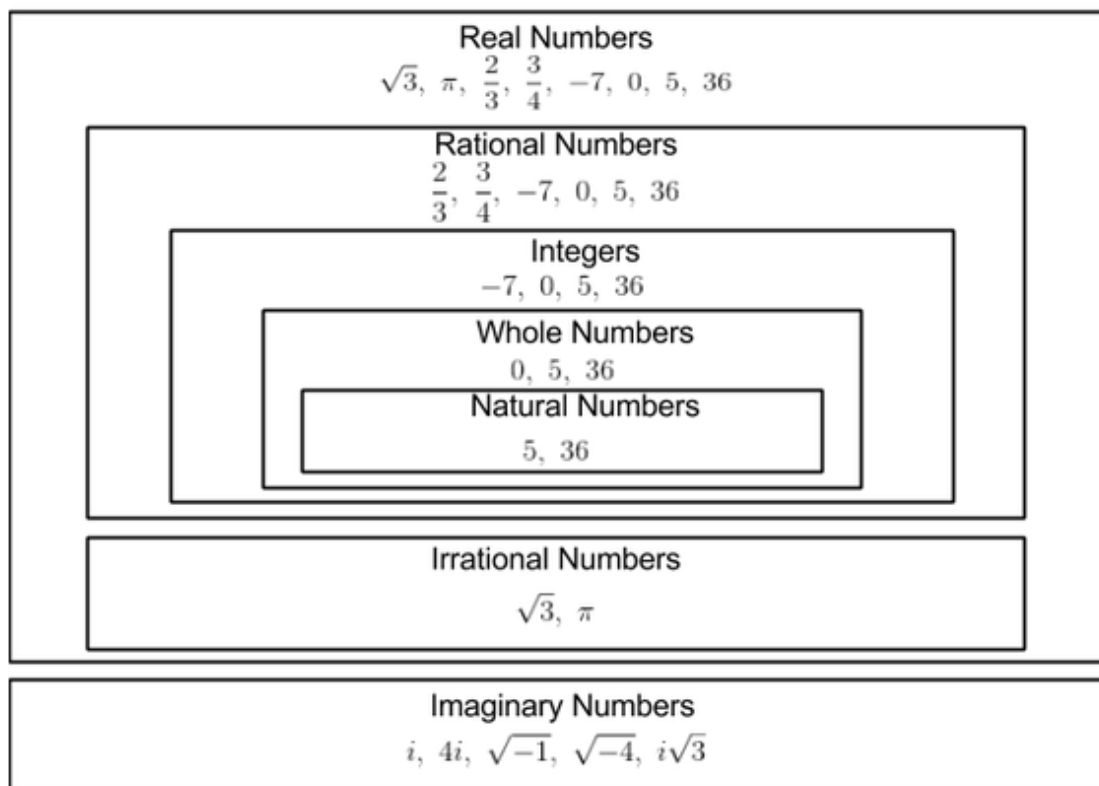
Numbers are fundamentally classified into a few different types. In arithmetic and algebra, it is important to pay attention to the type of number being addressed in the questions and equations you are dealing with. Certain numbers have specific properties that make them act differently mathematically, and they may require close attention in order to avoid miscalculations.

The following table provides a comprehensive breakdown of number classifications, starting with the most general class (real numbers) and ending with the most specific (natural numbers).

Classification	Description	Examples	
Real Numbers	Real numbers include all numbers that can be represented on a number line. This includes all rational and irrational numbers, but excludes all imaginary numbers.	Includes: $\sqrt{3}, \pi, \frac{2}{3}, \frac{3}{4}, -7, 0, 5, 36$	Excludes: $i, 4i, \sqrt{-1}, \sqrt{-4}, i\sqrt{3}$
Imaginary Numbers	Imaginary numbers involve the term “ i ,” which is indicative of the square root of -1 ($\sqrt{-1}$).	Includes: $i, 4i, \sqrt{-1}, \sqrt{-4}, i\sqrt{3}$	Excludes: $\sqrt{3}, \pi, \frac{2}{3}, \frac{3}{4}, -7, 0, 5, 36$
Rational Numbers	Rational numbers can be represented by a finite decimal. In other words, they can be represented as a ratio of two other numbers. This includes all integers, as well as all fractional numbers.	Includes: $\frac{3}{4}, -7, 0, 5, 36$	Excludes: $\sqrt{3}, \pi$
Irrational Numbers	Irrational numbers are numbers that cannot be expressed as a fraction of integers.	Includes: $\sqrt{3}, \pi$	Excludes: $\frac{3}{4}, -7, 0, 5, 36$
Integers	Integers are non-fractional numbers on the number line, including negative numbers, positive numbers, and zero. All whole numbers and all natural numbers are integers.	Includes: $-7, 0, 5, 36$	Excludes: $\sqrt{3}, \pi, \frac{2}{3}, \frac{3}{4}$
Whole Numbers	Whole numbers are non-fractional numbers on the number line, extending from zero upward. Whole numbers include all natural numbers, and exclude all negative numbers.	Includes: $0, 5, 36$	Excludes: $\sqrt{3}, \pi, \frac{2}{3}, \frac{3}{4}, -7$
Natural Numbers	Natural numbers include all non-fractional positive numbers and exclude zero.	Includes: $5, 36$	Excludes: $\sqrt{3}, \pi, \frac{2}{3}, \frac{3}{4}, -7, 0$



The figure below gives a visual representation of these classifications.



Order of Operations

Operations are the means of manipulating relationships between numbers. On the most fundamental level, there are four primary operations: addition, subtraction, multiplication, and division. Addition is used to find the sum; the sum is the total of two numbers, when one value adds to another. Subtraction is used to find the difference; the difference is the numerical distance between two numbers. Multiplication is used to find the product; the product is essentially the result of compound addition—adding a number to itself a predetermined number of times. Division is used to find the quotient; the quotient is the number of times a given number can be added to itself to result in an original term.

There are various additional operations, such as exponents and logarithms, that are introduced in higher level math, but they can always be reduced and explained in terms of addition, subtraction, multiplication, and division.

Order of operations refers to the rules governing the application of these relationships to a complex equation. Evaluating $5 \cdot 6$ involves only one step: multiplication. Things can get more complex when more operations become involved.

Order of operations dictates that when more than one operation is present in an expression or equation, they are evaluated in the following sequence:

Parentheses, Exponents, Multiplication/Division, Addition/Subtraction



This sequence gives the acronym PEMDAS, which is commonly taught alongside the mnemonic, “Please Excuse My Dear Aunt Sally.”

Parentheses and exponents are always evaluated first. Multiplication and division can be evaluated simultaneously, followed by simultaneous evaluation of addition and subtraction.

Example: $(5 \cdot 6) - 2^3 + 3 \cdot 4 - 1$

Parenthesis: $30 - 2^3 + 3 \cdot 4 - 1$

Exponent: $30 - 8 + 3 \cdot 4 - 1$

Multiplication/Division: $30 - 8 + 12 - 1$

Addition/Subtraction: 33

Key Tips

- Multiplication and division are inverse functions. This means that multiplying and dividing by the same number can cancel out. Example: $4 \cdot 6 \div 6 = 4$
- Addition and subtraction are inverse functions. This means that adding and subtracting the same number can cancel out. Example: $12 - 8 + 8 = 12$
- Fractions are just another way of writing division. For fractions, treat the numerator and denominator as fractions. Examples: $12 \div 4 = \frac{12}{4}$ and $\frac{12-6}{5-3} = \frac{6}{2} = 6 \div 2 = 3$
- When there are multiple sets of parentheses, always start from the innermost set and work outward. Example: $(5 \cdot 4(1 - 3)) + 4 = (5 \cdot 4(-2)) + 4 = (5 \cdot -8) + 4 = -40 + 4 = -36$

Try these practice problems:

1. $1 + 6 \div 3 \cdot 5 - 3^2$

2. $(8 + 4(5 - 2(3 \cdot 2))) + 5$

3. $\frac{1 - 4 \cdot 9}{2 + 3}$

Answers

1. $1 + 6 \div 3 \cdot 5 - 3^2$

Exponent: $1 + 6 \div 3 \cdot 5 - 9$

Multiply/Divide: $1 + 10 - 9$

Add/Subtract: 2

2. $(8 + 4(5 - 2(3 \cdot 2))) + 5$



Parenthesis 1: $(8 + 4(5 - 2(6))) + 5 = (8 + 4(5 - 12)) + 5$

Parenthesis 2: $(8 + 4(-7)) + 5 = (8 - 28) + 5$

Parenthesis 3: $-20 + 5$

Add/Subtract: -15

3. $\frac{1 - 4 \cdot 9}{2 + 3}$

Numerator/Denominator: $\frac{1 - 36}{2 + 3} = \frac{-35}{5}$

Divide: $\frac{-35}{5} = -35 \div 5 = -7$

Absolute Value

Absolute value is an application that measures the distance of a given value from zero. The most defining characteristic of absolute value is that it is a distance measure; as such, any absolute value must be positive. There is no such thing as a negative distance, and, therefore, no such thing as a negative absolute value.

When taking the absolute value of a positive number, the absolute value will be the same as the number itself. When taking the absolute value of a negative number, the absolute value will simply be the number without the negative indicator.

$$|5| = 5$$

$$|-5| = 5$$

In most cases, absolute value can be treated as a special form of parentheses that acts similarly in order of operations. To solve an absolute value question, evaluate all operations within the absolute value, and then determine its distance from zero.

Example: $|5 \cdot 4 - 9| = |20 - 9| = |11| = 11$

Practice:

1. $|-5| + |-8| \cdot |4|$

2. $8 \cdot |-4| + 4 \cdot |-8|$

3. $5 \cdot 2 - |6 - 4 \cdot 5|$



Answers

$$1. |-5| + |-8| \cdot |4|$$

$$5 + 8 \cdot 4$$

$$5 + 32$$

$$37$$

$$2. 8 \cdot |-4| + 4 \cdot |-8|$$

$$8 \cdot 4 + 4 \cdot 8$$

$$32 + 32$$

$$64$$

$$3. 5 \cdot 2 - |6 - 4 \cdot 5|$$

$$5 \cdot 2 - |6 - 20|$$

$$5 \cdot 2 - |-14|$$

$$5 \cdot 2 - 14$$

$$10 - 14$$

$$-4$$

Sets and Sequences

Sets and sequences are each a means of grouping specific numbers together. A set refers to a given list of any numbers. A sequence refers to a given list of numbers that follow a distinct, ordered pattern. Sets are formatted within closed brackets, while sequences are generally listed and end with an ellipsis.

Set of prime numbers between 1 and 10: {2, 3, 5, 7}

Sequence defined by an addition of 3: 1, 4, 7, 11, 14, ...

Note that the set of prime numbers between 1 and 10 can be written in any order, but the sequence must always remain ordered to preserve the pattern depicted. {2, 3, 5, 7} is the same as {3, 5, 2, 7}, but 1, 4, 7, 11, ... is not the same as 7, 4, 11, 1, ...

On the ACT, sets are commonly used for grouping problems or basic statistical analysis questions (such as mean, median, and mode), which will be discussed in-depth later in this book.

Sequences generally give rise to two different question types on the ACT: pattern identification and next-term identification. Pattern identification questions present a sequence and ask students to determine the pattern governing the sequence, while next-term identification questions present a sequence (with or



without the pattern given) and ask students to identify the next ordered term.

Sample Question

What is the next term in the sequence? 3, 18, 6, 36, 12, 72, 24, . . .

To start solving this problem, we will need to identify the pattern of the sequence before we can find the next term. Begin by identifying the change represented between each term, using addition, subtraction, multiplication, or division.

$$3(\cdot 6) = 18, 18(\div 3) = 6, 6(\cdot 6) = 36, 36(\div 3) = 12, 12(\cdot 6) = 72, 72(\div 3) = 24, 24(?), \dots$$

From this list of changes, we can see the pattern of the sequence: multiply a term by 6, and then divide the next term by 3. Since 72 is divided by 3, the next term, 24, must be multiplied by 6.

$$24 \cdot 6 = 144$$

The next term in the sequence is 144.

3, 18, 6, 36, 12, 72, 24, 144, . . .

Sequences generally do not end. As such, a variation of next-term questions is to ask for a distant term in the sequence. For example, what is the 12th term in the sequence from the previous example?

3, 18, 6, 36, 12, 72, 24, 144, 48, 288, 96, 576, . . .

The 12th term is 576.

To solve these questions, you can simply apply the pattern, but sometimes there are shortcuts! Let's look at a sequence that uses only addition. These sequences are known as arithmetic sequences, and are defined as sequences in which the difference between each subsequent term is equal.

Sample Question

12, 16, 20, 24, 28, . . .

What is the 22nd term in this sequence?

Rather than calculate over 20 terms, we can take a shortcut by looking at the pattern in terms of multiplication. Considering our starting value (12) as the variable n , we can rewrite the sequence: n , $n+4$, $n+8$, $n+12$, $n+16$, . . .

This can further be broken down by factoring out a 4 in each term: n , $n+4(1)$, $n+4(2)$, $n+4(3)$, $n+4(4)$, . . .



If the first term is n , the second term is $n+4(1)$, and the third term is $n+4(2)$, what will be the 22nd term? Reasonably, it follows that the 22nd term will be $n+4(21)$. Replace the n with its value (12), and evaluate: $12 + 4(21) = 12 + 84 = 96$.

Factors, Greatest Common Factor, and Least Common Multiple

Factors refer to the various combinations of integers that can be multiplied together to result in a particular term. For example, 6 and 2 are factors of 12 because $6 \cdot 2 = 12$.

Every integer will have the factors of 1 and itself, but certain terms can have numerous factors. Let's look at 100 as an example. Factors of 100 include: 1, 2, 4, 5, 10, 20, 25, 50, and 100.

$$\begin{aligned} 1 \cdot 100 &= 100 \\ 2 \cdot 50 &= 100 \\ 4 \cdot 25 &= 100 \\ 5 \cdot 20 &= 100 \\ 10 \cdot 10 &= 100 \end{aligned}$$

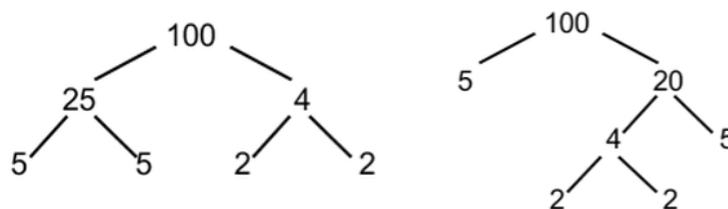
A term that is greater than one and has factors of only one and itself is known as a prime number. The prime numbers between 1 and 20 include 2, 3, 5, 7, 11, 13, 17, and 19. 2 is the only even prime number.

Every number can be reduced down to its prime factors, which, when multiplied together, result in the desired product.

For example, the prime factors of 100 are 2, 2, 5, and 5.

$$2 \cdot 2 \cdot 5 \cdot 5 = 100$$

To find the factors of a given term, it is usually best to build a factor tree. This will identify the prime factors of the number in question. A factor tree begins with any integer and visually deconstructs various factors of that number. A single term can have multiple variations of its factor tree (depending on what factors you choose for the highest level), but will always result in the same final prime factors.



The product of any combination of prime factors will also be a factor of the original number. For example, if the prime factors of 100 are 2, 2, 5, and 5, then we know that the product of any combination of these will also be a factor of 100. For example, 10 is the product of 2 and 5 and must, therefore, be a factor of 100.

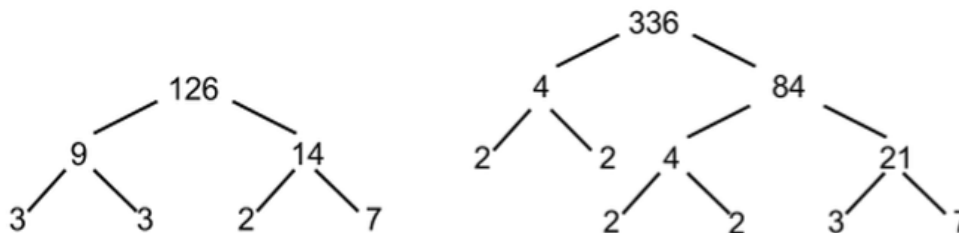
Comparing the factors of two different terms can provide useful information about the relationship between the given terms. Most importantly, we can find the greatest common factor. When dealing with two terms, the greatest common factor is the largest number that divides into both terms with no



remainder. To find the greatest common factor, you will need to start with two terms. Find the prime factors of each term individually, and then select all prime factors that the two terms have in common. Find the product of this combination of prime factors; this will be the greatest common factor. Note that sometimes the greatest common factor will be a prime factor if the terms in question have only one prime factor in common.

Sample Question

Find the greatest common factor of 126 and 336.



The prime factors of 126 are 2, 3, 3, and 7.

The prime factors of 336 are 2, 2, 2, 2, 3, and 7.

The common prime factors are 2, 3, and 7.

The greatest common factor of 126 and 336 is:

$$2 \cdot 3 \cdot 7 = 42$$

Once the greatest common factor has been identified, it can be factored out of the terms in question: $126 + 336 = 42(3 + 8)$. This is especially useful when simplifying fractions, as we will discuss later in this chapter.

The least common multiple is also particularly useful for manipulating fractions. When given two terms, their least common multiple is the smallest number that has both given terms as factors. For example, the least common multiple of 3 and 4 is 12. There are larger numbers that have both 3 and 4 as factors (such as 24); however, the least common multiple is the most applicable.

Finding the least common multiple can be a little bit complex; the least common multiple is equal to the product of the given terms divided by the greatest common factor.

Let's look at 126 and 336. Their greatest common factor is 42. The least common multiple will be:

$$(126 \cdot 336) \div 42$$

$$(42336) \div 42$$

$$1008$$

As we will see later, the greatest common factor is essential for simplifying fractions, while the least common multiple is essential for addition and subtraction of fractions.



Try these sample problems for extra practice:

1. What are the prime factors of 1638?
2. What is the greatest common factor of 378 and 288?
3. What is the least common multiple of 16 and 4?

Answers:

1. Prime factors of 1638 are 2, 3, 3, 7, and 13.
2. Prime factors of 378 are 2, 3, 3, 3, and 7. Prime factors of 288 are 2, 2, 2, 2, 2, 3, and 3. The greatest common factor is $2 \cdot 3 \cdot 3 = 18$.
3. Prime factors of 16 are 2, 2, 2, and 2. Prime factors of 4 are 2 and 2. The greatest common factor is $2 \cdot 2 = 4$. The least common multiple is $(16 \cdot 4) \div 4 = 16$.



Fractions and Decimals

Fractions and decimals are used to represent partial numbers. Fractions show partial numbers as a part of a whole, using division. Decimals show partial numbers in terms of exact distance between whole numbers. Percentages show partial numbers as a proportion of the partial distance in terms of 100.

Simplifying Fractions

Sometimes, fractions are made up of numbers that have factors in common. In the fraction below, both the numerator and the denominator are divisible by two. This means that the fraction is not in “lowest terms,” or, in other words, it can be reduced so that it represents the same thing using smaller numbers.

When using fractions in math or when coming up with an answer that is a fraction, it is common to hear “reduce to lowest terms” as the last thing in a question’s instructions. How do you reduce to lowest terms? You can start by analyzing the numerator and the denominator and identifying anything they have in common. Let’s reduce the following fraction:

$$\frac{9}{36}$$

The factors of 9 are 1, 3, and 9. The factors of 36 are 1, 3, 9, 4, 12, and 36. It looks like the numerator and the denominator have three factors in common: 1, 3, and 9. Let’s divide both the numerator and the denominator by their greatest common factor, 9:

$$\frac{9 \div 9 = 1}{36 \div 9 = 4} = \frac{1}{4}$$

This fraction is now in lowest terms. We can tell because the numerator is 1, and any fraction with 1 in the numerator can’t be reduced any more, since the only factor of 1 is 1.

Simplifying fractions is essential in algebra. Being able to factor out multipliers or factors can allow you to reduce fractions into their most basic components.

For example:

$$\frac{3x + 6}{5(x + 2)} = \frac{3(x + 2)}{5(x + 2)} = \frac{3}{5}$$

Mixed Fractions

Mixed fractions refer to whole numbers combined with a fractional component.

$$3\frac{1}{2}$$

Fundamentally, a mixed fraction is a form of addition.

$$3\frac{1}{2} = 3 + \frac{1}{2}$$



In most calculations, it is easiest to rewrite these terms as decimals or improper fractions (a fraction with a numerator that is greater than its denominator).

To convert to a decimal, simply divide the numerator of the fraction by the denominator and add to the given whole number.

$$3\frac{1}{2} = 3 + (1 \div 2) = 3 + 0.5 = 3.5$$

To convert to an improper fraction, you will need to convert the whole number to a fraction and add it to the fractional term in the mixed number. Converting the whole number to a fraction requires multiplying it by 1, using the format $\frac{x}{x}$, where x is equal to the denominator of the fractional term of the mixed number.

$$3\frac{1}{2} = 3\left(\frac{2}{2}\right) + \frac{1}{2} = \frac{3 \cdot 2}{2} + \frac{1}{2} = \frac{6+1}{2} = \frac{7}{2}$$

Once the mixed number is converted to a decimal or improper fraction, it is far easier to manipulate in calculations.

Operations with Fractions

As with whole numbers, fractions can be added, subtracted, multiplied, and divided, but certain manipulations must be applied before the operations can be completed. Fractions behave a little differently from whole numbers due to the presence of the denominator. Fractions with the same denominator represent equal divisions. $\frac{3}{11}$ is the same as $\frac{1}{11}$ three times.

When dealing with multiplication of fractions, simply apply the operation separately to the numerator terms and the denominator terms. Then, simplify the fraction (if possible).

Example:

$$\frac{2}{3} \cdot \frac{8}{9} = \frac{2 \cdot 8}{3 \cdot 9} = \frac{16}{27}$$

In this example, the fraction cannot be simplified, so the final answer is $\frac{16}{27}$.

When dividing fractions, the answer will be the same as multiplying the first term by the reciprocal of the second term. The reciprocal of a fraction switches the numerator and denominator terms. For example, the reciprocal of $\frac{7}{8}$ is $\frac{8}{7}$.

Example:

$$\frac{6}{7} \div \frac{3}{4} = \frac{6}{7} \cdot \frac{4}{3} = \frac{6 \cdot 4}{7 \cdot 3} = \frac{24}{21}$$

Simplify.

$$\frac{24}{21} = \frac{8}{7}$$

Try some practice problems that test multiplying and dividing fractions:



$$1. \frac{5}{6} \cdot \frac{-1}{12}$$

$$2. \frac{3}{4} \div \frac{5}{8}$$

Answers:

$$1. \frac{5}{6} \cdot \frac{-1}{12} = \frac{5 \cdot -1}{6 \cdot 12} = \frac{-5}{72}$$

$$2. \frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \cdot \frac{8}{5} = \frac{3 \cdot 8}{4 \cdot 5} = \frac{24}{20} = \frac{6}{5}$$

When adding or subtracting fractions, it is necessary to convert the given terms to have the same denominator. To find a common denominator for the given fractions, you will need to determine the least common multiple of the different denominators.

$$\text{Example: } \frac{3}{8} + \frac{5}{6}$$

To proceed, we will need to find the least common multiple of 8 and 6. In this case, the least common multiple is 24.

Next, multiply each fraction by a unit fraction (equal to 1) in order to convert to the common denominator.

$$\begin{aligned} & \frac{3}{8} + \frac{5}{6} \\ & \left(\frac{3}{8} \cdot \frac{3}{3} \right) + \left(\frac{5}{6} \cdot \frac{4}{4} \right) \\ & \left(\frac{3 \cdot 3}{8 \cdot 3} \right) + \left(\frac{5 \cdot 4}{6 \cdot 4} \right) \\ & \left(\frac{9}{24} \right) + \left(\frac{20}{24} \right) \end{aligned}$$

Once the fractions have the same denominator, they can be added. Only the numerator terms add together. The denominator remains constant.

$$\frac{9}{24} + \frac{20}{24} = \frac{9+20}{24} = \frac{29}{24}$$

Try some practice with adding and subtracting fractions:

$$1. \frac{2}{3} - \frac{5}{8}$$



$$2. \frac{5}{12} + \frac{1}{18}$$

Answers:

$$\begin{aligned} 1. \quad & \frac{2}{3} - \frac{5}{8} \\ & \frac{2}{3} \cdot \frac{8}{8} - \frac{5}{8} \cdot \frac{3}{3} \\ & \frac{16}{24} - \frac{15}{24} \\ & \frac{1}{24} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{5}{12} + \frac{1}{18} \\ & \frac{5}{12} \cdot \frac{3}{3} + \frac{1}{18} \cdot \frac{2}{2} \\ & \frac{15}{36} + \frac{2}{36} \\ & \frac{17}{36} \end{aligned}$$

Compound and Complex Fractions

Compound fractions, also called complex fractions, are fractions in which the numerator or the denominator itself contains a fraction.

Example:

$$\frac{\frac{1}{2}}{4}$$

For compound fractions, it is important to remember that a fraction is, fundamentally, an application of division.

$$\frac{\frac{1}{2}}{4} = \frac{1}{2} \div 4$$

Convert the denominator to a fraction.

$$\frac{1}{2} \div \frac{4}{1}$$



Divide the fractions by multiplying by the reciprocal.

$$\frac{1}{2} \div \frac{1}{4}$$

$$\frac{1}{2} \cdot \frac{4}{1}$$

Let's look at a more complex example.

$$\frac{7}{8} \div \frac{3}{4}$$

First, evaluate the numerator and the denominator.

$$\frac{7}{8} \div \frac{6}{8}$$

$$\frac{1}{8} \div \frac{1}{4}$$

Next, evaluate the compound fraction as a division problem.

$$\frac{1}{8} \div \frac{4}{18}$$

Simplify the result.

$$\frac{18}{32} = \frac{9}{16}$$

Ratios and Proportions

Ratios and proportions are often used as practical representations of fractional comparisons.

A ratio compares one group to another, after factoring out common factors to simplify the comparison.

Example:

The ratio of dogs to cats in a pet shop with 18 cats and 12 dogs is 2:3.



We can find this ratio by writing the comparison as a fraction, and then factoring out a 6.

$$\frac{12}{18} = \frac{2}{3} \rightarrow 2:3$$

Reversing the ratio means inverting the fraction. The ratio of cats to dogs would be:

$$\frac{18}{12} = \frac{3}{2} \rightarrow 3:2$$

Proportions are the ratio of one group to the total of all groups as a whole.

Example: In a pet shop with 18 cats and 12 dogs, what proportion of the animals are dogs?

To find this proportion, find the total number of animals in the pet shop, and then find the ratio of dogs to this total.

$$\begin{array}{r} 12 \\ 12 + 18 \\ \hline 30 \\ \hline 12 \\ 30 \\ \hline \frac{2}{5} \\ 2:5 \end{array}$$

The proportion of dogs in the pet shop is 2:5.

Decimals

A decimal measures the partial distance between two numbers as it would be read on a number line. Each decimal place represents an incremental division by ten.

$$\text{Tenths: } 0.1 = \frac{1}{10}$$

$$\text{Hundredths: } 0.01 = \frac{1}{100}$$

$$\text{Thousandths: } 0.001 = \frac{1}{1000}$$

In some cases, a decimal may be an irrational number. For example, certain decimals contain repeating patterns that are signified by a horizontal bar over the repeating pattern.

$$\text{Example: } 0.33\overline{3} = 0.333333333\ldots$$

Any fraction can be converted to a decimal simply by dividing the numerator by the denominator on a calculator. Some common fraction-to-decimal conversions are given below.

$$\frac{1}{8} = 0.125$$

$$\frac{1}{4} = 0.25$$

$$\frac{1}{2} = 0.5$$

$$\frac{3}{4} = 0.75$$



Combinations and Permutations

Combinations and permutations are ways of counting or calculating how to get a specific result given a certain number of objects to choose from. This construction is seen a lot in probability and statistics; however, basic forms of it are also seen in all levels of math. The key difference between a combination and a permutation is order. When dealing with a combination, the order does not matter; one can think of combinations as groups. On the other hand, when dealing with a permutation, order matters. Permutations are merely sequences. For example, say there are twenty children in a class, and the class is putting on a play. There is an opening for a director, a lighting coordinator, and a set designer. How many different combinations for director, lighting coordinator, and set designer can occur? This would be a combination question because the order does not matter; these three positions can be filled by any student, in any order. On the other hand, how many different ways the twenty students can stand in a line would be a permutation calculation. This is because the order matters. Students standing in line is a sequence, while filling the three positions is a grouping question. For example, say Sally, Joe, and Bob filled the three positions for the play. It doesn't matter who filled what specific position for the combination question. However, for the second question, if Sally, Joe, and Bob stand in a line, the different possibilities will be seen as a permutation:

Sally, Joe, Bob
Joe, Sally, Bob
Bob, Sally, Joe
Bob, Joe, Sally
Sally, Bob, Joe
Joe, Bob, Sally

Combinations

Basic combinations can be solved by multiplying out the possibilities. The formulas for combination and permutation deal with factorials. A factorial is when a number is multiplied by all integers that are less than itself. For example, $3!$ is $3 \cdot 2 \cdot 1$. The combination formula is as follows:

$${}_nC_k = \frac{n!}{k!(n-k)!}$$

where n =total number of options, k =the number of options desired

From the example above, say there are twenty children in a class, and the class is putting on a play. There is an opening for a director, a lighting coordinator, and a set designer. How many different combinations for director, lighting coordinator, and set designer can occur? To solve this combination, let's first write out the possibilities in English terms.

(Possibilities for Director)(Possibilities for Lighting Coordinator)(Possibilities for Set Designer)

To calculate the possibilities, recall that only one child can hold one position.

Therefore,

Possibilities for Director: 20

Possibilities for Lighting Coordinator: $20-1=19$



Possibilities for Set Designer: $19 - 1 = 18$

Thus the product becomes,

(Possibilities for Director)(Possibilities for Lighting Coordinator)(Possibilities for Set Designer)

$$(20)(19)(18) = 6840$$

Permutations

Permutations can be thought of as sequences. In other words, in how many specific orders can objects be arranged? The permutation formula is as follows:

$${}_nP_k = \frac{n!}{(n-k)!}$$

In this equation, n = total number of options, and k = the number of options desired.

For example, if Sally, Joe, and Bob stand in a line, how many different combinations are possible? Above, we see all the possibilities written out. Now let's use the permutation formula to solve for this question.

$${}_3P_3 = \frac{3!}{(3-3)!}$$
$${}_3P_3 = \frac{3 \cdot 2 \cdot 1}{0!}$$

Recall that zero factorial is equal to one.

$${}_3P_3 = 3 \cdot 2 \cdot 1 = 6$$



Squaring, Exponents, and Square Roots

When a given term is raised to the second power, it is said to be squared. In other words, when a number is squared, it has an exponent of 2 and is multiplied by itself. Three squared is equal to 3^2 , which is equal to $3 \cdot 3$.

Taking the square root of a term is the inverse function of squaring. In other words, the solution to a square root answers the question, “What number can be squared to get the starting value in this problem?”

Perfect Squares

A perfect square is a term that is equal to a single number squared. A short list of perfect squares is given below:

$$\begin{array}{ll} 1^2 = 1 & 6^2 = 36 \\ 2^2 = 4 & 7^2 = 49 \\ 3^2 = 9 & 8^2 = 64 \\ 4^2 = 16 & 9^2 = 81 \\ 5^2 = 25 & 10^2 = 100 \end{array}$$

A perfect square will always have a whole number square root. Since the square root operation is the inverse function of squaring, we can create a second list of equations showing the roots of each perfect square:

$$\begin{array}{ll} \sqrt{1} = 1 & \sqrt{36} = 6 \\ \sqrt{4} = 2 & \sqrt{49} = 7 \\ \sqrt{9} = 3 & \sqrt{64} = 8 \\ \sqrt{16} = 4 & \sqrt{81} = 9 \\ \sqrt{25} = 5 & \sqrt{100} = 10 \end{array}$$

Looking at a basic example, we can see how these operations function.

Sample Question

Solve: $5^2 - \sqrt{49}$

To find the final answer, simply evaluate each term. We can expand 5^2 to become $5 \cdot 5$, which is equal to 25.

$$5^2 - \sqrt{49} = 25 - \sqrt{49}$$

The square root of 49 is 7 because $7 \cdot 7 = 49$. 49 is a perfect square.

$$5^2 - \sqrt{49} = 25 - \sqrt{49} = 25 - 7 = 18$$

The final answer will be 18.



Simplifying Square Roots

The square root function is simplest when applied to perfect squares, but what about other numbers? On the ACT, you may be asked to simplify an expression that contains the square root of a number that is not a perfect square, but you will not be asked to solve for the value of this term. Most often, solving for these values results in an irregular decimal or other such inconvenient term. To avoid these scenarios, the exam will focus on simplification of terms as opposed to direct solutions.

Simplifying a square root will ultimately lead to factoring out terms from under the square root and extracting factors until the term cannot be further simplified. Remember that taking the square root is the inverse function of squaring a number. If we can identify instances where there is a square underneath a square root, we can factor out that term. For example:

$$\sqrt{4^2} = 4$$

The easiest method for simplifying a square root is to identify all of the prime factors of the term under the square root operation, and then identify duplicate factors. These duplicates indicate squared terms. Prime factors can be found by building a factor tree or using any other method to identify factors. Let's look at $\sqrt{75}$ as an example. The prime factors of 75 are 3, 5, and 5 because $3 \cdot 5 \cdot 5 = 75$. There are two ways to represent this in a factor tree.



Once we have identified the factors, look for any duplicates. In the case of 75, we can see that 5 appears twice. In other words, $5 \cdot 5 \cdot 3 = 5^2 \cdot 3 = 75$. Since taking the square root is the inverse of squaring a number, we can factor out the 5 from under the square root.

$$\sqrt{75} = \sqrt{5^2 \cdot 3} = 5\sqrt{3}$$

Note that only the 5 can be factored out of the square root. Since the 3 is not a duplicate in the factor tree, it cannot be simplified. $5\sqrt{3}$ is the most simplified answer possible.

Try a few more examples of simplifying square roots:

1. $\sqrt{99}$
2. $\sqrt{125}$
3. $\sqrt{32}$
4. $\sqrt{54}$



Answers:

1. Factors of 99 are 3, 3, and 11. The 3 can be factored out, but the 11 has no duplicate and must remain under the square root. Final answer: $\sqrt{99} = 3\sqrt{11}$.
2. Factors of 125 are 5, 5, and 5. There is one pair of 5's that can be factored out, but one unpaired 5 that must remain under the square root. Final answer: $\sqrt{125} = 5\sqrt{5}$.
3. Factors of 32 are 2, 2, 2, 2, and 2. There are two pairs of 2's that can be factored out, but one unpaired 2 that will remain under the square root. Final answer: $\sqrt{32} = 2 \cdot 2\sqrt{2} = 4\sqrt{2}$.
4. Factors of 54 are 3, 3, 3, and 2. A pair of 3's can be factored out, but the 2 and final 3 have no pairs and must remain under the square root. Final answer: $\sqrt{54} = 3\sqrt{3} \cdot 2 = 3\sqrt{6}$.

Equations with Square Roots

Now that we have looked at simplifying individual square root terms, we can start evaluating how these terms interact within equations and expressions. When a square root appears in an expression, the first step will always be to simplify the square root via the process we discussed previously. Once the square root cannot be simplified further, we can start applying other operations, such as addition, subtraction, multiplication, and division. Within expressions, the simplified square roots can be treated just like variables.

Let's start with a simple example of addition with square roots:

$$\sqrt{2} + \sqrt{2}$$

Treating the term " $\sqrt{2}$ " like a variable, we can imagine that this expression is the same as $x + x$, which simplifies to $2x$. In the same way, $\sqrt{2} + \sqrt{2}$ simplifies to $2\sqrt{2}$.

$$\sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

The same process is used for simplification of addition and subtraction. As long as the same number is under the square root, the terms can be combined. Let's look at a full problem:

$$\sqrt{9} - \sqrt{12} + \sqrt{75}$$

Simplify the square roots.

$$3 - 2\sqrt{3} + 5\sqrt{3}$$

Combine terms that contain $\sqrt{3}$.

$$3 + 3\sqrt{3}$$

This problem cannot be simplified any further because the leading term is not associated with a square root. Remember, you can only combine terms with the same value under the square root; in this problem, we can only combine the terms that contain $\sqrt{3}$.



When there are different values under the square roots, the terms cannot be simplified. For example, $2\sqrt{3} + \sqrt{5}$ cannot be simplified any further.

Example:

$$\sqrt{40} + \sqrt{18}$$

$$2\sqrt{10} + 3\sqrt{2}$$

Because the base terms cannot be combined, the expression cannot be simplified.

When dealing with multiplication and division of square roots, there is a bit more flexibility. When multiplying or dividing square roots, the terms under the square root are combined.

$$\sqrt{2} \cdot \sqrt{3} = \sqrt{2 \cdot 3} = \sqrt{6}$$

$$\sqrt{14} \div \sqrt{7} = \sqrt{14 \div 7} = \sqrt{2}$$

Since the terms under the square root can be combined, multiplication and division can lead to additional simplification of the final answer. Many times, you will be able to factor out a term after multiplying two square roots.

Example:

$$\sqrt{40} \cdot \sqrt{15}$$

Simplify the square roots.

$$2\sqrt{10} \cdot \sqrt{15}$$

Multiply.

$$2\sqrt{10 \cdot 15}$$

$$2\sqrt{150}$$

Simplify the final square root.

$$2 \cdot 5\sqrt{6}$$

$$10\sqrt{6}$$

In this example, combining $\sqrt{10}$ and $\sqrt{15}$ allowed us to factor out a 5 before arriving at our final answer.

Let's look at a few final practice problems:

$$1. \sqrt{90} \cdot \sqrt{45} - \sqrt{128}$$

$$2. \sqrt{216} \div \sqrt{24} - \sqrt{18} \cdot \sqrt{32}$$

$$3. \sqrt{12} - \sqrt{48} \cdot \sqrt{18}$$



Answers

1. Start by simplifying each term.

$$\sqrt{90} = 3\sqrt{10}$$

$$\sqrt{45} = 3\sqrt{5}$$

$$\sqrt{128} = 8\sqrt{2}$$

$$\sqrt{90} \cdot \sqrt{45} - \sqrt{128} = 3\sqrt{10} \cdot 3\sqrt{5} - 8\sqrt{2}$$

Multiply to combine the first two terms, and then simplify the result.

$$3 \cdot 3\sqrt{10 \cdot 5} - 8\sqrt{2}$$

$$9\sqrt{50} - 8\sqrt{2}$$

$$9 \cdot 5\sqrt{2} - 8\sqrt{2}$$

$$45\sqrt{2} - 8\sqrt{2}$$

Since the two terms have the same value under the square root, they can be combined.

The final answer is $37\sqrt{2}$.

2. Start by simplifying each term.

$$\sqrt{216} = 6\sqrt{6}$$

$$\sqrt{24} = 2\sqrt{6}$$

$$\sqrt{18} = 3\sqrt{2}$$

$$\sqrt{32} = 4\sqrt{2}$$

$$\sqrt{216} \div \sqrt{24} - \sqrt{18} \cdot \sqrt{32} = 6\sqrt{6} \div 2\sqrt{6} - 3\sqrt{2} \cdot 4\sqrt{2}$$

Divide the first two terms and multiply the last two terms, according to order of operations, and simplify the result.

$$6 \div 2\sqrt{6 \div 6} - 3 \cdot 4\sqrt{2 \cdot 2}$$

$$3\sqrt{1} - 12\sqrt{4}$$

Simplify the new square roots. Since both terms are perfect squares, we can factor out whole numbers.

$$3 \cdot 1 - 12 \cdot 2$$

$$3 - 24$$

$$-21$$



3. Start by simplifying each term.

$$\begin{aligned}\sqrt{12} &= 2\sqrt{3} \\ \sqrt{48} &= 4\sqrt{3} \\ \sqrt{18} &= 3\sqrt{2}\end{aligned}$$

$$\sqrt{12} - \sqrt{48} \cdot \sqrt{18} = 2\sqrt{3} - 4\sqrt{3} \cdot 3\sqrt{2}$$

Multiply the final two terms according to order of operations, and simplify the result.

$$\begin{aligned}2\sqrt{3} - 4 \cdot 3\sqrt{3} \cdot 2 \\ 2\sqrt{3} - 12\sqrt{6}\end{aligned}$$

Since the terms under the square roots are not the same, this expression cannot be simplified any further.

Squaring and Square Roots with Fractions

The critical element to recognize when dealing with squares, square roots, and fractions is that the exponent and square root both distribute to the terms in the fraction. In other words, the square root or the exponent can be applied separately to both the numerator and denominator.

For example:

$$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \frac{9}{25}$$

and

$$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

Squares and square roots can also cancel in fractions to produce a more reduced final answer. Treat the term as if it were a variable that can be factored out of the fraction.

For example:

$$\frac{5 \cdot 2^2}{2^2} = 5$$

$$\frac{\sqrt{21}}{\sqrt{63}} = \frac{\sqrt{3} \cdot \sqrt{7}}{3\sqrt{7}} = \frac{\sqrt{3}}{3}$$



In the first example, we are able to cancel out the 2^2 term. In the second example, we can cancel out the $\sqrt{7}$ term.

Try a more complex example:

$$\frac{(\sqrt{12} + \sqrt{27})(\sqrt{45} + \sqrt{20})^2}{\sqrt{14} \cdot \sqrt{28} \cdot \sqrt{75}}$$

Answer:

$$\frac{(\sqrt{12} + \sqrt{27})(\sqrt{45} + \sqrt{20})^2}{\sqrt{14} \cdot \sqrt{28} \cdot \sqrt{75}}$$

$$\frac{(2\sqrt{3} + 3\sqrt{3})(3\sqrt{5} + 2\sqrt{5})^2}{\sqrt{14} \cdot 2\sqrt{7} \cdot 5\sqrt{3}}$$

$$\frac{(5\sqrt{3})(5\sqrt{5})^2}{\sqrt{14} \cdot 2\sqrt{7} \cdot 5\sqrt{3}}$$

$$\frac{(5\sqrt{3})(5)^2(\sqrt{5})^2}{\sqrt{14} \cdot 2\sqrt{7} \cdot 5\sqrt{3}}$$

$$\frac{(5\sqrt{3})(25)(5)}{2\sqrt{98} \cdot 5\sqrt{3}}$$

$$\frac{(5\sqrt{3})(125)}{2\sqrt{98} \cdot 5\sqrt{3}}$$

$$\frac{125}{2\sqrt{98}}$$

$$\frac{125}{2 \cdot 7\sqrt{2}}$$

$$\frac{125}{14\sqrt{2}}$$



Squaring and Square Roots with Decimals

Generally speaking, there are very few differences between decimals and whole numbers when applying square roots and exponents. While the mechanics of the operations do not change, decimals can make things a bit more complex, and they also introduce a new set of perfect squares that may not be entirely intuitive. Just remember, if you ever feel stuck on a question with squares, square roots, and decimals, you are still only dealing with a single term multiplied by itself. Even if the numbers appear complex, you are still dealing with a relatively simple concept.

Let's look at the perfect squares of some decimal quantities. The values are given as both decimals and fractions to help show the values in multiple formats.

Perfect Squares with Decimals

$$0.5^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$$

$$0.25^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16} = 0.0625$$

$$0.2^2 = \left(\frac{1}{5}\right)^2 = \frac{1}{25} = 0.04$$

$$0.1^2 = \left(\frac{1}{10}\right)^2 = \frac{1}{100} = 0.01$$

Remember that, when multiplying one decimal by another, the answer will generally be equal to the product of the terms after the decimal, but with the decimal point shifted to the left.

Example: $0.3 \cdot 0.3 = 0.09$

This is especially important when dealing with squaring or taking the square root of decimals. Squaring the decimal will shift the decimal point to the left:

$$0.2^2 = 0.2 \cdot 0.2 = 0.04$$

$$0.6^2 = 0.6 \cdot 0.6 = 0.36$$

$$0.14^2 = 0.14 \cdot 0.14 = 0.0196$$

Taking the square root will shift the decimal point to the right:

$$\sqrt{0.0121} = 0.11$$

$$\sqrt{0.0001} = 0.01$$

$$\sqrt{0.0441} = 0.21$$

Try a simple example:

$$\sqrt{0.0625} + (\sqrt{0.25} - 0.02^2)$$



Answer:

Evaluate each term.

$$\sqrt{0.0625} = 0.25$$

$$\sqrt{0.25} = 0.50$$

$$0.02^2 = 0.4$$

$$\sqrt{0.0625} + (\sqrt{0.25} - 0.02^2)$$

$$0.25 + (0.5 - 0.4)$$

$$0.25 + 0.1$$

$$0.35$$



Data Analysis

Data analysis is the process of deriving meaning from a set of informative data. A sample may provide a wide range of collected data, but simply looking at the information is not always indicative of the trends present in the results. It is important to be able to mathematically evaluate the data to accurately establish the relevance of the information.

Section Outline

[Probability](#)

[Probability of Single Events](#)

[Probability of Multiple Events: “And” and “Or”](#)

[Dependent vs. Independent Events](#)

[Basic Statistics](#)

[Mean](#)

[Median](#)

[Mode](#)

[Range](#)

[Basic Statistics Review](#)



Probability**Probability of Single Events**

You can mathematically model the chances of a specified event happening using fractions or decimals with values between 0 and 1. 0 and 1 form the boundaries of the probability system: a probability of 1 means that the event is certain to happen, while a probability of 0 means that the event is certain not to happen.

Consider a fair coin. There are two possible outcomes: heads or tails. If we want to model the probability (the “chances”) of a fair coin landing on heads, we could do so by placing the number of outcomes we want to occur over the number of total outcomes. In this case, there are two total outcomes possible (heads or tails) and we want to calculate the odds of one of them occurring (heads), so we could model the situation using either the fraction $\frac{1}{2}$ or the decimal 0.5.

Let’s try a slightly more complex example:

Sample Question

You are eating lunch at a restaurant and want to order a sandwich. There are twenty-five sandwiches on the menu, and six of them are toasted. If you order a sandwich at random, what is the probability of you ordering a sandwich that is *not* toasted?

Let’s take stock of the situation described. To calculate probability, we need to know the total number of possible outcomes. In this case, that’s the total number of sandwiches the restaurant serves: 25. This will form the denominator of our fraction. But what do we put in the numerator? The only other number mentioned in the question is six, but before jumping to conclusions, be careful to look at the designation. The menu lists six sandwiches that *are* toasted; we want to calculate the probability of a sandwich that is *not* toasted. We can do this in one of two ways:

- 1.) Take the total number of sandwiches (25) and subtract the options that are toasted. Then, make the resulting number the numerator in a fraction with 25 in the denominator.
$$25 - 6 = 19$$
$$\frac{19}{25}$$
- 2.) Since we only have two options (toasted or not), we can subtract the probability of ordering a toasted sandwich from 1 to find the probability of ordering a sandwich that is not served toasted. Six out of the twenty-five possible sandwiches are served toasted, so the probability of ordering one of these is $\frac{6}{25}$.
$$1 - \frac{6}{25} = \frac{19}{25}$$

No matter which method you use, you get the same answer: if you order a sandwich at random, you have an $\frac{19}{25}$ chance of ordering a sandwich that is not served toasted.



If you need to apply a probability statistic to a larger group, you can do so using multiplication, as in the next example.

Sample Question

You are told that two-thirds of the incoming freshmen at your school are interested in helping out with the school play. There are one hundred and twenty-five freshmen in the incoming class. How many freshmen are interested in helping out with the school play?

We need to find out what $\frac{2}{3}$ of 125 is, and we can do so using multiplication:

$$\frac{2}{3} \cdot 125 = 83.\bar{3}$$

In this case, our calculation resulted in a repeating decimal. Since we are talking about people, we need to round to the nearest whole number: 83. 83 of the 125 incoming freshmen are interested in helping out with the school play.

A Note on Playing Cards

Many probability questions presume you have basic knowledge about what constitutes a “standard” deck of playing cards. Such a deck (without the jokers) contains 52 cards. There are 13 cards in each suit: hearts, diamonds, spades, and clubs. Hearts and diamonds are traditionally printed in red; spades and clubs are traditionally printed in black. In each suit, there are three face cards: jacks, queens, and kings. (Aces count as 1’s.)

Let’s try some sample problems:

1. There are seven flavors of ice cream available in an ice cream shop. Three of them contain nuts. What is the probability that if you order an ice cream cone at random, you’ll be able to share it with your friend, who is allergic to nuts?
2. If you pick a card from a standard deck of playing cards (without jokers) at random, what is the probability of drawing a queen?
3. If you pick a card from a standard deck of playing cards (without jokers) at random, what is the probability of drawing a red face card?
4. There are two dozen coffee shops within a mile of your school. $\frac{3}{4}$ of them also serve sandwiches.
How many of the coffee shops within a mile of your school serve sandwiches?
5. You have a set of fifty colored pencils. $\frac{1}{10}$ of them have broken leads and need to be sharpened, and $\frac{3}{10}$ of them aren’t sharpened. If you draw a pencil out of your colored pencil set at random, what is the probability that it will be sharpened?



Answers:

1. The denominator in our probability fraction will be the total number of options: 7. If your friend is allergic to nuts, we want to calculate the probability of picking out a flavor at random that does *not* contain nuts. We are told that three of the seven ice cream flavors contain nuts; that means that four do not. You could share four of the seven options with your friend, so four becomes the numerator in our probability fraction. The answer is $\frac{4}{7}$.
2. How many queens are in a standard deck of cards without jokers? Well, there are four suits, and each suit has one queen, so there are four queens. 52 will be the denominator of our probability fraction since there are 52 possible cards you could draw. We're looking for the probability of drawing a queen, so 4 becomes the numerator, and the answer is $\frac{4}{52}$.
3. How many red face cards are in a deck of cards? Be careful, this question is a little tricky. A "face card" refers to a jack, a queen, or a king. There are four suits, each with a jack, queen, and king, but only two of them are red (hearts and diamonds); the other two are black (spades and clubs). This means that there are six red face cards in the deck. 52 becomes the denominator of our probability fraction since there are 52 total cards from which to pick, and 6 becomes the numerator, making the answer $\frac{6}{52}$.

4. We are picking from 24 ("two dozen") coffee shops, and we're told that $\frac{3}{4}$ serve sandwiches.

Multiplying the probability by the number of items tells us how many items match up with the probability's event—in this case, serving sandwiches:

$$24 \cdot \frac{3}{4} = 18$$

18 of the coffee shops within a mile of your school also serve sandwiches.

- 5.) In this problem, we're dealing with three possible outcomes:

- 1.) You draw a colored pencil that is sharpened.
- 2.) You draw a colored pencil that has a broken lead and needs to be sharpened.
- 3.) You draw a colored pencil that is unsharpened.

We are given fractions denoting how many pencils fall into the second and third categories. By multiplying these fractions with the total number of colored pencils, we can figure out how many have broken leads and how many are unsharpened.

$$\text{Broken leads: } 50 \cdot \frac{1}{10} = 5$$

$$\text{Unsharpened: } 50 \cdot \frac{3}{10} = 15$$

By subtracting the resulting numbers of colored pencils with broken leads and unsharpened colored pencils from the total, we can figure out how many colored pencils are sharpened:

$$50 - 5 - 15 = 30$$

Finally, we can calculate our probability. Of a total of 50 colored pencils, 30 are sharpened, so 50 goes



in the denominator and 30 goes in the numerator, which (after reducing) results in a $\frac{3}{5}$ probability of drawing a sharpened and not broken colored pencil at random.

Probability of Multiple Events: “And” and “Or”

When considering multiple events, you have two things to keep track of:

- 1.) Whether the problem is asking about one event “and” another or one event “or” another
- 2.) Whether the events are independent or dependent

If a problem asks about the probability of one event happening **OR** the other event happening, calculate the probability of each event happening and then add them together. The two specified results are both valid, so you need to find one, more likely probability to represent either of them being the result.

If a problem asks about the probability of one event happening **AND** another event happening, calculate the probability of each event happening and then multiply them together. Multiplying a number by a fraction or decimal between 0 and 1 makes that number smaller. A smaller probability indicates a less likely result. If you want to calculate the probability of two events happening together, that is going to be less likely than either of them happening alone.

Dependent vs. Independent Events

Calculating probability requires you to determine whether the events that you are modeling are “dependent” on one another or “independent” of one another. Dependent events are events in which one event is necessary for the other event to happen, or in which you are considering the probability of both events as a pair. Independent events happen independently; the probability of one happening has no effect on the probability of another happening.

Let’s look at an independent event problem now.

Sample Question

You have a fair coin and a fair six-sided die. What is the probability of both getting tails on a coin flip and rolling a 4 on the die?

Whether you get heads or tails on the coin flip has nothing to do with which number you roll on the die: these events are independent. This means that we’re going to multiply their discrete probabilities together to figure out the probability of them both happening.

$$P_{\text{Tails}} = \frac{1}{2}$$

$$P_{\text{Rolling a 4}} = \frac{1}{6}$$



$$P_{\text{Both}} = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

The probability of both getting tails on a coin flip and rolling a 4 on the die is $\frac{1}{12}$.

In contrast to independent events, dependent events affect one another, often making the order of actions significant. A good example of dependent events would be drawing cards out of a standard deck and not replacing them. Order becomes important, because if you draw a card out first and then ask about the probability of drawing out another card, the deck has one less card in it.

Watch out for questions about drawing multiple cards! Unlike questions about flipping multiple coins or rolling multiple dice (independent events), cards that are not replaced affect how large the deck is from which you're drawing the next card. Similar problems might also involve drawing items out of a bag and not replacing them. If you replace them, the denominator in your fraction will stay the same, and you won't have to adjust the starting ratio of items. But if you pull cards out of a deck or items from a bag and don't put them back, you have to reconsider the probability of drawing the next item, and the denominator of your fraction will change. For instance, if you draw a card from a standard deck of cards without jokers, the probability of drawing that particular card is $\frac{1}{52}$. But if you don't put it back, you only have 51 cards in the deck for the next time you draw a card!

Some questions may specify a specific order of events, while others do not. Let's consider a two-part sample question that demonstrates the importance of realizing whether order is specified or not.

Sample Question

1. What is the probability of drawing one face card and one four from a standard deck of cards (without jokers)?
2. What is the probability of drawing one four and then one face card from a standard deck of cards (without jokers)?

Question 1 does not imply a specific order: you could either draw a face card and then a four, or a four and then a face card. Question 2 specifies a specific order: first you draw a four, and then you draw a face card.

Question 1 gets a bit complex; since either order would fulfill the criteria, we have to calculate the probability of each order and then add together the results.

$$P_{\text{Face card and then four}} = \frac{12}{52} \cdot \frac{4}{51} = \frac{48}{2652} = \frac{24}{1326} = \frac{12}{663} = \frac{4}{221}$$

$$P_{\text{Four and then face card}} = \frac{4}{52} \cdot \frac{12}{51} = \frac{48}{2652} = \frac{24}{1326} = \frac{12}{663} = \frac{4}{221}$$

$$P_{\text{Drawing one face card and one four in either order}} = \frac{4}{221} + \frac{4}{221} = \frac{8}{221}$$



The second question is a lot more straightforward to solve because it provides a more specific set of circumstances:

$$P_{\text{Four and then face card}} = \frac{4}{52} \cdot \frac{12}{51} = \frac{48}{2652} = \frac{24}{1326} = \frac{12}{663} = \frac{4}{221}$$

Now that you know what to watch out for in probability problems, let's try out a few sample questions:

- 1.) Your friend has a bag containing twelve marbles. Four are red, four are blue, and four are yellow. What is the probability of your drawing out of the bag first a blue marble, and then a yellow marble, if you don't replace the first marble you draw?
- 2.) You and your friend have entered a raffle. The first person to draw the winning ticket from a bag of tickets wins the raffle prize. You and your friend will each draw one ticket at a time, not replacing them, and will alternate until one of you draws the winning ticket. Your friend offers to let you go first. Statistically, should you do this if you want to win the raffle prize?
- 3.) A hotel offers complementary breakfast that includes a guest's choice of one of five types of breakfast cereal, eggs, either bacon, ham, or sausage, and either orange juice or milk. If you order a breakfast combination at random, what are the odds that you will receive a breakfast that includes ham?
- 4.) At your local no-kill animal shelter, $\frac{1}{3}$ of the pets available for adoption are cats, and of those cats, $\frac{2}{5}$ are kittens and the rest are adult cats. Half of the cats have green eyes, and a third of them are declawed. If someone adopts a pet at random from this shelter, what are the odds that they will adopt an adult cat with green eyes that has not been declawed?
- 5.) To win the largest prize at a fair booth, you have to pick the ace of hearts out of a deck of cards and then roll snake eyes (two ones) on two six-sided dice. What is the probability of winning the largest prize at this fair booth?



Answers:

- 1.) This is an “and” problem in disguise—we could rephrase it as “first a blue marble and then a yellow marble.” So, we’ll be multiplying. The odds of drawing a blue marble from the bag first are $\frac{4}{12}$, but you don’t replace it, so this affects the odds of drawing a yellow marble. There are as many yellow marbles in the bag as before you drew the blue marble, so the numerator will be 4, but there is one fewer marble in the bag, so the total marbles (the denominator) will be reduced by 1, resulting in odds of $\frac{4}{11}$. Multiplying these fractions together results in the answer:

$$\frac{4}{12} \cdot \frac{4}{11} = \frac{16}{132} = \frac{4}{33}$$
- 2.) Statistically, you should let your friend go first if you want to have the better odds of winning the raffle prize. Let’s say there are fifty tickets in the bag at the start of this scenario. The person who draws the first ticket has a $\frac{1}{50}$ chance of drawing the winning ticket. But, because the tickets aren’t getting replaced, the second person to draw out a ticket has a greater chance, $\frac{1}{49}$, of drawing the winning ticket.
3. This question isn’t as tough as it looks! You don’t have to take into account anything other than the breakfast meat selections in order to calculate the probability in this case. While the breakfast choices will differ in other regards (orange juice vs. milk, type of cereal), we are only interested in picking out how many will include ham, so we can simply ignore the rest of the information presented. Of the three choices (bacon, ham, and sausage), we are only interested in one (ham), so that means that $\frac{1}{3}$ of the breakfast options will include ham. Probability questions can be complicated, but don’t overcomplicate them!
4. Let’s write out all of the information we’ve been given:

$$P_{cat} = \frac{1}{3}$$

$$P_{kitten} = \frac{2}{5}$$

$$P_{cat \text{ with green eyes}} = P_{cat} \cdot \frac{1}{2}$$

$$P_{declawed \ cat} = P_{cat} \cdot \frac{1}{3}$$

That’s a lot of information! Next, let’s construct an expression of the probability we need to calculate: we’re looking for the odds of someone adopting at random an adult declawed cat with green eyes.

$$P_{declawed \ adult \ cat \ with \ green \ eyes} = P_{cat} \cdot (1 - P_{kitten}) \cdot \frac{1}{3} \cdot \frac{1}{2}$$



Since declawed cats and cats with green eyes are drawn from the group of cats, and we started the equation with P_{cat} , we don't need to put P_{cat} in the equation multiple times. This is because we want to multiply the fractions representing green-eyed cats and declawed cats with the initial fraction of cats, since we're narrowing in our focus on that one particular group of animals.

At this point, we can substitute in the data presented in the problem and solve by multiplying:

$$P_{\text{declawed adult cat with green eyes}} = \frac{1}{3} \cdot \left(1 - \frac{3}{5}\right) \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{5} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{2}{60} = \frac{1}{30}$$

5. Let's figure out what constitutes the probability of winning the largest prize at this booth. You have to draw the ace of hearts out of a standard card deck without jokers *and* roll snake eyes, so we're going to be multiplying: put a different way, we have to draw the ace of hearts *and* roll a one on a six-sided die *and* roll another one on another six-sided die.

$$P_{\text{Winning largest prize}} = P_{\text{Drawing the ace of hearts}} \cdot P_{\text{Rolling a 1 on first die}} \cdot P_{\text{Rolling a 1 on second die}}$$

Now that we've got the equation worked out, we can simply substitute in the correct probabilities for each of the constituent events occurring. Drawing the ace out of a standard card deck without jokers has a probability of $\frac{1}{52}$, and rolling a 1 on a six-sided die has a probability of $\frac{1}{6}$.

$$P_{\text{Winning largest prize}} = \frac{1}{52} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{1872}$$



Basic Statistics

Statistics is the primary field of study used to derive meaningful information from aggregate data. By applying statistical analyses to a given data set, we can easily determine trends and patterns in the data. These findings can help us draw conclusions about the applicable meanings of the data set. Basic statistical principles can help determine a “normal” value for a set, and, subsequently, can help us identify outliers. In a practical sense, these measurements are useful in recognizing changes and abnormalities (or lack thereof) in collected data, allowing us to draw meaningful conclusions about the information from a given study or sample.

We will look at four common statistical measures. The mean, median, and mode of a data set are each used to evaluate a central measure of the data, while the range is used to give a general sense of the dispersion of the data measures.

Mean

The mean of a data set—also called the average—is used to find the exact center point of the data set. In a mathematical sense, finding the center point of the data requires you to find the sum of all data points, and then divide by the number of data points.

Using summation notation, the formula for the mean looks like:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$\bar{x}$ = mean

n = number of terms in set

Let's try applying this formula. For the set 3, 19, 6, 2, 9, 12, the mean will be equal to $(3+19+6+2+9+12) \div 6$. We divide by 6 because there are six terms in the data set. The mean of 3, 19, 6, 2, 9, 12 is 8.5.

$$(3+19+6+2+9+12) \div 6 = (51) \div 6 = 8.5$$

In a practical sense, finding the mean answers the question, “What would the value of each data point be if all of the data points were exactly equal?” In a study that is looking for consistency of data, the values in the data set should be extremely close to the mean. Values that range significant amounts around the mean demonstrate variability and inconsistency.

Try some basic practice problems to find the mean for the given data sets:

1. {4, 5, 6}
2. {18, 32, 5, 46}
3. {13.5, 4, 19.3, 26.25}
4. $\{x, 5x, \frac{x}{2}\}$



Answers:

$$1. \frac{4+5+6}{3} = \frac{15}{3} = 5$$

$$2. \frac{18+32+5+46}{4} = \frac{101}{4} = 25.25$$

$$3. \frac{13.5+4+19.3+26.25}{4} = 15.7625$$

$$4. \frac{x+5x+\frac{x}{2}}{3} = \frac{6.5x}{3} = \frac{13x}{6}$$

The mean can also be used to determine a value missing from a data set. Consider the following practice problem:

Sample Question

Jeanine needs to get an average of 88 in her biology class in order to maintain her GPA. If her previous test scores were 92, 74, 86, and 97, what score does she need to get on her final test in order to achieve her goal?

We can set up a mathematical model of this problem:

$$\frac{92+74+86+97+x}{5} = 88$$

x represents the missing test score. Together, the four past test scores and the final test show that there are a total of five data points in the set, so we need to divide by 5. Finally, we are told that the average must be equal to 88. Now, use the equation to solve for the missing test score, x .

$$\frac{92+74+86+97+x}{5} = 88$$

$$\frac{349+x}{5} = 88$$

$$349+x = 5 \cdot 88$$

$$349+x = 440$$

$$x = 440 - 349$$

$$x = 91$$

Jeanine needs to get a score of 91 on her final test in order to achieve her goal.

Algebraically, the mean is a relatively simple representation. Just remember the formula: the mean is equal to the sum of the data, divided by the number of terms.



Practice

1. A scientist is doing a study to determine the typical levels of carbon monoxide in the air of automotive garages. She measures the air quality at eight different garages and gets the following results: 13ppm, 26ppm, 4ppm, 36ppm, 12ppm, 19ppm, 5ppm, and 9ppm. What is the mean of her data set?
2. Ryan and Thomas want to see who can jump higher. They each jump six times. Ryan jumps 5.1 inches, 6.5 inches, 5.4 inches, 4.9 inches, 5.8 inches, and 6.2 inches. Thomas jumps 6.0 inches, 5.8 inches, 5.0 inches, 4.7 inches, 5.4 inches, and 6.1 inches. Based on each boy's average, who can jump higher?
3. In order to meet community demand, a local bakery must produce an average of 42 loaves of bread per day. If they produced 44 loaves on Monday, 36 loaves on Tuesday, 35 loaves on Wednesday, 46 loaves on Thursday, 31 loaves on Friday, and 39 loaves on Saturday, how much bread must they bake on Sunday to meet demand?
4. Lillian has a potted plant that grows exactly 3 inches each day. If the plant has a height of h on the first day, what is its average height between days 1 and 5?

Answers:

$$1. \frac{13 + 26 + 4 + 36 + 12 + 19 + 5 + 9}{8} = \frac{124}{8} = 15.5 \text{ ppm}$$

$$2. \text{Ryan: } \frac{5.1 + 6.5 + 5.4 + 4.9 + 5.8 + 6.2}{6} = \frac{33.9}{6} = 5.65$$

$$\text{Thomas: } \frac{6.0 + 5.8 + 5.0 + 4.7 + 5.4 + 6.1}{6} = \frac{33.0}{6} = 5.50$$

Ryan can jump higher.

$$3. \frac{44 + 36 + 35 + 46 + 31 + 39 + x}{7} = 42$$

$$\frac{231 + x}{7} = 42$$

$$231 + x = 7 \cdot 42$$

$$231 + x = 294$$

$$x = 294 - 231$$

$$x = 63$$

4. We know that the plant grows 3 inches each day, and begins at height h .

Day 1: h , Day 2: $h+3$, Day 3: $h+3+3$, Day 4: $h+3+3+3$, Day 5: $h+3+3+3+3$

$$\frac{(h) + (h + 3) + (h + 3 + 3) + (h + 3 + 3 + 3) + (h + 3 + 3 + 3 + 3)}{5}$$

$$\frac{5h + 30}{5}$$

$$\frac{5h}{5} + \frac{30}{5}$$

$$h + 6$$



Median

While the mean is designed to identify the most central value of a data set, the median is used to identify the most central term in the given data. The mean is indiscriminate of data distribution and can be severely impacted by an outlier. In contrast, the median is much more robust to outliers, as it relies less on aggregate data and more on the individual terms present in the set.

To find the median, the data must first be arranged in order from smallest to largest. Next, identify the central term in the set. In the instance of a set with an even number of terms, take the mean of the two centermost terms.

Sample Question

Find the median for the set: {93, 45, 61, 18, 109, 22, 88}

First, order the terms.

{18, 22, 45, 61, 88, 93, 109}

Next, identify the centermost term.

~~{18, 22, 45, 61, 88, 93, 109}~~

The median of this set is 61.

When the number of terms in the set is even, there will be two central terms.

{1, 4, 4, 6, 9, 13, 17, 19, 23, 23}

~~{1, 4, 4, 6, 9, 13, 17, 19, 23, 23}~~

In this case, find the mean of the two central terms.

$$\frac{9+13}{2} = \frac{22}{2} = 11$$

The median of this set is 11.

To compare mean and median, consider the following set of data:

{34, 36, 39, 28, 436}



The average of this set is:

$$\frac{34 + 36 + 39 + 28 + 436}{5} = \frac{573}{5} = 114.6$$

The median of this set is:

{28, 34, 36, 39, 436}

{~~28, 34~~, 36, ~~39, 436~~}

Clearly, in this case, the average is affected by the outlier and becomes almost entirely meaningless. In contrast, the median of 36 is still relatively representative of the data at hand.

Try a few more practice problems:

1. Find the median: {5, 8, 14, 14, 19}
2. Find the median: {-1, -4, -10, -25}
3. Find the median: {56, 72, 65, 98, 83, 96, 104, 72, 82}
4. Find the median: {4.6, 5.9, 12.7, 3.0, 4.2, 1.6}

Answers:

1. {~~5, 8~~, 14, ~~14, 19~~} . Final answer: 14
2. {-1, -4, -10, -25}

Order from least to greatest: {-25, -10, -4, -1}

Identify center: {~~-25~~, -10, -4, ~~-1~~}

Find mean of central terms: $\frac{(-10) + (-4)}{2} = \frac{-14}{2} = -7$

Final answer: -7

3. {56, 72, 65, 98, 83, 96, 104, 72, 82}

Order from least to greatest: {56, 65, 72, 72, 82, 83, 96, 98, 104}

Identify center: {~~56, 65, 72, 72~~, 82, ~~83, 96, 98, 104~~}



Final answer: 82

4. {4.6, 5.9, 12.7, 3.0, 4.2, 1.6}

Order from least to greatest: {1.6, 3.0, 4.2, 4.6, 5.9, 12.7}

Identify center: {~~1.6, 3.0~~, 4.2, 4.6, ~~5.9, 12.7~~}

Find mean of central terms: $\frac{4.2 + 4.6}{2} = \frac{8.8}{2} = 4.4$

Final answer: 4.4

Mode

The mode of a data set is arguably the easiest statistical measure to calculate: it is simply the term that appears with the greatest frequency in the data set. In other words, the mode is the number that appears the most.

Example:

{5, 8, 12, 5, 7, 3}

The mode of this set is 5. The value 5 appears twice, while every other term appears only once.

When there are two terms that appear with the same high frequency, the data set is said to have multiple modes. Do not average the modes!

Example:

{105, 119, 136, 121, 119, 133, 136}

The modes are 119 and 136. Both of these values appear twice, while every other term appears only once. Thus, they must both be included in the answer.

When dealing with larger data sets, remember that the mode is the term that appears with the greatest frequency. Sometimes the mode will appear twice; other times there will be more complex distributions.

Example:

{43, 45, 43, 47, 47, 44, 45, 43, 45, 44, 43, 47, 42}

Rearrange the set to make the distribution more apparent.

{42, 43, 43, 43, 43, 44, 44, 45, 45, 45, 47, 47, 47}

The mode of this set is 43. The value 43 appears four times, the value 45 appears three times, the value 47 appears three times, and the value 44 appears two times. Despite the multiplicity of other terms, 43 is the only mode of this set because it appears the most.



In cases where every term in the set appears once, the set is said to have no mode.

Example:

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

In this set, each term appears exactly once, so there is no mode.

Let's look at some practice problems:

1. Find the mode: $\{1, 3, 5, 6, 7, 8, 8, 10\}$
2. Find the mode: $\{55, 55, 56, 58, 58, 61, 63, 63, 65, 67\}$
3. Find the mode: $\{112, 118, 111, 97, 113, 119, 122, 118, 105\}$
4. Find the mode: $\{34, 33, 36, 34, 35, 33, 35, 34, 36, 36, 33, 35, 37, 34\}$

Answers:

1. $\{1, 3, 5, 6, 7, 8, 8, 10\}$

The value 8 appears twice, while every other value appears only once.

Final answer: 8

2. $\{55, 55, 56, 58, 58, 61, 63, 63, 65, 67\}$

The values 55, 58, and 63 appear twice, while every other value appears only once.

Final answer: $\{55, 58, 63\}$

3. $\{112, 118, 111, 97, 113, 119, 122, 118, 105\}$

Reorder the set from least to greatest.

$\{97, 105, 111, 112, 113, 118, 118, 119, 122\}$

The value 118 appears twice, while every other value appears only once.

Final answer: 118

4. $\{34, 33, 36, 34, 35, 33, 35, 34, 36, 36, 33, 35, 37, 34\}$

Reorder the set from least to greatest.



5. {33, 33, 33, 34, 34, 34, 34, 35, 35, 35, 36, 36, 36, 37}

The value 34 appears four times. The values 33, 35, and 36 each appear three times.

Final answer: 34

Range

Unlike mean, median, and mode—which are designed to illustrate the centrality of a set—range is designed to illustrate the set distribution. To calculate the range, first order the data from smallest value to largest value. Then, subtract the smallest value from the largest value.

Example:

{5, 12, 19, 23, 24, 24, 34, 38, 41}

The range will be:

$$41 - 5 = 36$$

A small range is related to a tighter distribution, while a large range is related to a wider distribution. Consider the range of the two sets given below:

Set 1: {2.5, 2.8, 2.5, 2.7, 2.9}

Set 2: {48, 125, 191, 214, 258, 164}

Reorder each set.

Set 1: {2.5, 2.5, 2.7, 2.8, 2.9}

Set 2: {48, 125, 164, 191, 214, 258}

Calculate the range.

$$\text{Set 1: } 2.9 - 2.5 = 0.4$$

$$\text{Set 2: } 258 - 48 = 210$$

We can see from these values that the data in Set 1 has a very tight distribution with a very small range, while the data in Set 2 has a very wide distribution with a very large range.



Try some practice problems!

1. Find the range: {86, 92, 104, 88, 79, 95}
2. Find the range: {0.15, 0.30, 0.61, 0.22, 0.12, 0.54}
3. Find the range: {11, 14, 15, 15, 11, 12, 13, 14, 13, 11, 12}
4. Find the range: {1, 2, 3, 4, 5, 5, 4, 3, 2, 1}

Answers

1. {86, 92, 104, 88, 79, 95}
Reorder the set from least to greatest.
{79, 86, 86, 92, 95, 104}
Subtract the smallest term from the largest.
 $104 - 79 = 25$
Final answer: 25
2. {0.15, 0.30, 0.61, 0.22, 0.12, 0.54}
Reorder the set from least to greatest.
{0.12, 0.15, 0.22, 0.30, 0.54, 0.61}
Subtract the smallest term from the largest.
 $0.61 - 0.12 = 0.49$
Final answer: 0.49
3. {11, 14, 15, 15, 11, 12, 13, 14, 13, 11, 12}
Reorder the set from least to greatest.
{11, 11, 11, 12, 12, 13, 13, 14, 14, 15, 15}
Subtract the smallest term from the largest.
 $15 - 11 = 4$
Final answer: 4
4. {1, 2, 3, 4, 5, 5, 4, 3, 2, 1}
Reorder the set from least to greatest.
{1, 1, 2, 2, 3, 3, 4, 4, 5, 5}
Subtract the smallest term from the largest.
 $5 - 1 = 4$
Final answer: 4



Basic Statistics Review

1. Find the range, mean, median, and mode for the data set.
 $\{46, 72, 66, 98, 31, 44, 53, 66, 91\}$
2. Find the range, mean, median, and mode for the data set.
 $\{0.8, 1.2, 2.6, 5.0, 0.2, 1.7, 1.9, 2.3\}$
3. Find the range, mean, median, and mode for the data set.
 $\{333, 334, 334, 332, 335, 333, 334, 335, 336, 333\}$
4. Find the range, mean, median, and mode for the data set.
 $\{214, 81, 1127, 108, 834, 591, 13, 602\}$

[See Correct Answers and Explanations](#)

4. {214, 81, 1127, 108, 834, 591, 13, 602}

Reorder: {13, 81, 108, 214, 591, 602, 834, 1127}

Range: $1127 - 13 = 1114$

Mean: $\frac{13 + 81 + 108 + 214 + 591 + 602 + 834 + 1127}{8} = \frac{3570}{8} = 446.25$

Median: {~~13, 81, 108~~, 214, 591, ~~602, 834, 1127~~}

$\frac{214 + 591}{2} = \frac{805}{2} = 402.5$

Mode: No mode



Algebra

The ACT's algebra questions encompass a wide variety of mathematical challenges. Some questions will be relatively simple, asking you to solve for a single variable in a simple equation. Others may present greater challenges, asking you to deal with inequalities, exponents, and systems of equations. No matter how confident you feel in your algebra skills, working through practice problems can help you identify the areas in which you could use the most practice and focus your attention efficiently.

Section Outline

- [Rules of Exponents](#)

- [Monomials](#)

 - [Adding and Subtracting Monomials](#)

 - [Multiplying and Dividing Monomials](#)

 - [Factoring Monomials](#)

 - [Practice Problems: Monomials](#)

- [Binomials](#)

 - [Adding and Subtracting Binomials](#)

 - [Multiplying and Dividing Binomials](#)

 - [Factoring Binomials](#)

 - [FOIL](#)

 - [Practice Problems: Binomials](#)

- [Polynomials](#)

 - [Adding and Subtracting Polynomials](#)

 - [Multiplying and Dividing Polynomials](#)

 - [Factoring Polynomials](#)

 - [Practice Problems: Polynomials](#)

- [Equations and Inequalities](#)

 - [Linear Equations](#)

 - [Absolute Value Equations](#)

 - [Direct and Inverse Variation](#)

 - [Rational Equations](#)

 - [Radical Equations](#)

 - [Quadratic Equations](#)

 - [Inequalities](#)

 - [Rules of Inequalities](#)

 - [Linear Inequalities](#)

 - [Non-Linear Inequalities](#)

- [Function Notation](#)

- [Solving Systems of Equations](#)

 - [Solving Using Substitution](#)

 - [Solving Using Elimination](#)



Rules of Exponents

The rules of exponents are a major component of the ACT Math test. Sometimes, the rules of exponents are tested as straightforward questions about the rules themselves (ex: $x^2 \cdot x^3 = ?$). More often, they are incorporated into broader questions about simplifying expressions.

For the purposes of the ACT Math section, it is important to know 6 rules of exponents. These 6 rules can be divided into two groups of 3 rules. The first group includes basic rules that students need to be aware of. The second group includes the more complex rules that students need to apply to different values. We will refer to these as “Awareness” and “Application” rules:

Awareness Rules

$$x^1 = x \quad \text{Identity Property of Exponents}$$

$$x^0 = 1 \quad \text{Zero Exponents}$$

$$x^{-n} = \frac{1}{x^n} \quad \text{Negative Exponents}$$

Application Rules

$$x^a \cdot x^b = x^{(a+b)} \quad \text{Product of Exponents with the Same Base}$$

$$\frac{x^a}{x^b} = x^{(a-b)} \quad \text{Quotient of Exponents with the Same Base}$$

$$(x^a)^b = x^{(a \cdot b)} \quad \text{Power-to-a-Power Rule}$$

Let's look at the Awareness Rules first.

The **Identity Property** and the **Zero Exponent** rules are pretty straightforward. They can be broken down as:

“Any number raised to the power of 1 is equal to itself.”

$$\begin{aligned} \text{Examples: } 2^1 &= 2 \\ 1,000,000^1 &= 1,000,000 \end{aligned}$$

“Any number raised to the power of 0 is equal to 1.”

$$\begin{aligned} \text{Examples: } 2^0 &= 1 \\ 5^0 &= 1 \\ 1,000,000^0 &= 1 \end{aligned}$$



Finally, the **Negative Exponent** rule is a little trickier. This rule states that if you raise a number to a negative power, you need to divide the expression by 1 and change the sign on the exponent.

So for instance:

$$5^{-3}$$

becomes

$$\frac{1}{5^3} = \frac{1}{125}$$

Similarly, if you have a negative exponent in the denominator, that expression moves to the numerator (dividing by 1):

$$\frac{1}{3^{-2}} = 3^2 = 9$$

Perhaps the most basic way to view a negative exponent is to look at your negative exponential expression and:

1. Flip it
2. Change the sign of the exponent

*It is worth noting that it is extremely common for high-school students to forget or misapply the rule for negative exponents. Students generally want to change the sign of the base rather than the sign of the exponent. **Negative exponents do not affect whether the final answer is positive or negative!**

Examples:

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100}$$

$$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$$

$$\frac{1}{4^{-3}} = 4^3 = 64$$

$$\frac{1}{2^{-2}} = 2^2 = 4$$

Now let's look at the rules of exponents that require some application. These rules will be the most frequently encountered on an ACT Math section, as they are prominent features of simplification questions.

The **Product Rule** and the **Quotient Rule** can be summed up as follows:

"If you are MULTIPLYING exponential terms that have the same base, you can simply add the exponents, and raise the common base to that exponent."

and

"If you are DIVIDING exponential terms that have the same base, you can subtract the exponent of the denominator from the exponent of the numerator."



Examples:

$$x^2 \cdot x^3 = x^{(2+3)} = x^5$$

$$2^5 \cdot 2^4 = 2^{(5+4)} = 2^9 = 512$$

$$\frac{2^5}{2^4} = 2^{(5-4)} = 2^1 = 2$$

$$\frac{x^2}{x^3} = x^{(2-3)} = x^{-1} = \frac{1}{x^1} = \frac{1}{x}$$

Finally, we have the Power to a Power Rule. This rule states:

“When raising a power to a power, you multiply the exponents.”

Example:

$$(2^3)^2 = 2^{(3 \cdot 2)} = 2^6 = 64$$

$$((3^2)^3)^2 = 3^{(2 \cdot 3 \cdot 2)} = 3^{12} = 531,441$$

*Another important reason to know the rules of exponents for the ACT is because of your calculator. It is easy for test makers to construct questions that, while not difficult, are easy to miss if you rely too heavily on a calculator. And the more complicated the exponential expression, the more likely you are to input the expression incorrectly.



Variables and Algebraic Notation

The core of algebra lies in the symbolic representation of various unknown numbers by a given letter, known as a variable. The inherent “unknown” nature in algebraic calculations can be particularly intimidating. This section will present an overview of variables and algebraic notation, focused on the manipulation of algebraic expressions. To properly approach this content, it is essential to leave the desire to “solve for x ” behind, and focus instead on the interactions of values—both known and unknown—in a mathematical context.

Monomials

A monomial is a single algebraic term that consists of variables with positive exponents and is usually accompanied by integers. Algebraic operations can be applied to these monomial terms. This includes addition, subtraction, multiplication, and division. It is important to remember the rules for the various algebraic operations as they apply to monomials with like terms. Some examples of monomials are as follows:

$$5x, y, x^3, 10xy$$

Adding and Subtracting Monomials

When adding and subtracting monomials it is important to have like bases. The base of a monomial is the variable or the combination of variables it has; these must be the same to be considered like bases. The degree on the variables must also be equal in order for them to be like bases. For example, x and $3x$ are like bases but $3x$ and $3x^2$ are not like bases because the exponent on the variable is not the same. The bases must be the same in order to add or subtract them. Recall that integers are constants associated with how many of the variables they accompany. Some examples of like bases are as follows.

$$\begin{aligned} 17x + 3x \\ 20y + y - 12y \\ 14x^2y - 2x^2y + 5x^2y \end{aligned}$$

To add monomials, first identify which terms have like bases. For example, $2x + 3y + 6x + x$ has like bases of x . Then, identify how many of each monomial term exists in the expression. To accomplish this, look at the constant coefficient that is in front of each like base term. Once this has been completed add the coefficients of the like base terms from left to right. For the above example this would lead to $2 + 6 + 1 = 9$; this is the new coefficient that will go in front of the x term resulting in the following final expression $9x + 3y$. This same process is done for the subtraction of monomials.

Sample Question

Add the following monomials.

$$2x + x + 9x$$

Step 1: Identify terms with like bases: x

Step 2: Identify and add the amount of each like base monomial term:



$$2 + 1 + 9 = 12$$

Step 3: Combine new coefficient with variable, making the final answer:

$$12x$$

Sample Question

Subtract the following monomials.

$$20xy - 10yx - 3x$$

Step 1: Identify terms with like bases:

Since xy is the same as yx the like base is xy .

Step 2: Identify and subtract the amount of each like base monomial term from left to right:

$$20 - 10 = 10$$

Step 3: Combine new coefficient with variable:

$$10xy - 3x$$

Multiplying and Dividing Monomials

When multiplying and dividing monomials, it is important to remember the rules of exponents and operations with like bases, and multiplication and division with integers. Expressions can be written in a few different formats for multiplication, but what is important to remember is the distributive property. For example, $2x \cdot 15$ and $2x(15)$ may look different, but they are the same. In this particular case, we will multiply the coefficients together first, which results in $2 \cdot 15 = 30$. From here, we multiply the exponents together. Since there is only one variable, x , it just stays the same. This results in a final answer of $30x$.

Division can also appear in different layouts such as negative exponents; remember, a negative exponent is the same as that term with a positive exponent in the denominator of a fraction. For example, $3x^{-2}$ is the same as $\frac{3}{x^2}$. When dividing monomials that include integers and variables, the first step is to divide the integer part. Next, the like base variables will get divided. If a variable of the same base is located in both the numerator and denominator of a fraction, subtract the exponents. The term in the denominator will be subtracted from the term in the numerator.

For example, $18x^2 \div 2x$ is the same as $\frac{18x^2}{2x}$. In English terms this means, "How many times can $2x$ go into $18x^2$?"

First we will divide the integer part of the fraction:



$$18 \div 2 = 9$$

Next, we will divide the variable with like bases, x . Since $x \cdot x = x^2$, that means that:

$$x^2 \cdot x = x$$

Combining the terms back together results in the final answer:

$$18x^2 \div 2x = 9x$$

Sample Question

Multiply to simplify the following expression.

$$3x(4x^2)$$

Step 1: Multiply the integers together:

$$3 \cdot 4 = 12$$

Step 2: Identify and multiply the like base variables together:

$$x \cdot x^2 = x^{1+2} = x^3$$

Step 3: Combine the integer value with the variable:

$$12x^3$$

Sample Question

Simplify the follow expression.

$$36x^4 \div 2x^7$$

Step 1: Divide the integer values:

$$36 \div 2 = 18$$

Step 2: Divide the like base variable term. When dividing like terms subtract the exponents:

$$x^4 \div x^7 = x^{4-7} = x^{-3}$$

Since there is a negative exponent on the variable, make this into a fraction where the variable is moved to the denominator and the exponent is made positive. Therefore:



$$x^{-3} = \frac{1}{x^3}$$

Step 3: Combine the integer and variable terms:

$$18x^{-3} \text{ or } \frac{18}{x^3}$$

Factoring Monomials

Factoring monomials is a key concept that aids in simplifying expressions which ultimately leads to solving problems. Each value in an expression whether it be the integer portion or variable portion, is composed of factors. By separating out these factors there is the potential to simplify expressions. For example, $70x^2y^3$ can be factored by first factoring the integer portion. This becomes $70 = 7 \cdot 10$, which can be factored further to get, $70 = 7 \cdot 10 = 7 \cdot 2 \cdot 5$. From here, each variable term will need to be factored. The x variable can be factored as follows: $x^2 = x \cdot x$. Now factor the y variable: $y^3 = y \cdot y \cdot y$. Lastly, combine all the factored portions together. This results in the final factored form, $7 \cdot 2 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$.

Sample Question

Factor the following expression.

$$40xy^2$$

Step 1: Factor the integer portion of the expression.

$$40 = 4 \cdot 10$$

$$40 = 4 \cdot 2 \cdot 5$$

$$40 = 2 \cdot 2 \cdot 2 \cdot 5$$

Step 2: Identify and factor the variable portions of the expression.

Since the x variable has an exponent of one it cannot be factored.

Since the y variable has an exponent of two it can be factored, which results in: $y^2 = y \cdot y$

Step 3: Combine the integer and variable terms together.

$$2 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot y \cdot y$$



Sample Question

Factor the following expression.

$$10x$$

Step 1: Factor the integer portion of the expression. $10 = 2 \cdot 5$

Step 2: Identify and factor the variable portion of the expression.

Since the variable has an exponent of one, it cannot be factored any further.

Step 3: Combine the integer and variable portions together.

$$2 \cdot 5 \cdot x$$



Practice Problems: Monomials

1. $23x + 4x =$

- A. $27x$
- B. $19x$
- C. $92x$
- D. $27x^2$

2. $2xy + 12x + 7yx =$

- A. $23x^3y$
- B. $9xy + 12x$
- C. $14yx + 9y$
- D. $21x^2y$

3. $10xy - 4xy + 3 =$

- A. $5yx + 3$
- B. $17x^2y$
- C. $6xy + 3$
- D. $21x^2y$

4. $12x - 2x - 13x =$

- A. $4x$
- B. $1x$
- C. $-3x^3$
- D. $-3x$

5. $100x^2 \div 25x =$

- A. $5x$
- B. $20x^2$
- C. $125x^3$
- D. $25x^2$

6. $39xy^2 \div 3x^2y =$

- A. $39xy$
- B. $3yx^2$
- C. $\frac{13y}{x}$
- D. $\frac{3x}{y}$

7. $2x(3y + 4z - 10) =$

- A. $6xy + 4x^2 - 10$
- B. $6xy + 8xz - 20x$
- C. $6zy + 8xy - 10$
- D. $8zx + 4xy + 10x$

8. $4y^2(x + 3y) =$

- A. $4xy^2 + 12y^3$
- B. $7y^3 + 4y^2 + 4x$
- C. $12y + 7x^2y$
- D. $8x + 12xy$

9. Factor $27x^2y^3$.

- A. $3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y$
- B. $3 \cdot 3 \cdot x \cdot y \cdot y \cdot y$
- C. $3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y$
- D. $3 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y$

10. Factor $3x^2y$.

- A. $3xy$
- B. $2 \cdot 3x \cdot y$
- C. $3 \cdot x \cdot x \cdot y$
- D. $3 \cdot y \cdot x \cdot y$

11. Factor and simplify $\frac{10x^2}{2x}$.

- A. $4x$
- B. $6x$
- C. $5x$
- D. $5x^2$

12. Factor and simplify $\frac{25x^2y^3}{5xy}$.

- A. $5xy^2$
- B. $10x^2y$
- C. $5yx^2$
- D. $5 \cdot x \cdot y \cdot y$



Practice Problems: Monomials: Answer Key

1. $23x + 4x =$

- A. $27x$
- B. $19x$
- C. $92x$
- D. $27x^2$

Step 1: Identify the like bases: x

Step 2: Add the coefficients of the like bases: $23 + 4 = 27$

Step 3: Combine the integer value and the variable value.

Answer: $27x$

2. $2xy + 12x + 7yx =$

- A. $23x^3y$
- B. $9xy + 12x$
- C. $14yx + 9y$
- D. $21x^2y$

Step 1: Identify the like bases: xy

Step 2: Add the coefficients of the like bases: $2 + 7 = 9$

Step 3: Combine the integer value, variable, and the non-like base variable term.

Answer: $9xy + 12x$

3. $10xy - 4xy + 3 =$

- A. $5yx + 3$
- B. $17x^2y$
- C. $6xy + 3$
- D. $21x^2y$

Step 1: Identify the like bases: xy

Step 2: Subtract the coefficients of the like bases from left to right.

$$10 - 4 = 6$$

Step 3: Combine the integer value, variable, and the non-like-base term.

Answer: $6xy + 3$

4. $12x - 2x - 13x =$

- A. $5x$
- B. $1x$
- C. $-3x^3$
- D. $-3x$

Step 1: Identify the like base: x

Step 2: Subtract the coefficients of the like bases from left to right.

$$12 - 2 - 13 = -3$$

Step 3: Combine the new coefficient value with the variable.

Answer: $-3x$

5. $100x^2 \div 25x =$

- A. $4x$
- B. $20x^2$
- C. $125x^3$
- D. $25x^2$

Step 1: Divide the integer values: $100 \div 25 = 4$

Step 2: Divide the like variable portion of the expression.

When dividing like variables, subtract the exponents: $x^2 \div x = x^{2-1} = x$

Step 3: Combine the integer and variable portion together.

Answer: $4x$

6. $39xy^2 \div 3x^2y =$

- A. $39xy$
- B. $3yx^2$
- C. $\frac{13y}{x}$
- D. y

Step 1: Divide the integer values. $39 \div 3 = 13$

Step 2: Divide the variable portion of the expression. When dividing like variables subtract the exponents.

First, divide the x variable from both terms: $x \div x^2 = x^{1-2} = x^{-1} = \frac{1}{x}$

Now, divide the y variable from both terms: $y^2 \div y = y^{2-1} = y$

Step 3: Combine the integer and variable portions of the expression.

$$13x^{-1}y = \frac{13y}{x}$$

$$\frac{13y}{x}$$

Answer: $\frac{13y}{x}$



7. $2x(3y + 4z - 10) =$

A. $6xy + 4x^2 - 10$

→ B. $6xy + 8xz - 20x$

C. $6zy + 8xy - 10$

D. $8zx + 4xy + 10x$

8. $4y^2(x + 3y) =$

→ A. $4xy^2 + 12y^3$

B. $7y^3 + 4y^2 + 4x$

C. $12y + 7x^2y$

D. $8x + 12xy$

9. Factor $27x^2y^3$.

→ A. $3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y$

B. $3 \cdot 3 \cdot x \cdot y \cdot y \cdot y$

C. $3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y$

D. $3 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y$

Step 1: Distribute the integer that is out in front to each integer within the parentheses.

$$2 \cdot 3 = 6$$

$$2 \cdot 4 = 8$$

$$2 \cdot (-10) = -20$$

This creates the following expression:

$$x(6y + 8z - 20)$$

Step 2: Distribute the x to each term within the parentheses.

$$6 \cdot x \cdot y + 8 \cdot x \cdot z - 20 \cdot x$$

Since the variables in each term are different, the expression is in its simplest form.

Answer: $6xy + 8xz - 20x$

Step 1: Distribute the integer value out front to each term within the parentheses.

$$4 \cdot 1 = 4$$

$$4 \cdot 3 = 12$$

$$y^2(4x + 12y)$$

Step 2: Distribute the variable out front to each term within the parentheses.

For variables with like bases that are being multiplied together, simply add their exponents.

$$4x \cdot y^2 + 12y \cdot y^2$$

$$4xy^2 + 12y^{1+2}$$

Answer: $4xy^2 + 12y^3$

Step 1: Factor the integer portion of the expression.

$$27 = 9 \cdot 3 = 3 \cdot 3 \cdot 3$$

Step 2: Factor each variable portion of the expression.

$$x^2 = x \cdot x$$

$$y^3 = y \cdot y \cdot y$$

Step 3: Combine the integer and variable portions to create the factored expression.

Answer: $3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y$



10. Factor $3x^2y$.

A. $3xy$

B. $2 \cdot 3x \cdot y$

→ C. $3 \cdot x \cdot x \cdot y$

D. $3 \cdot y \cdot x \cdot y$

11. Factor and simplify $\frac{10x^2}{2x}$.

A. $4x$

B. $6x$

→ C. $5x$

D. $5x^2$

12. Factor and simplify $\frac{25x^2y^3}{5xy}$.

A. $5xy^2$

B. $10x^2y$

C. $5yx^2$

→ D. $5 \cdot x \cdot y \cdot y$

Step 1: Factor the integer portion of the expression.

In this particular case, the integer is a prime number, and therefore it cannot be factored.

Step 2: Factor each variable portion of the expression.

$$x^2 = x \cdot x$$

$$y = y$$

Step 3: Combine the integer and variable factored portions into one term.

Answer: $3 \cdot x \cdot x \cdot y$

Step 1: Factor the integer portion of the numerator and denominator.

$$\frac{2 \cdot 5 \cdot x^2}{2 \cdot x}$$

Step 2: Factor the variable portion of the numerator and denominator.

$$\frac{2 \cdot 5 \cdot x \cdot x}{2 \cdot x}$$

Step 3: Simplify the expression by canceling like factors that occur in both the numerator and denominator. In this particular problem, the two in the numerator cancels the two in the denominator. Also, one of the x variables in the numerator cancels the x variable in the denominator. This results in the final answer.

Answer: $5x$

Step 1: Factor the integer portion of the expression in both the numerator and denominator.

In this particular problem, the integer value in the denominator cannot be factored, as it is a prime number.

$$\frac{5 \cdot 5 \cdot x^2y^3}{5xy}$$

Step 2: Factor the variables in both the numerator and denominator.

$$\frac{5 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y}{5xy}$$

Step 3: Simplify the expression by canceling common factors that exist in both the numerator and denominator. For this particular problem, one 5, x , and y will cancel out.

Answer: $5 \cdot x \cdot y \cdot y$



Binomials

Binomials are a special type of algebraic polynomial that are comprised of the addition or subtraction of two monomials. At least one term in a binomial must be a variable monomial. Algebraic operations can also be applied to these binomials and it is important to remember the various algebraic operations and properties. Such examples of binomials are as follows.

$$\begin{aligned}(x + 14) \\ (3 + 2x) \\ (2x + 3y)\end{aligned}$$

Adding and Subtracting Binomials

When adding binomials, the parentheses that separate each binomial can be removed and the addition of terms with like bases can occur from left to right. For example, $(x + 14) + (3 + 2x)$ becomes $x + 14 + 3 + 2x$. From here, identify the like bases to be added. In this particular example there are two type of monomials: the terms containing the x variable, and the integer terms. First, identify the coefficients that accompany the x -variable terms and add them; $1 + 2 = 3$, so the variable term becomes $3x$. Next, add the integer terms together: $14 + 3 = 17$. Now, combine the variable term and the integer term to create the final binomial, $3x + 17$.

Sample Question

Add the following binomials.

$$(x + 4) + (15 + 4x)$$

Step 1: Remove the parentheses from each binomial.

$$x + 4 + 15 + 4x$$

Step 2: Identify like terms and add the coefficients.

Like variable base: x

Coefficients of variable base: $1 + 4 = 5$

Integer terms: $4 + 15 = 19$

Step 3: Combine the coefficient with the variable base.

$$5x$$

Step 4: Combine the variable term and the integer term to create the final binomial.

$$5x + 19$$



Sample Question

Add the following binomials.

$$(2x + 3) + (6 + 3x) + (5 + 5x)$$

Step 1: Remove the parentheses from each binomial.

$$2x + 3 + 6 + 3x + 5 + 5x$$

Step 2: Identify like terms and add the coefficients.

Like variable base: x

$$\text{Coefficients of variable base: } 2 + 3 + 5 = 10$$

$$\text{Integer terms: } 3 + 6 + 5 = 14$$

Step 3: Combine the coefficient with the variable base.

$$10x$$

Step 4: Combine the variable term and the integer term to create the final binomial.

$$10x + 14$$

When subtracting binomials, it is important to distribute the negative sign that is in front of the second binomial to each term within the binomial. After the negative is distributed, the parentheses can be removed from all binomials and the algebraic order of operations can occur from left to right. For example, $(2x + 1) - (3 + 3x)$ becomes $2x + 1 - 3 - 3x$. From here, identify the like bases and add their coefficients. In this particular case, x is the like base variable and has two terms containing it, $2x$ and $-3x$. The variable term after addition of the coefficients becomes, $2x - 3x = 1x = -x$. From here, perform the algebraic operations for the integer terms, $1 - 3 = -2$. Now combine the variable term with the integer term to create the final binomial, $-x - 2$.

Sample Question

Subtract the following binomials.

$$(3x - 14) - (7x + 10)$$

Step 1: Distribute the negative sign to all terms in the second binomial and remove parentheses.

$$3x - 14 - 7x - 10$$

Step 2: Identify the like bases and add their coefficients.



Like base: x

Coefficients of like bases: $3 + -7 = -4$

New variable term is, $-4x$.

Step 3: Add the integer terms.

$$-14 + (-10) = -24$$

Step 4: Combine the variable term and the integer term to create the final binomial.

$$-4x - 24$$

Sample Question

Subtract the following binomials.

$$(7x - 12) - (x + 3) - (x - 4)$$

Step 1: Distribute the negative sign to each term in the second and third binomials. Then, remove the parentheses. Remember that when a negative sign is distributed to a negative term, that term becomes positive.

$$7x - 12 - x - 3 - x + 4$$

Step 2: Identify the like bases and add their coefficients.

Like base: x

Coefficients of like base: $7 + (-1) + (-1) = 5$

Integer terms: $-12 - 3 + 4 = -11$

Step 3: Combine the variable term and the integer term to create the final binomial.

$$5x - 11$$



Multiplying and Dividing Binomials

When multiplying a monomial and binomial together it is important to distribute the outside monomial to all terms in the binomial. When doing so, use the properties of exponents and order of operations to correctly multiply the terms. Take the following expression, $2x(3x + 5)$. In this particular case, the monomial term is $2x$ and the binomial it is being multiplied with is $(3x + 5)$. When multiplying these terms, distribute the monomial to each term within the binomial; this is as follows: $2x \cdot 3x + 2x \cdot 5$. Now, using the rules of exponents to multiply the x variable and then multiply the coefficients, the resulting binomial becomes $2 \cdot 3 \cdot x^{1+1} + 2 \cdot 5x = 6x^2 + 10x$.

When dividing binomials by monomials, it is important to separate the expression into two terms. The denominator will be the monomial that the binomial is being divided by. Each term's numerator will

be a single monomial from the original binomial. Take the expression $\frac{4x + 12}{2x}$. In this particular case,

separating the expression into two yields $\frac{4x}{2x} + \frac{12x}{2x}$. From here, divide each term. The first term has a common factor of $2x$ which cancels out in the numerator and denominator leaving 2. The second term has a common factor of 2 in the numerator and denominator that cancel out leaving $6x$. Combining these two terms together results in the final binomial, $2 + 6x$.

Sample Question

Simplify the following expression.

$$x(5x - 10)$$

Step 1: Distribute the monomial term to each term in the binomial.

$$x \cdot 5x - x \cdot 10$$

Step 2: Use the rules for exponents to simplify further.

$$5x^{1+1} - 10x$$

$$5x^2 - 10x$$

Sample Question

Divide $\frac{(10 + 24x)}{2}$

Step 1: Separate the expression into two terms.

$$\frac{10}{2} + \frac{24x}{2}$$



Step 2: Divide each term by identifying common factors.

$$\frac{2 \cdot 5}{2} + \frac{2 \cdot 12x}{2}$$

Step 3: Cancel common factors and simplify.

$$5 + 12x$$

Factoring Binomials

Factoring binomials is a key concept that is beneficial for solving more complex functions and even finding roots. To factor a binomial, find a common integer and/or variable that exists in both terms. Pulling out a common factor is the same as dividing each term by that factor to find the new terms of the binomial portion of the factors. The outside monomial portion of the factors is the common factor originally found. Examine the binomial $(30x + 18)$. In this particular case, find the factors of $30x$ and the factors of 18 to find the common part that will be factored out of the binomial. The factors of $30x$ are $6 \cdot 5 \cdot x$; the factors of 18 are $6 \cdot 3$. Since 6 exists in both factored forms of the binomial terms, it becomes the common factor to be pulled out. Dividing $30x$ by 6 leaves $5x$, and dividing 18 by 6 leaves 3. Therefore, the new factored form of the binomial is $6(5x+3)$.

Sample Question

Factor $(x^2 + x)$.

Step 1: Factor each term in the binomial.

$$(x \cdot x + x \cdot 1)$$

Step 2: Identify the common factor.

The common factor that exists in both terms is x .

Step 3: Divide each term in the binomial by the common factor.

$$x^2 \div x = x \text{ and } x \div x = 1$$

Step 4: Write the final factored form with the new factored terms.

$$x(x+1)$$



Sample Question

Factor $(16x + 8x^2)$.

Step 1: Factor each term in the binomial.

$$(8 \cdot 2 \cdot x + 8 \cdot 1 \cdot x \cdot x)$$

Step 2: Identify the common factor.

The common factor that exists in both terms is $8x$.

Step 3: Divide each term in the binomial by the common factor.

$$16x \div 8x = 2 \text{ and } 8x^2 \div 8x = x$$

Step 4: Write the final factored form with the new factored terms.

$$8x(2 + x)$$

FOIL

FOIL is an acronym that represents the order to multiply binomials together to create one simplified polynomial. FOIL is short for: Firsts, Outers, Inners, and Lasts. When multiplying two binomials together it is important to distribute both terms in each binomial to each other and FOIL helps accomplish this. More specifically, FOIL states to multiply the first terms of each binomial together. Then, multiply the first term of the first binomial with the last term of the second binomial; this is referred to as the outer pieces being multiplied together. Next, multiply the second term in the first binomial with the first term in the second binomial; in other words, multiply the inner terms together. After that, multiply the second terms in each binomial together—the last terms. From here create the polynomial by adding all the products together. Take for instance, $(x + 3)(2x - 5)$. In this particular case the first terms are x and $2x$, the outer terms are x and -5 , the inner terms are 3 and $2x$, and the last terms are 3 and -5 . Find each product:

$$\begin{aligned} x \cdot 2x &= 2x^2 \\ x \cdot -5 &= -5x \\ 3 \cdot 2x &= 6x \\ 3 \cdot -5 &= -15 \end{aligned}$$

Now add all the products together, $2x^2 - 5x + 6x - 15$. Notice that $-5x$ and $6x$ have like bases and thus can be added together and simplify to one term, $-5x + 6x = x$. Reflecting this combination in the final polynomial results in $2x^2 + x - 15$.



Sample Question

Use FOIL to multiply the following binomials: $(7x-10)(2x+4)$

Step 1: Identify the FOIL terms of each binomial and multiply them together.

First: $7x \cdot 2x = 14x^2$

Outer: $7x \cdot 4 = 28x$

Inner: $-10x \cdot 2x = -20x$

Last: $-10 \cdot 4 = -40$

Step 2: Identify like bases and combine into one term.

$28x$ and $-20x$ have like bases.

$$28x + -20x = 8x$$

Step 3: Combine all terms to create final polynomial.

$$14x^2 + 8x - 40$$

Sample Question

Use FOIL to multiply the following binomials: $(3-x)(7x+11)$

Step 1: Identify the FOIL terms and multiply them together.

First: $3 \cdot 7x = 21x$

Outer: $3 \cdot 11 = 33$

Inner: $-x \cdot 7x = -7x^2$

Last: $-x \cdot 11 = -11x$

Step 2: Identify like bases and combine into one term.

$21x$ and $-11x$ have like bases and can be combined to form $21x + -11x = 10x$.

Step 3: Combine products to create final polynomial.

$$-7x^2 + 10x + 33$$



Practice Problems: Binomials

1. $(x + 7) + (9 + 2x)$

- A. $3x + 16$
- B. $2x - 12$
- C. $x + 16$
- D. $x^2 + 16$

2. $(3x + 5) - (9 - 2x) + (7x + 2)$

- A. $12x^2 - 2$
- B. $12x - 2$
- C. $8x + 2$
- D. $8x - 2$

3. $(5x + 11) + (8x + 14) + (x + 1)$

- A. $4x + 22$
- B. $14x^2 + 26$
- C. $14x + 26$
- D. $11x + 3$

4. $(-x + 3) - (x + 6)$

- A. $2x - 3$
- B. $-2x + 3$
- C. $2x + 3$
- D. $-2x - 3$

5. $2x(16 - 6x)$

- A. $32 - 12x^2$
- B. $32x - 2x^2$
- C. $32x + 12x^2$
- D. $32x - 12x^2$

6. $(18x - 2x^2) \div 2x$

- A. $9 + x$
- B. $-9 - x$
- C. $9 - x$
- D. $9 + 2x$

7. $3(x + 2x^2)$

- A. $3x - 6x^2$
- B. $3x + 6x^2$
- C. $3x + 6x$
- D. $x - 6x^2$

8.
$$\frac{75x^3 - 25x^2}{5x}$$

- A. $15x^2 + 5x$
- B. $15x^2 - 5x$
- C. $15x^2 + x$
- D. $10x^2 - 5x$

9. Factor $(9x^3 - 3x^2)$

- A. $3x^2(3x + 1)$
- B. $x^2(3x - 1)$
- C. $3x^2(3x - 1)$
- D. $x^2(3x + 1)$

10. Factor $(0.25x^2 + 0.5x)$

- A. $0.25x(x + 2)$
- B. $1.25x(x + 2)$
- C. $0.25x(x - 2)$
- D. $1.25x(x - 2)$

11. Factor $(10 - 120x^2)$

- A. $10(1 - 12x^2)$
- B. $10(1 + 12x^2)$
- C. $5(1 - 12x^2)$
- D. $5(1 + 12x)$



12. Factor $(27x^4 - 9x^2)$

- A. $9x^2(3x^2 + 1)$
- B. $9x^2(3x - 1)$
- C. $9x^2(x^2 - 1)$
- D. $9x^2(3x^2 - 1)$

13. Factor $(9 - x)(8 + 2x)$

- A. $72 + 10x - 2x^2$
- B. $72 + x - 2x^2$
- C. $72 + 10x - x^2$
- D. $72 - 10x - 2x^2$

14. Factor $(x^2 - 2x)(x + 3)$

- A. $x^3 + x^2 - 6x$
- B. $x^3 - x^2 - 6x$
- C. $x^3 + x^2 + 6x$
- D. $x^3 + x^2 + 6x$

15. Factor $(5x - 10)(4x + 2)$

- A. $20x^2 + 30x - 20$
- B. $20x^2 + 30x + 20$
- C. $20x^2 - 30x - 20$
- D. $20x^2 - 30x + 20$

16. Factor $(3 - 12x)(2x + 7)$

- A. $-24x^2 + 78x + 21$
- B. $-24x^2 - 78x + 21$
- C. $-24x^2 + 78x - 21$
- D. $24x^2 - 78x + 21$



Practice Problems: Binomials: Answer Key

1. $(x + 7) + (9 + 2x)$

- A. $3x + 16$
 B. $2x - 12$
 C. $x + 16$
 D. $x^2 + 16$

Step 1: Drop the parentheses and identify the like terms.

$$x + 7 + 9 + 2x$$

Like bases: x and integer terms.

Step 2: Combine the coefficients of the like bases.

$$2x + x = 3x$$

$$7 + 9 = 16$$

Step 3: Add the variable and integer terms together.

$$3x + 16$$

2. $(3x + 5) - (9 - 2x) + (7x + 2)$

- A. $12x^2 - 2$
 → B. $12x - 2$
 C. $8x + 2$
 D. $8x - 2$

Step 1: Drop the parentheses and distribute the negative term in front of the second binomial to each term within the binomial. Recall that two negatives when multiplied together become a positive.

$$3x + 5 - 9 + 2x + 7x + 2$$

Step 2: Identify like bases.

Like bases: x and the integer terms

Step 3: Combine the coefficients of the like terms.

$$3x + 2x + 7x = 12x$$

$$5 - 9 + 2 = -2$$

Step 4: Add the variable and the integer terms together.

$$12x - 2$$

3. $(5x + 11) + (8x + 14) + (x + 1)$

- A. $4x + 22$
 B. $14x^2 + 26$
 → C. $14x + 26$
 D. $11x + 3$

Step 1: Drop the parentheses and identify like bases.

$$5x + 11 + 8x + 14 + x + 1$$

Like bases: x and integer terms

Step 2: Combine the coefficients of the like bases.

$$5x + 8x + x = 14x$$

$$11 + 14 + 1 = 26$$

Step 3: Add the variable and integer terms together.

$$14x + 26$$

4. $(-x + 3) - (x + 6)$

- A. $2x - 3$
 B. $-2x + 3$
 C. $2x + 3$
 → D. $-2x - 3$

Step 1: Drop parentheses and distribute the negative sign in front of the second binomial to each term within the binomial.

$$-x + 3 - x - 6$$

Step 2: Identify like bases and combine their coefficients.

Like bases: x and integer terms

$$-x - x = -2x$$

$$3 - 6 = -3$$

Step 3: Add the variable and integer terms together.

$$-2x - 3$$



5. $2x(16 - 6x)$

A. $32 - 12x^2$

B. $32x - 2x^2$

C. $32x + 12x^2$

→ D. $32x - 12x^2$

6. $(18x - 2x^2) \div 2x$

A. $9 + x$

B. $-9 - x$

→ C. $9 - x$

D. $9 + 2x$

7. $3(x + 2x^2)$

A. $3x - 6x^2$

→ B. $3x + 6x^2$

C. $3x + 6x$

D. $x - 6x^2$

8. $\frac{75x^3 - 25x^2}{5x}$

A. $15x^2 + 5x$

→ B. $15x^2 - 5x$

C. $15x^2 + x$

D. $10x^2 - 5x$

Step 1: Distribute the monomial to each term within the binomial.

$$2x(16) - 2x(6x)$$

Step 2: Multiply the coefficients together. Recall when multiplying like variables, add the exponents.

$$32x - 12x^2$$

Step 1: Step up division as a fraction.

$$\frac{18x - 2x^2}{2x}$$

Step 2: Separate the fraction into two fractions.

$$\frac{18x}{2x} - \frac{2x^2}{2x}$$

Step 3: Find common factors in each fraction and reduce to the simplest form.

In the first and second fraction, $2x$ is a common factor that can cancel out.

$$\frac{9(2x)}{2x} - \frac{2x(x)}{2x}$$

$$9 - x$$

Step 1: Distribute the constant to each term within the binomial.

$$3(x) = 3x$$

$$3(2x^2) = 6x^2$$

Step 2: Add the two terms together.

$$3x + 6x^2$$

Step 1: Separate fraction into two fractions.

$$\frac{75x^3}{5x} - \frac{25x^2}{5x}$$

Step 2: Find common factors in each fraction and reduce to the simplest form.

In both fractions $5x$ is a common factor that can cancel out.

$$\frac{5x(15x^2)}{5x} - \frac{5x(5x)}{5x}$$

$$15x^2 - 5x$$



9. Factor $(9x^3 - 3x^2)$

A. $3x^2(3x + 1)$

B. $x^2(3x - 1)$

→ C. $3x^2(3x - 1)$

D. $x^2(3x + 1)$

Step 1: Identify common integer factors that can be found in each term.

$$9 = 3(3)$$

$$3 = 3(1)$$

Step 2: Identify common variable factors that can be found in each term.

$$x^3 = x^2(x)$$

$$x^2 = x^2(1)$$

Step 3: Pull out the common integer and variable factors from both terms.

In this particular case, $3x^2$ is a factor of both terms.

$$3x^2(3x - 1)$$

10. Factor $(0.25x^2 + 0.5x)$

→ A. $0.25x(x + 2)$

B. $1.25x(x + 2)$

C. $0.25x(x - 2)$

D. $1.25x(x - 2)$

Step 1: Identify common integer factors that can be found in each term.

$$0.25 = 0.25(1)$$

$$0.5 = 0.25(2)$$

Step 2: Identify common variable factors that can be found in each term.

$$x^2 = x(x)$$

$$x = x(1)$$

Step 3: Pull out the common integer and variable factors from both terms.

In this particular case, $0.25x$ is a factor of both terms.

$$0.25x(x + 2)$$

11. Factor $(10 - 120x^2)$

→ A. $10(1 - 12x^2)$

B. $10(1 + 12x^2)$

C. $5(1 - 12x^2)$

D. $5(1 + 12x)$

Step 1: Identify common integer factors that can be found in each term.

$$10 = 10(1)$$

$$120 = 10(12)$$

Step 2: Identify common variable factors that can be found in each term.

There are no common variables in this expression.

Step 3: Pull out the common integer and variable factors from both terms.

In this particular case 10 is a factor of both terms.

$$10(1 - 12x^2)$$

12. Factor $(27x^4 - 9x^2)$

A. $9x^2(3x^2 + 1)$

B. $9x^2(3x - 1)$

C. $9x^2(x^2 - 1)$

→ D. $9x^2(3x^2 - 1)$

Step 1: Identify common integer factors that can be found in each term.

$$27 = 9(3)$$

$$9 = 9(1)$$

Step 2: Identify common variable factors that can be found in each term.

$$x^4 = x^2(x^2)$$

$$x^2 = x^2(1)$$

Step 3: Pull out the common integer and variable factors from both terms.

In this particular case, $9x^2$ is a factor of both terms.

$$9x^2(3x^2 - 1)$$



13. Factor $(9 - x)(8 + 2x)$

- A. $72 + 10x - 2x^2$
 B. $72 + x - 2x^2$
 C. $72 + 10x - x^2$
 D. $72 - 10x - 2x^2$

Step 1: Multiply the first terms from each binomial together.

$$9(8) = 72$$

Step 2: Multiply the outer terms from each binomial together.

$$9(2x) = 18x$$

Step 3: Multiply the inner terms from each binomial together.

$$-x(8) = -8x$$

Step 4: Multiply the last terms from each binomial together.

$$-x(2x) = -2x^2$$

Step 5: Add all new terms together and combine like terms.

$$72 + 10x - 2x^2$$

14. Factor $(x^2 - 2x)(x + 3)$

- A. $x^3 + x^2 - 6x$
 B. $x^3 - x^2 - 6x$
 C. $x^3 + x^2 + 6x$
 D. $x^3 + x^2 + 6x$

Step 1: Multiply the first terms from each binomial together.

$$x^2(x) = x^3$$

Step 2: Multiply the outer terms from each binomial together.

$$x^2(3) = 3x^2$$

Step 3: Multiply the inner terms from each binomial together.

$$-2x(x) = -2x^2$$

Step 4: Multiply the last terms from each binomial together.

$$-2x(3) = -6x$$

Step 5: Add all new terms together and combine like terms.

$$x^3 + x^2 - 6x$$

15. Factor $(5x - 10)(4x + 2)$

- A. $20x^2 + 30x - 20$
 B. $20x^2 + 30x + 20$
 → C. $20x^2 - 30x - 20$
 D. $20x^2 - 30x + 20$

Step 1: Multiply the first terms from each binomial together.

$$5x(4x) = 20x^2$$

Step 2: Multiply the outer terms from each binomial together.

$$5x(2) = 10x$$

Step 3: Multiply the inner terms from each binomial together.

$$-10(4x) = -40x$$

Step 4: Multiply the last terms from each binomial together.

$$-10(2) = -20$$

Step 5: Add all new terms together and combine like terms.

$$20x^2 - 30x - 20$$

16. Factor $(3 - 12x)(2x + 7)$

- A. $-24x^2 + 78x + 21$
 → B. $-24x^2 - 78x + 21$
 C. $-24x^2 + 78x - 21$
 D. $24x^2 - 78x + 21$

Step 1: Multiply the first terms from each binomial together.

$$3(2x) = 6x$$

Step 2: Multiply the outer terms from each binomial together.

$$3(7) = 21$$

Step 3: Multiply the inner terms from each binomial together.

$$-12x(2x) = -24x^2$$

Step 4: Multiply the last terms from each binomial together.

$$-12x(7) = -84x$$

Step 5: Add all new terms together and combine like terms.

$$-24x^2 - 78x + 21$$



Polynomials

A polynomial is an algebraic expression comprised of more than two terms usually of like bases and different powers. Polynomials frequently have integer terms in them as well. Some examples of polynomial are as follows.

$$\begin{aligned}(3x^2 + 2x - 4) \\ (x^4 - 6x^2 + x - 10) \\ (7x^3 + x^2 + x + 1)\end{aligned}$$

Adding Polynomials

The process used to add polynomials together is the same one that is used to add monomials and binomials together. The key when adding polynomials is identifying the various like bases and then proceeding to add their coefficients. For instance, when adding $(x^2 + 2x - 1) + (3x - 2x^2 + 4)$, the first step is to drop the parentheses, making the expression $x^2 + 2x - 1 + 3x - 2x^2 + 4$. Next, identify the like terms. In this particular case, the like terms include x^2 , x , and the integer terms. Add the coefficient of each like base together to create the new term. For the like base of x^2 , the coefficients are 1 and -2. After adding them, this results in $-x^2$. Now, for the like base of x , the coefficients are 2 and 3. After adding them, this results in $5x$. From here, add the integer portions of the polynomials, which in this case are, -1 and 4: $-1 + 4 = 3$. The final step is to combine all the terms to create the new simplified polynomial, $-x^2 + 5x + 3$.

Sample Question

Simplify the following expression: $(3x^4 + x^2 - 12) + (x^4 + x^3 - 3x^2 + 10)$

Step 1: Drop the parentheses and identify like bases.

$$3x^4 + x^2 - 12 + x^4 + x^3 - 3x^2 + 10$$

Like bases: x^4 , x^2 , and the integer terms

Step 2: Add the coefficients of like bases.

Coefficients of x^4 : $3 + 1 = 4$, so $3x^4 + x^4 = 4x^4$

Coefficients of x^2 : $1 + -3 = -2x^2$

Integer terms: $-12 + 10 = -2$

Step 3: Combine the terms together to get simplified polynomial.

In this problem, it is important to notice that x^3 was only in the second polynomial and still needs to be included in the final polynomial.

$$4x^4 + x^3 - 2x^2 - 2$$



Subtracting Polynomials

The process to subtract polynomials is the same as the one used to subtract monomials and binomials. In order to drop the parentheses and simply combine like terms, one must first distribute the negative sign in front of the second polynomial through to each term within the polynomial. Take, for example, $(x^3 + 4x^2 - x + 3) - (x^3 + 2x^2 - 2x - 7)$. Distributing the negative sign to each term in the second polynomial will change each term's sign. Remember, when a negative is multiplied by a positive, the term becomes negative, and if a negative is multiplied by another negative, the term becomes positive. Applying these rules to the example above results in $(x^3 + 4x^2 - x + 3) + (-x^3 - 2x^2 + 2x + 7)$; from here, the parentheses can be dropped, and combining like terms can occur: $x^3 + 4x^2 - x + 3 - x^3 - 2x^2 + 2x + 7$ has like terms of x^3 , x^2 , x , and the integer terms. To combine like terms, add their coefficients. The coefficients on the cubed terms are 1 and -1. When added, these coefficients cancel each other out, resulting in a zero cubed term. The coefficients on the squared term are 4 and -2, which when added together, yield a sum of 2: $4 + (-2) = 4 - 2 = 2$. The coefficients on the x term are -1 and 2, which when added together sum to $-1 + 2 = 1$. Adding the integer terms gives $3 + 7 = 10$. From here, all the coefficients, bases, and integer portions are combined to give the final polynomial, $2x^2 + x + 10$.

Sample Question

Simplify the following expression: $(12x^2 + 3x - 4) - (x^3 + 2x^2 - x)$

Step 1: Distribute the negative sign to each term in the second polynomial. Once the negative sign is distributed, the parentheses can be dropped.

After the negative is distributed, the second polynomial becomes $-x^3 - 2x^2 + x$. Remember, when two negatives are multiplied together, they create a positive number.

$$12x^2 + 3x - 4 - x^3 - 2x^2 + x$$

Step 3: Identify like bases and add their coefficients.

Like bases: x^2 and x

The coefficients for x^2 are 12 and -2. $12 + (-2) = 10$, so $12x^2 + (-2x^2) = 10x^2$.

The coefficients for x are 3 and 1. $3 + 1 = 4$, so $3x + x = 4x$.

Step 4: Combine terms to create a simplified polynomial.

In this problem, remember to include the terms that were not in both polynomials: -4 and $-x^3$.

$$-x^3 + 10x^2 + 4x - 4$$

Multiplying Polynomials

Multiplying polynomials is extremely beneficial when it comes to simplifying an expression. When it comes to multiplying polynomials, the most important thing to remember is that each term from one polynomial will need to be multiplied with each term from the other polynomials. Rule of multiplying exponents and order of operations are key in correctly multiplying polynomials as well. The larger the polynomials is, more computations are done to arrive at the simplified polynomial. Such an example of polynomial multiplication is $(x + 3 - x^2)(2x - 1 + 2x^2)$. First, multiply the first term from the first polynomial with each term in the second polynomial; this results in



$x \cdot 2x = 2x^2$, $x \cdot -1 = -x$, and $x \cdot 2x^2 = 2x^3$. Next, multiply the second term in the first polynomial with each term in the second polynomial; this results in $3 \cdot 2x = 6x$, $3 \cdot -1 = -3$, and $3 \cdot 2x^2 = 6x^2$. Then, multiply the third term in the first polynomial with each term in the second polynomial; this results in $-x^2 \cdot 2x = -2x^3$, $-x^2 \cdot -1 = x^2$, and $-x^2 \cdot 2x^2 = -2x^4$. From here, we combine all terms into a polynomial, $2x^2 - x + 2x^3 - 2x^3 + x^2 - 2x^4$. Now, combine like base terms to simplify further. In this particular case, the x^3 terms will cancel out since they have coefficients that add up to zero. The final simplified polynomial is $3x^2 - x - 2x^4$.

Sample Question

Simplify the following expression: $(3x^3 + 2x^2 - 5)(x^3 + 5x^2 - x + 6)$

Step 1: Distribute the first term in the first polynomial to each term in the second polynomial.

$$\begin{aligned}3x^3 \cdot x^3 &= 3x^6 \\3x^3 \cdot 5x^2 &= 15x^5 \\3x^3 \cdot -x &= -3x^4 \\3x^3 \cdot 6 &= 18x^3\end{aligned}$$

Step 2: Distribute the second term from the first polynomial to each term in the second polynomial.

$$\begin{aligned}2x^2 \cdot x^3 &= 2x^5 \\2x^2 \cdot 5x^2 &= 10x^4 \\2x^2 \cdot -x &= -2x^3 \\2x^2 \cdot 6 &= 12x^2\end{aligned}$$

Step 3: Distribute the third term in the first polynomial to each term in the second polynomial.

$$\begin{aligned}-5 \cdot x^3 &= -5x^3 \\-5 \cdot 5x^2 &= -25x^2 \\-5 \cdot -x &= 5x \\-5 \cdot 6 &= -30\end{aligned}$$

Step 4: Identify and combine like terms.

$$\begin{aligned}3x^6 + 15x^5 + 2x^5 - 3x^4 + 10x^4 + 18x^3 - 2x^3 - 5x^3 + 12x^2 - 25x^2 + 5x - 30 \\3x^6 + 17x^5 + 7x^4 + 11x^3 - 13x^2 + 5x - 30\end{aligned}$$



Dividing Polynomials

When dividing polynomials, it is important to identify like bases and use the rule of exponents to simplify the terms which contain variables. Similarly to multiplying polynomials, when dividing polynomials each term in the dividend must be divided by each term in the divisor. One way to accomplish division with polynomials is to perform long division, which is also known as synthetic division. An example of using long division with polynomials is as follows: in $(12x^2 + 2x + 4) \div (2x + 1)$, the dividend is $(12x^2 + 2x + 4)$ and the divisor is $(2x + 1)$. Performing this division will result in finding the quotient.

$$(2x + 1) \overline{) 12x^2 + 2x + 4}$$

To start dividing, look at the first term in the divisor and the first term in the dividend. In this particular case, $2x$ goes into $12x^2$ $6x$ times. In other words $2x \cdot 6x = 12x^2$. Now multiply $6x$ to each term in the divisor.

$$(2x + 1) \overline{) 12x^2 + 2x + 4} \quad \begin{array}{r} 6x \\ \hline \end{array}$$

From here, subtract the multiplied-out terms from the terms in the dividend.

$$\begin{array}{r} (2x + 1) \overline{) 12x^2 + 2x + 4} \\ \underline{-12x^2 - 6x} \\ 0 - 4x + 4 \end{array}$$

Now, look at the second term in the dividend and see how many times the first term in the divisor can go into it. In this particular case, $2x$ goes into $-4x$ -2 times. Thus, we have:

$$\begin{array}{r} (2x + 1) \overline{) 12x^2 + 2x + 4} \\ \underline{-12x^2 - 6x} \\ 0 - 4x + 4 \end{array}$$

From here, multiply the second term of the quotient with the divisor and subtract from the dividend.

$$\begin{array}{r} (2x + 1) \overline{) 12x^2 + 2x + 4} \\ \underline{-12x^2 - 6x} \\ 0 - 4x + 4 \\ \underline{-2x - 1} \\ -6x + 3 \end{array}$$

Since there are no terms left to subtract from, we place the remainder $(-6x + 3)$ over the divisor in the quotient to create the final polynomial, $6x + 1 + \frac{-6x + 3}{2x + 1}$.

$$\begin{array}{r} (2x + 1) \overline{) 12x^2 + 2x + 4} \\ \underline{-12x^2 - 6x} \\ 0 - 4x + 4 \\ \underline{-2x - 1} \\ -6x + 3 \end{array} \quad \begin{array}{r} 6x + 1 + \frac{-6x + 3}{2x + 1} \end{array}$$



Factoring Polynomials

Factoring polynomials is a technique which is beneficial to simplify expressions. When dividing polynomials, factoring requires less computations and time than long division and thus is more efficient. Factoring polynomials is also the opposite operation of FOIL. Polynomials with the highest degree being a squared term, results in two binomial factors. When factoring a second degree polynomial, the factors of the integer term when multiplied and then added to the factors of the squared term should result in the coefficient of the single variable term. It is also important to know that if both signs are negative that comes from one binomial that has addition in it, and one binomial that has subtraction in it. If the middle sign is negative and the last sign is positive then both binomial factors have subtraction within them. If both signs in the polynomial are positive then both signs in the binomial factors are positive. For example, $x^2 - 3x - 10$, has a squared term that has factors x, x . The integer term has factors of $2 \cdot 5$ and $1 \cdot 10$. Now, find the combination of the factors that when added together give -3 as an answer. In this particular case, one binomial factor will have addition, and the other will have subtraction. What this means is that one of the factors from ten will be negative and one will be positive. From here it can be seen that $-5 + 2 = -3$; therefore, the binomial factored form of this particular polynomial is $(x + 2)(x - 5)$. To double check that this is the correct factorization, FOIL the two binomials and see if it is the original polynomial.

Sample Question

Factor $x^2 - 6x + 8$.

Step 1: Identify the factors of the squared and integer terms.

$$x^2 : x, x$$

$$8 : 1, 8 \text{ and } 2, 4$$

Step 2: Identify the signs in the binomial factors.

Since the second term in the polynomial is a negative sign and the last term is a positive, that means both binomial factors will have negative signs.

Step 3: Find the terms from the integer factors that will add to give the coefficient of the middle term.

Since both signs in the binomial factors will be negative that means each of our factors are negative. Which factors of eight added together give negative 6?

$$-2 + -4 = -6$$

Step 4: Create the binomial factors.

$$(x - 2)(x - 4)$$

Sample Question

Factor $x^2 + 2x + 1$.



Step 1: Identify the factors of the squared and integer term.

$$\begin{array}{l} x^2 : x, x \\ 1 : 1, 1 \end{array}$$

Step 2: Identify the signs in the binomial factors.

Since both signs in the polynomial are positive, then both signs in the binomial factors are positive.

Step 3: Find which factors add together to get the middle term in the polynomial.

$$1 + 1 = 2$$

Step 4: Create the binomial factors.

$$(x + 1)(x + 1)$$



Practice Problems: Polynomials

1. $(x^2 + 3x + 10) + (2x^2 + 7x - 4)$

- A. $3x^2 + 10x + 6$
- B. $3x^2 + 4x + 6$
- C. $x^2 + 10x + 6$
- D. $3x^2 + 10x + 14$

2. $(3x^3 + 2x + 1) + (2x^2 + 4x + 3)$

- A. $x^2 + 6x + 4$
- B. $5x^2 + 6x + 4$
- C. $5x^2 + 3x + 4$
- D. $5x^2 + 6x + 2$

3. $(x^2 + 5x - 11) - (x^3 - 2x^2 + 1)$

- A. $x^3 + 3x^2 + 5x - 12$
- B. $-x^3 + 3x^2 - 2x - 10$
- C. $-x^3 + 3x^2 + 5x - 12$
- D. $-x^3 + 3x^2 - 5x - 12$

4. $(4x^2 - 2x + 8) - (2x^2 + 8x - 2)$

- A. $2x^2 + 6x - 2$
- B. $2x^2 - 10x + 6$
- C. $x^2 - 10x + 10$
- D. $2x^2 - 10x + 10$

5. $(x^2 + 2x + 1)(3x^2 + 2x - 4)$

- A. $3x^4 + 6x^3 + 3x^2 - 6x - 4$
- B. $3x^4 + 8x^3 + 4x^2 - 6x - 4$
- C. $3x^4 + 8x^3 + 3x^2 - 10x - 4$
- D. $3x^4 + 8x^3 + 3x^2 - 6x - 4$

6. $(3x^2 - 4x + 2)(x^3 + 2x^2 + x)$

- A. $3x^5 + 2x^4 - 3x^3 + 2x$
- B. $3x^5 + 6x^4 - 3x^3 + 2x$
- C. $2x^5 + 2x^4 - 3x^3 + 2x$
- D. $3x^5 + 2x^4 - 3x^3 + 2x^2$

7. $(x^2 + x + 1) \div (x + 1)$

- A. $x - 1$
- B. $\frac{1}{x+1} + x$
- C. $x + 1$
- D. $x^2 + 1$

8. $(3x^2 + 2x + 3) \div (x - 3)$

- A. $3x + 11 + \frac{36}{x-3}$
- B. $3x - 1$
- C. $3x + 11 + \frac{3}{x-3}$
- D. $3x + \frac{36}{x-3}$

9. Factor $x^2 - 10x + 21$.

- A. $(x - 2)(x - 9)$
- B. $(x + 7)(x - 3)$
- C. $(x - 7)(x - 3)$
- D. $(x - 7)(x + 3)$

10. Factor $x^2 - 81$.

- A. $(x + 9)(x + 9)$
- B. $(x - 9)(x + 9)$
- C. $(x - 7)(x + 8)$
- D. $(x - 11)(x + 8)$

11. Factor $16x^2 - 4$.

- A. $(4x - 2)(4x + 2)$
- B. $(4x + 2)(4x + 2)$
- C. $(4x - 2)(4x - 2)$
- D. $(-4x - 2)(4x + 1)$

12. Factor $x^2 - 2x - 8$.

- A. $(x - 4)(x + 2)$
- B. $(x + 4)(x + 2)$
- C. $(x - 4)(x - 2)$
- D. $(x + 4)(x - 2)$



Practice Problems: Polynomials: Answer Key

1. $(x^2 + 3x + 10) + (2x^2 + 7x - 4)$

→ A. $3x^2 + 10x + 6$

B. $3x^2 + 4x + 6$

C. $x^2 + 10x + 6$

D. $3x^2 + 10x + 14$

Step 1: Drop the parentheses and identify terms with like bases.

$$x^2 + 2x^2 + 3x + 7x + 10 - 4$$

Like terms: x^2 , x , and the integer terms.

Step 2: Combine the coefficients of the like terms.

$$3x^2 + 10x + 6$$

2. $(3x^3 + 2x + 1) + (2x^2 + 4x + 3)$

A. $x^2 + 6x + 4$

→ B. $5x^2 + 6x + 4$

C. $5x^2 + 3x + 4$

D. $5x^2 + 6x + 2$

Step 1: Drop the parentheses and identify terms with like bases.

$$3x^3 + 2x^2 + 4x + 2x + 1 + 3$$

Like terms: x and the integer terms.

Step 2: Combine the coefficients of the like terms.

$$3x^3 + 2x^2 + 6x + 4$$

3. $(x^2 + 5x - 11) - (x^3 - 2x^2 + 1)$

A. $x^3 + 3x^2 + 5x - 12$

B. $-x^3 + 3x^2 - 2x - 10$

→ C. $-x^3 + 3x^2 + 5x - 12$

D. $-x^3 + 3x^2 - 5x - 12$

Step 1: Drop the parentheses and identify terms with like bases.

$$x^2 + 5x - 11 - x^3 + 2x^2 - 1$$

Like terms: x and the integer terms.

Step 2: Combine the coefficients of the like terms.

$$3x^3 + 2x^2 + 6x + 4$$

4. $(4x^2 - 2x + 8) - (2x^2 + 8x - 2)$

A. $2x^2 + 6x - 2$

B. $2x^2 - 10x + 6$

C. $x^2 - 10x + 10$

→ D. $2x^2 - 10x + 10$

Step 1: Drop the parentheses and distribute the negative sign to each term within the second polynomial. Recall that two negatives create a positive number while one positive and one negative create a negative value.

$$4x^2 - 2x + 8 - 2x^2 - 8x + 2$$

Step 2: Identify like terms.

Like terms: x^2 , x , and integer terms.

Step 3: Combine the coefficients of like terms.

$$2x^2 - 10x + 10$$



5. $(x^2 + 2x + 1)(3x^2 + 2x - 4)$

A. $3x^4 + 6x^3 + 3x^2 - 6x - 4$

B. $3x^4 + 8x^3 + 4x^2 - 6x - 4$

C. $3x^4 + 8x^3 + 3x^2 - 10x - 4$

→ D. $3x^4 + 8x^3 + 3x^2 - 6x - 4$

Step 1: Multiply each term in the first polynomial with each term in the second polynomial.

Recall that when multiplying like bases, add the exponents.

$$x^2(3x^2) = 3x^4$$

$$x^2(2x) = 2x^3$$

$$x^2(-4) = -4x^2$$

$$2x(3x^2) = 6x^3$$

$$2x(2x) = 4x^2$$

$$2x(-4) = -8x$$

$$1(3x^2) = 3x^2$$

$$1(2x) = 2x$$

$$1(-4) = -4$$

Step 2: Identify and combine like terms.

$$3x^4$$

$$2x^3 + 6x^3 = 8x^3$$

$$-4x^2 + 4x^2 + 3x^2 = 3x^2$$

$$-8x + 2x = -6x$$

$$-4$$

Step 3: Add all terms together to create the simplified expression.

$$3x^4 + 8x^3 + 3x^2 - 6x - 4$$

6. $(3x^2 - 4x + 2)(x^3 + 2x^2 + x)$

→ A. $3x^5 + 2x^4 - 3x^3 + 2x$

B. $3x^5 + 6x^4 - 3x^3 + 2x$

C. $2x^5 + 2x^4 - 3x^3 + 2x$

D. $3x^5 + 2x^4 - 3x^3 + 2x^2$

Step 1: Multiply each term in the first polynomial with each term in the second polynomial.

Recall that when multiplying like bases, add the exponents.

$$3x^2(x^3) = 3x^5$$

$$3x^2(2x^2) = 6x^4$$

$$3x^2(x) = 3x^3$$

$$-4x(x^3) = -4x^4$$

$$-4x(2x^2) = -8x^3$$

$$-4x(x) = -4x^2$$

$$2(x^3) = 2x^3$$

$$2(2x^2) = 4x^2$$

$$2(x) = 2x$$

Step 2: Identify and combine like terms.

$$3x^5$$

$$6x^4 - 4x^4 = 2x^4$$

$$3x^3 - 8x^3 + 2x^3 = -3x^3$$

$$-4x^2 + 4x^2 = 0$$

$$2x$$

Step 3: Add all terms together to create the simplified expression.

$$3x^5 + 2x^4 - 3x^3 + 2x$$



7. $(x^2 + x + 1) \div (x + 1)$

A. $x - 1$

→ B. $\frac{1}{x+1} + x$

C. $x + 1$

D. $x^2 + 1$

8. $(3x^2 + 2x + 3) \div (x - 3)$

→ A. $3x + 11 + \frac{36}{x-3}$

B. $3x - 1$

C. $3x + 11 + \frac{3}{x-3}$

D. $3x + \frac{36}{x-3}$

9. Factor $x^2 - 10x + 21$.

A. $(x - 2)(x - 9)$

B. $(x + 7)(x - 3)$

→ C. $(x - 7)(x - 3)$

D. $(x - 7)(x + 3)$

Step 1: Setup the expression using long division.

Step 2: Perform long division until the remainder is the only term left.

Multiply x to each term in the divisor.

From here, subtract the multiplied out terms from the terms in the dividend.

$$\begin{array}{r} x + \frac{1}{x+1} \\ (x+1)\overline{)x^2 + x + 1} \\ \underline{-x^2 + x} \\ 0 + 0 + 1 \end{array}$$

Step 1: Setup the expression using long division.

Step 2: Perform long division until the remainder is the only term left.

Multiply x to each term in the divisor.

From here, subtract the multiplied out terms from the terms in the dividend.

$$\begin{array}{r} 3x + 11 + \frac{36}{x-3} \\ (x-3)\overline{)3x^2 + 2x + 3} \\ \underline{-3x^2 - 9x} \\ -11x + 3 \\ \underline{-(11x - 33)} \\ 36 \end{array}$$

Step 1: Identify the factors of the squared and integer terms.

$$x^2 = x \cdot x$$

$$21 = 7 \cdot 3$$

Step 2: Identify the signs in the binomial factors.

Since the middle term is negative and the last term is positive, then both signs in the binomial factors are negative.

Step 3: Find which factors add together to get the middle term in the polynomial.

$$-7 + -3 = -10$$

Step 4: Create the binomial factors.

$$(x - 7)(x - 3)$$



10. Factor $x^2 - 81$.

A. $(x+9)(x+9)$

→ B. $(x-9)(x+9)$

C. $(x-7)(x+8)$

D. $(x-11)(x+8)$

11. Factor $16x^2 - 4$.

→ A. $(4x-2)(4x+2)$

B. $(4x+2)(4x+2)$

C. $(4x-2)(4x-2)$

D. $(-4x-2)(4x+1)$

12. Factor $x^2 - 2x - 8$.

→ A. $(x-4)(x+2)$

B. $(x+4)(x+2)$

C. $(x-4)(x-2)$

D. $(x+4)(x-2)$

Step 1: Identify the factors of the squared and integer terms.

$$x^2 = x \cdot x$$

$$81 = 9 \cdot 9$$

Step 2: Identify the signs in the binomial factors.

Since there is no middle term and the last term is negative, then one sign is negative and the other is positive in the binomial factors.

Step 3: Find which factors add together to get the middle term in the polynomial.

This particular function when factored is known as the difference of perfect squares.

$$(x-9)(x+9)$$

Step 1: Identify the factors of the squared and integer terms.

$$16x^2 = 4x \cdot 4x$$

$$4 = 2 \cdot 2$$

Step 2: Identify the signs in the binomial factors.

Since there is no middle term and the last term is negative, then one sign is negative and the other is positive in the binomial factors.

Step 3: Find which factors add together to get the middle term in the polynomial.

This particular function when factored is known as the difference of perfect squares.

$$(4x-2)(4x+2)$$

Step 1: Identify the factors of the squared and integer terms.

$$x^2 = x \cdot x$$

$$8 = 4 \cdot 2$$

Step 2: Identify the signs in the binomial factors.

Since the middle term is negative and the last term is negative, then one sign in the binomial factor is positive and the other is negative.

Step 3: Find which factors add together to get the middle term in the polynomial.

$$-4 + 2 = -2$$

Step 4: Create the binomial factors.

$$(x-4)(x+2)$$



Equations and Inequalities

The ACT Math section features all sorts of equations and inequalities from the most basic linear equations of Algebra 1, to the challenging quadratic equations of Algebra 2, and even some basic trigonometric equations.

This section will briefly survey many of the different types of equations and inequalities that a student will see on a typical ACT Math section. Unlike a typical math class, we won't focus on how to "do" (factor, solve, simplify, etc.) different equations. Instead, we will focus on many scenarios that are specific to the ACT. In other words, we will explore questions that are common and maybe even somewhat unique to the ACT. In many cases, these are questions that are slightly different than what students have experienced in math class.

Linear Equations

Linear equations are a basic concept for purposes of the ACT, but even though the concept is easy, the questions may not be. In fact, you should expect to see easy, medium, and difficult linear equation questions on your ACT Math section.

Are you stuck on a question? Here are some important tips:

1. Find the slope!

There are lots of ways to do this, depending on your question. The challenge will be to apply a certain technique for finding slope based on the information you are given. Have a dialogue with yourself:

Do I have two points?

Use the formula for the slope between two points: $\frac{y_2 - y_1}{x_2 - x_1}$

Is the given line parallel to the line I need to find?

The slopes are the same.

Is the given line perpendicular to the line I need to find?

The slopes are opposite reciprocals. Flip the fraction and change the sign.

Is the equation given to you? Is it in slope-intercept form ($y = mx + b$)?

The slope is the coefficient on the x term.

Is the equation given to you in ANYTHING other than slope-intercept form?

Solve the equation for y , which will put the equation in slope-intercept form, and you can then look at the coefficient on the x term.



2. The word “origin” means the point (0,0).

This might seem like a no-brainer. Nevertheless the ACT loves to use “origin” instead of “(0,0).” Why? Because it is harder to identify! Here is a quick example, which shows the difference between a problem you might have seen in an Algebra 1 class and the SAME question, worded differently, on the ACT:

Algebra 1 test: Find the slope of a line that passes between the points (0,0) and (7,6).

ACT Math section: What is the slope of a line that travels through the origin and the point (7,6)?

It is a small, subtle difference, but it matters. On test day, when the clock is running, students who can quickly and effectively identify what to do on the first version routinely miss what to do on the second version.

3. Even if you don’t know how to find out exactly what the slope *is*, you very likely know a few values that the slope *isn’t*.

This is especially important on more difficult problems. Are you looking at the graph of a line and completely stumped on what to do? Chances are, you can tell if the line has a positive or a negative slope or, more importantly, that the slope is *not* positive or *not* negative. Thus, any answer that has an equation with a slope of an opposite sign to what you have identified *must* be wrong. Suddenly, a difficult, multiple-choice question with 5 possibilities can be whittled down to 3 or even 2 possibilities without performing a single calculation.

Linear Inequalities

One basic rule to keep in mind here: Do not forget to switch the direction of the inequality symbol if you multiply OR divide by a negative number.

Absolute Value Equations

Two important things to know about absolute value equations and inequalities:

1. They are an ACT favorite.
2. Students routinely miss these questions.

Remember that absolute value represents the distance that a number is from 0. This value is always a positive value or zero. Thus,

$$|6|=6$$

and

$$|-6| = 6$$



Direct and Inverse Variation

For direct and inverse variation, it is best to memorize the formulas:

$$\text{Direct Variation: } y = kx$$

$$\text{Inverse Variation: } y = \frac{k}{x}$$

The variables mean the same thing in each of these equations, but they have different relationships. k is what is known as the “constant of proportionality.”

In a direct variation equation, as the value of x increases, the value of y increases as well. As the value of x decreases, the value of y decreases as well. Thus, x and y vary *directly*, or in the same direction.

In an inverse variation equation, as the value of x increases, the value of y decreases. As the value of x decreases, the value of y increases. Thus, x and y vary *inversely*, or in opposite directions.

These problems almost always follow the same pattern. You are given an x -value and a y -value and told that they “vary” directly or inversely, or you are told that they are “directly proportional” or “inversely proportional.” The word “direct” or “directly” signifies that you use the Direct Variation equation. The word “inverse” or “inversely” signifies that you use the Inverse Variation equation.

Once you have decided on which equation you need, you will follow the same two steps:

1. Solve the direct/inverse equation for k .
2. Substitute your new value for k into a second equation with the given x - or y -value.

Let’s try an example:

Sample Question

x is directly proportional to y . When x equals 3, y equals 6. What is the value of x when $y = 18$?

Our first sentence tells us that x and y are “directly proportional,” so we will use the Direct Variation equation, $y = kx$.

Now, we will perform the first step, which is to solve for k using our x and y values.

$$6 = k \cdot 3$$

$$6/3 = \frac{k \cdot 3}{3}$$

$$2 = k$$

$$k = 2$$

Now that we have solved for k , we plug that value into a second Direct Variation equation, with our second y value, and solve for x :



$$18 = 2 \cdot x$$

$$\frac{18}{2} = \frac{2 \cdot x}{2}$$

$$9 = x$$

$$x = 9$$

*Note on this particular problem that y increased from 6 to 18. Since y varies directly with x , x would have to increase as well. If you understand the relationship of direct (x and y go in the same direction) and inverse (x and y go in opposite directions) variations, you will be able to eliminate some incorrect answers *even if you don't know the equations*.

Let's try an inverse variation problem. The steps are the same, but the equation is different.

Sample Question

x and y vary inversely. When $x = 6$, $y = 2$. What is the value of y when $x = 4$?

This time we use the inverse variation equation:

$$y = \frac{k}{x}$$

$$2 = \frac{k}{6}$$

$$6 \cdot 2 = \frac{k}{6} \cdot 6$$

$$12 = k$$

$$k = 12$$

Now that we have found k , we plug that value into a second Inverse Variation equation using our second x value:

$$y = \frac{12}{4}$$

$$y = 3$$

*Note on this particular problem that x decreased in value from 6 to 4. Since this is an inverse variation problem we know that y must do the opposite or INCREASE in value. In this problem, y increased from 2 to 3.

Direct variation and inverse variation are important concepts on both the ACT Math section and the ACT Science section. It is important to know what they mean at their most basic: "direct" = "same direction" and "inverse" = "opposite directions."



Rational Equations

For purposes of the ACT, a rational equation is an equation that involves polynomials as part of the a fraction. In other words, there is a quotient of polynomials. For example:

$$\frac{x+1}{x-1} + \frac{1}{x+1} = \frac{1}{(x+1)(x-1)}$$

or

$$\frac{2s-3}{s+2} = 4$$

Rational equations frustrate many students. This is mostly because they involve fractions, a common source of frustration for high-school students. Many student are out of practice, lack confidence, or both when it comes to dealing with fractions; however, it is important to focus on the fraction skills that students learn in elementary and middle school in order to make rational equations manageable. Here are a few simple rules to keep in mind:

1. Any number divided by itself is equal to 1.

This is an extremely basic rule, but very important. This is an identity property. So:

$$\frac{1}{1} = 1$$

and

$$\frac{4}{4} = 1$$

and

$$\frac{1,343,565}{1,343,565} = 1$$

and perhaps most importantly,

$$\frac{x}{x} = 1$$

and

$$\frac{x+1}{x+1} = 1$$



2. You can only add and subtract fractions that have a common denominator.

This is another basic rule. When you add or subtract fractions with a common denominator, you simply rewrite the denominator, and add or subtract the numerators. For example:

$$\frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$$

or a rational expression:

$$\frac{1}{x} + \frac{1}{x} = \frac{1+1}{x} = \frac{2}{x}$$

If you have two fractions with different denominators, you need to find the least common denominator for the two denominators before you can add or subtract. For example, given the expression:

$$\frac{1}{2} + \frac{1}{3} = ?$$

We cannot add them together without finding the least common denominator for the two fractions. An example of a similar rational expression would be:

$$\frac{1}{x} + \frac{1}{y} = ?$$

3. In order to get a common denominator, you can multiply EACH term by the identity fraction you need.

Let's go straight to our previous example:

$$\frac{1}{2} + \frac{1}{3} = ?$$

We know our common denominator is 6. We need to multiply 2 by 3 and 3 by 2 in order to get 6.

Therefore, we multiply by the identity fractions $\frac{3}{3}$ and $\frac{2}{2}$:

$$\left(\frac{3}{3}\right) \cdot \frac{1}{2} + \frac{1}{3} \cdot \left(\frac{2}{2}\right) = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$$

Now, you can apply the same principle to a rational expression, where there are variables instead of numbers. Your identity fraction is just either of the variables divided by itself. For example:

$$\frac{1}{x} + \frac{1}{y} = ?$$

$$\left(\frac{y}{y}\right) \cdot \frac{1}{x} + \frac{1}{y} \cdot \left(\frac{x}{x}\right) = \frac{y+x}{xy}$$



4. If a denominator has a “+” or “-” sign, you cannot break it up.

This is a more difficult rule. An example will help demonstrate this:

$$\frac{1}{2+3} \neq \frac{1}{2} + \frac{1}{3}$$

As we demonstrated above, $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

If we add the 2 and the 3 first, we get $\frac{1}{5}$, a different value. The same principle holds true with variables:

$$\frac{1}{x+3} \neq \frac{1}{x} + \frac{1}{3}$$

Now, let's put this all together for rational equations.

Let's say we have an equation that looks like this:

$$\frac{1}{x} + \frac{1}{x^2} = \frac{1}{2x^2}$$

First, we see that this equation is a rational equation with addition. To add these fractions, we need to have a common denominator, which we don't have. We have 3 different denominators: x , x^2 , $2x^2$.

Our lowest common denominator here is $2x^2$. Now we need to multiply each term by the identity fraction that will give us the lowest common denominator:

$$\left(\frac{2x}{2x}\right) \cdot \frac{1}{x} + \frac{1}{x^2} \cdot \left(\frac{2}{2}\right) = \frac{1}{2x^2} \cdot \left(\frac{1}{1}\right)$$

This is where students tend to get confused. We multiplied each of our 3 original terms by 3 *different* fractions. However, these fractions only appear different, as their value is the same. Remember, any number divided by itself is equal to one.

When working with rational equations, we change the appearance of the equation without changing its value.

Now, let's finish with the example above:

$$\frac{2x}{2x^2} + \frac{2}{x^2} = \frac{1}{2x^2}$$

We have a common denominator, so we can combine the left side of the equation into one fraction:

$$\frac{2x+2}{2x^2} = \frac{1}{2x^2}$$



Now, since we can multiply both sides by the common denominator in order to simplify the equation (and get rid of the fractions):

$$\frac{2x^2}{1} \cdot \frac{2x+2}{2x^2} = \frac{1}{2x^2} \cdot \frac{2x^2}{1}$$

At this point, we are left with our numerators:

$$2x + 2 = 1$$

Now, finish the simplification:

$$2x + 2 - 2 = 1 - 2$$

$$2x = -1$$

$$\frac{2x}{2} = -\frac{1}{2}$$

$$x = -\frac{1}{2}$$

Radical Equations

Radical equations are, quite simply, equations that involve a radical. Radicals can take the form of any root (square root, 4th root, 10th root). On the ACT, however, you are most likely to encounter a square root symbol ($\sqrt{\quad}$), or possibly a cube root.

There are three steps to apply to every question that involves radicals.

1. Isolate the radical

2. Get rid of the radical

3. Solve the equation

Isolating the Radical

Isolating the radical means that the radical expression is the only thing left on the left or right side of the equation. For easy radical equations, the radical is already isolated. For instance, $\sqrt{x+3} = 10$ and $\sqrt{x^2+1} = 2$ are already isolated.

For more complex equations, you use the same skills that you use to isolate a variable in an algebraic equation to isolate a radical in a radical equation. Thus, in order to isolate the radical in the equation $2\sqrt{x^2-5} + 3 = 11$, we would use the same steps as if we were isolating the “x” in $2x + 3 = 11$, which is to subtract 3 from both sides, and then divide by two on both sides.



Get Rid of the Radical

Once you isolate the radical, you can get rid of the radical by raising both sides of the equation to the exponent that corresponds to the root of the radical. Thus, if you have a square root (most likely), you raise both sides of the equation to a power of 2. If you have a cube root (less likely) you raise both sides of the equation to a power of 3. If you have a higher root (unlikely) you raise both sides to corresponding power (4th root to a power of 4, 5th root to a power of 5, and so on).

Once this is accomplished, you can then solve the equation as any other algebraic equation. However, once you solve the equation, make sure you plug your solution(s) back into the original equation to make sure they aren't extraneous solutions.

Example:

$$4\sqrt{x+3}+8=4$$

$$(4\sqrt{x+3}+8)-8=(4)-8$$

$$4\sqrt{x+3}=-4$$

$$\frac{4\sqrt{x+3}}{4}=\frac{-4}{4}$$

$$\sqrt{x+3}=-1$$

$$(\sqrt{x+3})^2=(-1)^2$$

$$x+3=1$$

$$(x+3)-3=1-3$$

$$x=-2$$

Plug $x = -2$ back into the original equation:

$$4\sqrt{-2+3}+8=4$$

$$4\sqrt{1}+8=4$$

$$4 \cdot 1 + 8 = 4$$

$$4 + 8 = 4$$

$$12 = 4$$

This is a false statement, so the solution is extraneous.



Quadratic Equations

Quadratic equations are a special type of polynomial with a highest degree of two. These equations graphically create smooth parabolas and algebraically can be solved by factoring or through the use of the quadratic formula. Quadratic equations can have up to three terms: one squared term, one single variable term, and one constant term. Although quadratics can have three terms, doesn't mean that they always do. The important characteristic to remember, which identifies a quadratic equation, is the highest term is two. The following are all examples of quadratic equations.

$$y = x^2$$

$$y = 3x^2 - 1$$

$$y = x^2 + 2x + 4$$

Methods to Solve Quadratic Equations

When solving quadratic equations, there are three methods to use: using quadratic formula, factoring, and graphing. Depending on the specific quadratic equation, factoring may not be the easiest method. However, all quadratic equations can be solved quite easily using the quadratic formula. Graphing can be used to solve the quadratic equation by simply plugging in various x -values to solve for the y -values and then plotting them on a graph. This method is often referred to as "plug and chug."

Quadratic Formula

The quadratic formula is used to solve an quadratic equation that is in the form of $ax^2 + bx + c$, where a is the coefficient on the quadratic term, b is the coefficient on the single variable term, and c is the constant term. The quadratic formula is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To solve the equation, plug in the coefficients of the quadratic equation into the above formula.

For example, use the quadratic formula to solve $3x^2 - 4x + 1$. In this particular case, let's first identify the coefficients to be used in the quadratic formula: $a = 3$, $b = -4$, and $c = 1$. Now, plug these values into the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(1)}}{2(3)}$$

Recall that when a negative sign is applied to a negative number, the number becomes positive. This is also true when a negative number is squared—it becomes a positive number. Therefore, this particular solution becomes:



$$x = \frac{4 \pm \sqrt{16 - 12}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{4}}{6}$$

$$x = \frac{4 \pm 2}{6}$$

With quadratic equations, there will be two solutions.

$$x = \frac{6}{6}, \frac{2}{6}$$

$$x = 1, \frac{1}{3}$$

Sample Question

Solve $6x^2 - 2x - 7$.

Step 1: Identify the coefficients:

$$a = 6, b = -2, c = -7$$

Step 2: Plug coefficients into the quadratic formula:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(-7)}}{2(6)}$$

Step 3: Simplify. Recall that a negative multiplied by a negative becomes a positive:

$$x = \frac{2 \pm \sqrt{4 + 168}}{12}$$

$$x = \frac{2 \pm \sqrt{172}}{12}$$

$$x = \frac{2 \pm 2\sqrt{43}}{12}$$

$$x = \frac{1 \pm \sqrt{43}}{6}$$



Sample Question

Solve $x^2 - 18$.

Step 1: Identify the coefficients:

$$a = 1, b = 0, c = -18$$

Step 2: Plug coefficients into the quadratic formula:

$$x = \frac{-0 \pm \sqrt{(0)^2 - 4(1)(-18)}}{2(1)}$$

Step 3: Simplify:

$$x = \frac{\pm\sqrt{72}}{2}$$

$$x = \frac{\pm 2\sqrt{18}}{2}$$

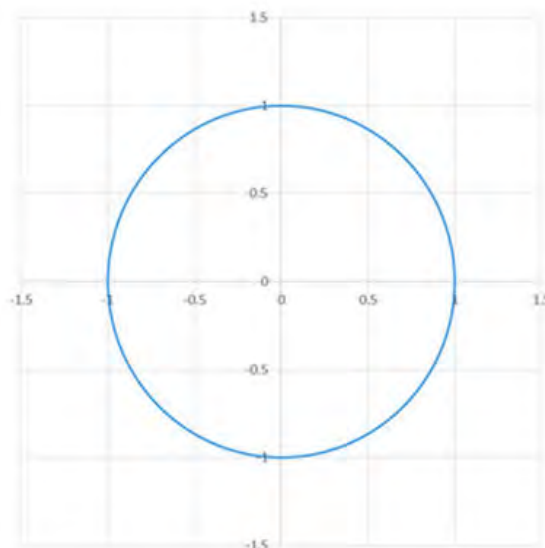
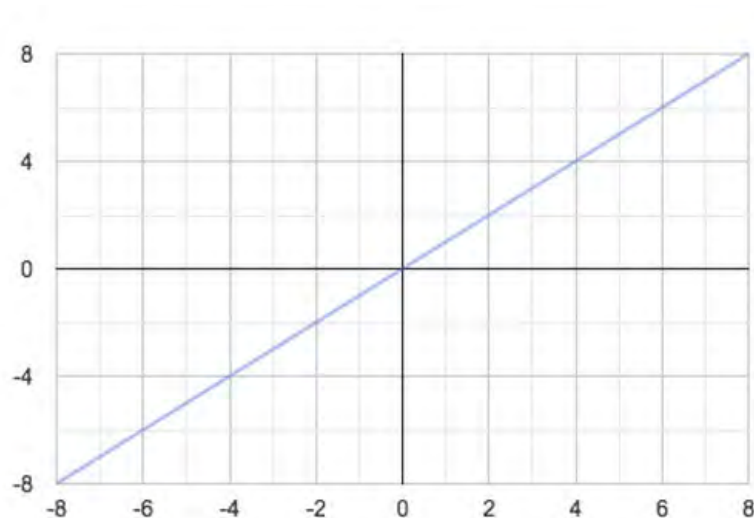
$$x = \pm\sqrt{18}$$



Function Notation

Function notation is an alternative form of formatting certain equations. A function is defined as an algebraic equation for which each input term has exactly one output term. In general, this means that for every value of x , there is only one possible solution for y . Multiple inputs can result in equal outputs (multiple values of x can give the same value of y), but a single input cannot give two solutions.

By this definition, all functions will adhere to the vertical line test. In other words, when a function is graphed, it will be impossible for a vertical line on the graph to intersect multiple points.



Note that for the equation $x = y$, a vertical line cannot possibly intersect the graph more than once. In contrast, there are values on the graph of a circle where a vertical line will intersect the graph twice. All linear equations are functions, with the exception of those that generate a vertical line such as $x = 5$. By definition, these graphs would fail the vertical line test. All vertically-oriented parabolas (those that open upwards or downwards) are also functions.

Function notation provides a different method of writing these special equations. Instead of the traditional format using an input variable (typically x) and an output variable (typically y), function notation implies a function being applied to a set of input variables, generating a set of outputs.

Let's examine a simple linear function:

$$y = 4x + 3$$

We can rewrite this equation using function notation:

$$f(x) = 4x + 3$$

The implication is that for any value of x , the function can be applied to give a single output. $f(x)$ denotes “the function of x ,” or the set of all output values corresponding to the available set of input values. In many cases, it is still practical to think of $f(x)$ as equal to y , but function notation provides a framework to look at trends in the graph of the function as opposed to singular values.



Consider evaluating the function above when x is equal to 5. In function notation, this would be written as $f(5)$. On the ACT, this question would be asked in the following format:

Sample Question

$$f(x) = 4x + 3 \text{ Find } f(5).$$

To solve this question, you need to plug in 5 as the value of x and evaluate the result.

$$f(5) = 4(5) + 3 = 20 + 3 = 23$$

In function notation, the term in parentheses is the input. Though this may seem like an obvious statement, consider the following example:

Sample Question

$$\text{Find } f(x^2 + 3) = x^2 - x + 7.$$

Instead of a single variable input, function notation allows us to evaluate more complex applications of the given equation. To solve this problem, we need to replace every instance of x in the right side of the equation with the term given as the input: $x^2 + 3$.

$$f(x^2 + 3) = (x^2 + 3)^2 - (x^2 + 3) + 7$$

Simplify to find the final answer.

$$f(x^2 + 3) = (x^2 + 3)(x^2 + 3) - (x^2 + 3) + 7$$

$$f(x^2 + 3) = x^4 + 6x^2 + 9 - x^2 - 3 + 7$$

$$f(x^2 + 3) = x^4 + 5x^2 + 4$$

In some instances, the ACT may ask you to apply one function to another. These questions will provide two separate functions, then ask you to evaluate an input for one function in terms of another. The format of these questions will appear as $f(g(x))$.

Sample Question

$$f(x) = x^2 - 14$$

$$g(x) = -x + 4$$

$$\text{Find } g(f(6)).$$

What does $g(f(6))$ mean? We need to apply the function $g(x)$ to the outcome of $f(6)$. To start, we need to find $f(6)$.



$$f(6) = (6)^2 - 14 = 36 - 14 = 22$$

Now, we can rewrite $g(f(6))$ as $g(22)$, because $f(6)=22$. To find the final answer to this question, simply solve for $g(22)$.

$$g(22) = -(22) + 4 = -18$$

So, the final answer will be $g(f(6)) = -18$.



Inequalities

Inequalities are types of mathematical expressions in which two quantities are not equal. Solving inequalities is very similar to solving equations, with a few key differences; as the name suggests, the quantities are not equal to each other. Rather, they are compared using relative terms such as greater than and less than. Other inequalities can be related using the terms greater than or equal to and less than or equal to. Below is the list of the symbols used when representing inequalities mathematically and their definitions.

$<$ Quantities on the left side of this symbol are said to be “less than” quantities on the right side of this symbol

Ex: $5 < 7$

$>$ Quantities on the left side of this symbol are said to be “greater than” quantities on the right side of this symbol

Ex: $20 > 3$

$\leq$ Quantities on the left side of this symbol are said to be “less than or equal to” quantities on the right side of this symbol

Ex: $9 \leq 15$

Ex: $15 \leq 15$

$\geq$ Quantities on the left side of this symbol are said to be “greater than or equal to” quantities on the right side of this symbol

Ex: $11 \geq 6$

Ex: $11 \geq 11$

Note that all of these inequalities are true, and still uphold the definitions of each symbol. For example, 9 is indeed less than or equal to 15; it is less than it. Similarly, 15 is indeed less than or equal to 15; it is equal to it. One way to remember the definitions of these inequalities is to think of the symbols as the open mouth of a hungry alligator. The alligator always opens his mouth toward and “eats” the larger number.



Rules of Inequalities

The two symbols $<$ and $>$ are said to be non-inclusive, or exclusive. This means that if we have the inequality:

$$x < 4$$

For this inequality to be true, x can be equal to any values less than 4. Presented in a different way,

$$x \leq 3.\overline{9}$$

Like equations, whatever operation is done to one side of the inequality must be done to the other side.

$$\text{Ex: } x + 2 < 10$$

To solve this inequality, we must subtract 2 from both sides of the inequality.

$$(x + 2) - 2 < (10) - 2$$

Simplify.

$$x < 8$$

The same is true for multiplication and division.

$$\text{Ex: } 2x > 20$$

To solve this inequality, we must divide both sides by 2.

$$\frac{2x}{2} > \frac{20}{2}$$

Simplify.

$$x > 10$$

However, one important rule when solving inequalities that is different from solving equations is that *if we multiply or divide by a negative number, then the direction of the inequality switches*.

$$\text{Ex: } -\frac{1}{2}x > 14$$

To solve this inequality for x , we must multiply both sides by (-2) :

$$(-2) \cdot -\frac{1}{2}x > 14(-2)$$



Remember that when we multiply or divide by a negative number, our inequality changes direction! So:

$$x < -28$$

Note that this is true (although easy to miss) for multiplying both sides by (-1) .

Ex: $-x \geq 7$

This means that:

$$x \leq -7$$

To illustrate this property, let's pick a number that satisfies the second inequality, say, -10 . -10 is indeed less than or equal to -7 (it is less). Now if we plug in -10 to the original inequality, we are left with the inequality $-(-10) \geq 7$ or just $10 \geq 7$, which are both true.

Linear Inequalities

Solving linear inequalities is just like solving linear equations, with the exception listed above. Refer to the Inequalities section in the chapter on Coordinate Geometry for details on graphing linear inequalities.

Sample Question

Solve for x : $3x - 23 < 4$

Start by adding 23 to both sides of the inequality:

$$3x < 27$$

Next, divide each side by 3:

$$\frac{3x}{3} < \frac{27}{3}$$

Simplify.

$$x < 9$$



Sample Question

$$\text{Solve for } x: -\frac{1}{6}x - 16 \geq -4$$

We notice that all terms have a negative sign in front of them, so we can multiply both sides of the inequality by -1 .

$$\frac{1}{6}x + 16 \leq 4$$

Remember to switch the sign of the inequality since we multiplied by a negative number! Next, subtract 16 from each side:

$$\frac{1}{6}x \leq -12$$

Multiply each side by 6 and simplify.

$$x \leq -72$$

If you have any time left over in the mathematics section, be sure to check your work. Let's check our answer for Example 2 by plugging in any value for x that is less than or equal to -72 to ensure that the inequality holds true. Say we choose -90 .

$$-\frac{1}{6}(-90) - 16 \geq -4$$

Simplify.

$$\begin{aligned} 15 - 16 &\geq -4 \\ -1 &\geq -4 \end{aligned}$$

Is -1 greater than or equal to -4 ? Yes, it is greater. Thus, we see our answer is correct.

Sample Question

$$\text{Solve for } x: 2(5x + 8) < 100 - 4x$$

Start by distributing the multiplication on the left side of the inequality:

$$10x + 16 < 100 - 4x$$

Next, add $4x$ to both sides:

$$14x + 16 < 100$$

Subtract 16 from both sides:

$$14x < 84$$

Divide both sides by 14:

$$x < 6$$

Note that since we did not divide or multiply by any negative numbers, we never had to change the direction of the inequality symbol.



Sample Question

$$\text{Solve for } x: 2(5x + 8) < 100 - 4x$$

Treat this inequality as if it were an equation and solve.

$$2y < 6x + 4$$

This is our final answer. Refer to the section in Coordinate Geometry for details on how to graph this linear inequality.

$$y < 3x + 2$$

Non-Linear Inequalities

Nonlinear inequalities are easy to recognize in that they involve variables raised to exponents. Solving non-linear inequalities can be done by factoring or by using the quadratic formula. See the section in Coordinate Geometry for details on how to graph non-linear inequalities.

Sample Question

$$\text{Solve for } x: x^2 + 3 < -2x$$

First, arrange the inequality such that one of the sides is zero. In this case, this involves adding $2x$ to both sides of the inequality.

$$x^2 + 2x + 3 < 0$$

Here, we can factor to find the critical values.

$$(x + 2)(x + 1) < 0$$

Set both factors equal to zero.

$$(x + 2) = 0 \text{ and } (x + 1) = 0$$

This gives us our critical values:

$$x = -2 \text{ and } x = -1$$



Now, this divides our number line into three distinct areas, with two open circles at $x = -2$ and $x = -1$. To determine for which values of x this inequality will be true, plug in any number found within each distinct area. For example, for the area of the number line that is less than -2 , let's choose -4 . If when we plug in -4 to our two equations for the critical values: $(x + 2) = 0$ and $(x + 1) = 0$, and their quotient is positive, then that value will be true for the inequality and values in this distinct range are included in the range of our solutions.

$$(-4 + 2) = -2$$

Our answer, -2 , is negative. Plug in -4 to the other equation:

$$(-4 + 1) = -3$$

Our answer, -3 , is also negative. Thus when we multiply two negatives, we end up with a positive. This means that all values less than -2 will be included in the range of answers for x .

Repeat this procedure with the two other distinct areas of the number line using -1.5 and 0 .

$$(-1.5 + 1) = -0.5$$

Our answer, -0.5 , is negative. Plug in -1.5 into the other equation:

$$(-1.5 + 2) = 0.5$$

Our answer, 0.5 , is positive. Thus, when we multiply a negative by a positive, we end up with a negative. This means that values in between -2 and -1 will not be included in the range of answers for x .

$$(0 + 1) = 1$$

Our answer, 1 , is positive. Plug in 0 to the other equation:

$$(0 + 2) = 2$$

Our answer, 2 , is also positive. Thus, when we multiply two positives, we end up with a positive. This means that all values greater than -1 will be included in the range of answers for x .

Thus our final answer for this non-linear inequality is: $x < -2$ or $x > -1$



Solving Systems of Equations

A system of equations is comprised of two or more algebraic equations which contain the same number of unknown variables. These equations can be linear or nonlinear; however, the number of unknowns must be the same for every equation in the system. For a system of equations to exist the solution to one equation must be the solution to each and every equation in the system. To solve a system of equations, the number of equations must be greater than or equal to the number of unknowns one is trying to solve. For example, if there are two unknowns in an equation, say x and y , then there must be at least two equations in the system in order to find the solution. If an equation has x , y , and z as unknown variables, then there must be at least three equations in the system in order to solve the problem. Examples of system of equations are as follows.

Example 1

$$\begin{aligned}y + x &= 7 \\ 2y + 4x &= 22\end{aligned}$$

Example 2

$$\begin{aligned}x + y &= 5 \\ z &= 2 \\ 2x + 3z &= 18\end{aligned}$$

There are two primary ways to go about solving a system of equations: substitution and elimination.

Solving Using Substitution

Substitution refers to the process of solving one equation for one variable and then substituting that new equation in place of that variable in one of the other equations of the system. This substitution creates a new equation comprised of one variable. From here, one can perform algebraic operations to solve for the one variable. Once one variable is solved for, the solution can be substituted back into the original equation to solve for the other unknown variable. This process may need to be repeated depending on the number of unknown variables in the system. Working with example one from earlier, let's use substitution to solve the system of equations.

$$\begin{aligned}y + x &= 7 \\ 2y + 4x &= 22\end{aligned}$$

Step 1: Solve the first equation for y and then subtract x from both sides:

$$\text{The new equation for equation one is: } y = 7 - x$$

Step 2: Substitute the new equation $7 - x$ in for each y in equation two:

$$2y + 4x = 22 \rightarrow 2(7 - x) + 4x = 22$$

Step 3: Use algebraic operations to solve for x . First distribute the two to each term inside the parentheses:

$$2(7 - x) + 4x = 22$$



Combine like terms:

$$14 - 2x + 4x = 22$$

$$14 + 2x = 22$$

Subtract 14 from both sides:

$$2x = 8$$

Divide by 2 on both sides:

$$x = 4$$

Step 3: Substitute the value found for x into the y equation and solve for y :

$$y = 7 - 4$$

$$y = 3$$

Step 4: State the solution:

$$x = 4, y = 3$$

Solving Using Elimination

Elimination is the process by which adding or subtracting one equation from another equation will cancel one of the variables out, allowing one to solve for the other variable. With elimination, sometimes multiplication to one equation must occur prior to adding or subtracting the equations. Once one variable is solved for by the process of elimination, then substitute that value into the equation and solve for the other unknown variable. If there are more than two equations this process may need to be repeated in order to solve for all the unknown variables. Working with example two from earlier, let's use elimination to solve the system of equations.

$$x + y = 5$$

$$z = 2$$

$$2x + 3z = 18$$

Step 1: Subtract two times the first equation from the third equation. Notice that the x 's cancel out. (This is the elimination step.)

$$2x - 3z = 18$$

$$\underline{-2(x + y = 5)}$$

$$2x - 3z = 18$$

$$\underline{-2x - 2y = -10}$$

$$-3z - 2y = 8$$



Step 2: Substitute the second equation into the new equation found in the previous step:

$$\text{Given: } z = 2$$

$$-3z - 2y = 8 \rightarrow -3(2) - 2y = 8$$

Multiply 2 and 3 together:

$$-6 - 2y = 8$$

Subtract six from both sides:

$$-2y = 14$$

Divide by negative two on both sides:

$$y = -7$$

Step 3: Substitute the value found for y into the first equation to solve for x :

$$x + y = 5$$

$$x + (-7) = 5$$

$$x - 7 = 5$$

$$x = 12$$

Step 4: State the solution.

$$x = 12, y = -7, z = 2$$



Geometry

Geometry questions on the ACT test your ability to calculate the measurements of a wide variety of two-dimensional and three-dimensional shapes, from simple ones such as squares to complex ones that involve combinations of shapes, e.g. spheres circumscribed in cubes. While certain formulae are provided for your use on the exam, it is helpful not only to be familiar with these formulae, but to understand how they can be employed to solve a wide variety of posed problems. Working through a varied sample of geometry problems can help you reinforce what you already know and identify and remedy any gaps in your knowledge well before your test date.

Section Outline

[Plane Geometry](#)

[Squares](#)

[Area of a Square](#)

[Perimeter of a Square](#)

[Diagonal of a Square](#)

[Rectangles](#)

[Rectangular Area](#)

[Rectangular Diagonals](#)

[Triangles](#)

[Hypotenuse](#)

[Area of a Triangle](#)

[Equilateral Triangles](#)

[Isosceles Triangles](#)

[Circles](#)

[Bringing It All Together](#)

[Complex Shapes](#)

[Solid Geometry](#)

[Cubes](#)

[Cylinders](#)

[Spheres](#)

[Angles](#)

[Triangular Angles](#)

[Circular Angles](#)

[Clock Math](#)



Plane Geometry

Geometry is an ancient discipline in mathematics. If you're taking the ACT, it may seem ancient to you, as you may have taken it as your first high-school math class (or even earlier, tackling it in middle school). It might not seem quite fair that you're being required to look back over a few years and remember how to do it, since you've moved on to "bigger and better things," but geometry has many practical applications.

Then again, perhaps you think you're "not a math person." If this is the case, you may find that geometry is a fun and interesting topic by which you may gain confidence in your abilities: there's something compelling about seeing the abstractions of math in the form of shapes you can see.

Let's have a look at some of the basic geometrical forms.

Squares: In many ways, squares are the simplest of geometrical shapes. They have four sides of equal length connected by four ninety-degree angles. This means that there's very little to figure out regarding a simple square: if you know the length of one side, you know the length of all sides. Draw a diagonal line between two opposing corners of any square, and you've divided it into two identical right triangles. This diagonal will always have the same length, relative to the sides of the square. Simply multiply any side of a square by the square root of two to find the length of its diagonal.

For example, the diagonal of a square with a side length of 5 will always be $5\sqrt{2}$; a square with a side length of 41 will have a diagonal of $41\sqrt{2}$.

Triangles: These are a bit more complex than squares. A good starting place with triangles is to realize that every square can be divided along its diagonal into two identical isosceles right triangles, and that the $\sqrt{2}$ ratio actually gives us insight into something really interesting: the Pythagorean Theorem.

The Pythagorean Theorem: $a^2 + b^2 = c^2$

So, let's take a look at a triangle.

An isosceles right triangle has two sides of 3, ergo in this case, $a = b = 3$. We can substitute these values into the equation and then solve for c , the hypotenuse of the triangle and the diagonal of the square:

$$3^2 + 3^2 = c^2$$

$$18 = c^2$$

$$c = \sqrt{18}$$

$$c = \sqrt{9 \cdot 2}$$

$$c = \sqrt{9} \cdot \sqrt{2}$$

$$c = 3 \cdot \sqrt{2}$$

$$c = 3\sqrt{2}$$

The length of the diagonal is $3\sqrt{2}$.



Using Properties of 45-45-90 Triangles

The second approach relies on recognizing a 45-45-90 triangle. Although one could solve this rather easily with the Pythagorean Theorem, the following method could be faster.

45-45-90 triangles have side length ratios of $x : x : x^2$, where x represents the side lengths of the triangle's legs and x^2 represents the length of the hypotenuse.

Therefore, using the 45-45-90 triangle ratios, we have $3\sqrt{2}$ for the hypotenuse of our triangle, which is also the diagonal of our square.

As you can see, it is often the case with geometry questions that you can solve them multiple ways. Don't think of this as burdensome: having many tools in your toolbox means that you can choose the way that works best for you. It means that if you cannot remember one way of doing something, another might occur to you.

Circles: If the Pythagorean Theorem is the definitive formula for triangles, then π is the defining characteristic of circles. So what is it?

Quite simply, π is the ratio of the circumference of a circle to its diameter. It is roughly equivalent to 3.14, and it is the key to calculating the area of any circle. More than that, π enables you to calculate the volume of three-dimensional forms, like cylinders and spheres, but we'll talk about that more later.

The most important basic function of π is in calculating the area or circumference of any circle, as long as you know (or can calculate) the diameter / radius of that circle. The inverse of this statement is also true: if you know the area or circumference of a circle, you can calculate its diameter and radius. The formula for the area of a circle is $A = \pi r^2$.

Let's have a look at an example of what you can do with circles:

Sample Question

Stan is making some giant circular pies for his friend's wedding. The pies each have a diameter of 6 ft and will be cut into 64 equal pieces. Stan wants to have three pieces to himself. What is the surface area of the pie that Stan will eat if he eats three pieces?

To begin with, we need to break down the word problem to figure out exactly what is being asked of us. We are given a diameter and some clues about how much pie Stan wants to eat.

To solve this problem, we will go through a few steps. First, we have to find the area of a whole pie. Then, we have to find the area of one slice of pie. Finally, we have to multiply the area of a slice of pie by 3, since Stan wants to eat three slices of pie, not just one.

To find the area of a whole pie, we will need to recall the formula for area of a circle: $A = \pi r^2$

In this case, we aren't given r , but we do know d , which is equal to $2r$. Since $d = 6\text{ft}$, $r = 3\text{ft}$. Substituting these values into the equation for an area of a circle, we get:



$$A = \pi 3^2 = 9\pi$$

Each of the answer choices includes π , so don't change 9π into a decimal.

Next, to find the area of one slice of pie, we want to multiply the total area of the circle by the fractional area we are interested in.

We are told that one slice is equivalent to $\frac{1}{64}$ of the pie since each pie will be cut into 64 slices. We also know that Stan wants to eat three slices of pie. So, we will need to multiply the total area by $\frac{3}{64}$.

To find the total area of pie that Stan wants to eat, we will perform the following calculation:

$$9\pi \cdot \frac{3}{64} = \frac{27\pi}{64}$$

So, Stan wants to eat $\frac{27\pi}{64}$ ft² of pie.

These are some of the basic things you can do with the fundamental geometric shapes: squares, triangles, and rectangles. Let's look at each shape a little bit more in depth. Once you understand how to work with these basic shapes, you will be able to apply this knowledge practically, in order to solve more complex problems in geometry.



Squares

Let's start with some practice calculating area and perimeter, and then address the length of the diagonal.

Area of a Square

Sample Question

Jim uses 160 feet of fence to enclose a square parking lot he manages. What is the area of the lot?

The area of a square is equal to its length times its width, so we need to figure out how long each side of the parking lot is. Since a square has four sides, we calculate each side by dividing its perimeter by four.

$$\frac{160}{4} = 40$$

Each side of the square lot will use 40 feet of fence. To find the area, we multiply the length (40) by the width (40).

$$40 \text{ ft} \cdot 40 \text{ ft} = 1600 \text{ ft}^2$$

That's pretty straightforward. You can find the side of any square if you know its perimeter simply by dividing the perimeter by four. You can calculate the perimeter of any square if you know the length of a single side (or a fraction of that side, from which you derive the total length). To find the perimeter, you multiply that side by four.

You can also work this formula "backwards," finding the length of the side of a square from its area:

Sample Question

If the area of the square is 100 square units, what is, in units, the length of one side of the square?

$$\text{Area} = \text{Length} \cdot \text{Length}$$

$$100 = (\text{Length})^2$$

$$\text{Length} = \sqrt{100} = 10$$

In other words, the length of a side of a square is the square root of its area. It's easy to figure out that a square with an area of 100 also has a perimeter of 40 since, as we saw in that last problem, it has a side length of 10. Four sides of 10 makes a perimeter of 40.



Perimeter of a Square

Let's take a look at finding the area of a square from its perimeter:

Sample Question

If the perimeter of a square is 44 centimeters, what is the area of the square in square centimeters?

Since the square's perimeter is 44, each side is $\frac{44}{4} = 11$.

In order to find the area, use the definition that the $\text{Area} = \text{side}^2$.

$$11^2 = 121$$

One of the key elements in understanding geometry is realizing that what you are doing is calculating an unknown quantity from a known quantity (or quantities). With a simple shape like a square, perimeter, side length, and area work together in such a way that if you know one quantity, you can always calculate the other two.

One important distinction for the ACT, when compared to the math you've done in high school, is the fact that you don't have to worry about proofs. You don't have to prove that the side, perimeter, and area of a square exist always in a certain ratio to one another; you only have to know how to calculate an answer from a quantity of known data. This is much easier than writing proofs, and if you remember your geometry class as being less than fun, you're probably remembering the part in which you had to do proofs. We're not saying that by studying how to prove that the elements of squares stand in certain determinable ratios to each other you weren't doing something useful—it's just that the ACT isn't interested in proofs. It doesn't test you in this area. The ACT wants to test your practical knowledge of geometry, like how you can use some shapes to solve problems with other shapes, or how you use incomplete information to paint a higher-resolution picture.

Diagonal of a Square

We've looked at three elements of the square: side length, perimeter, and area. Now let's look at a fourth, slightly more complex element: the diagonal of a square.

Just like side length, perimeter, and area, the diagonal of a square stands in a direct relationship with the elements of a square. This gives us four elements. If we know any of these elements, we can calculate the other three. The key to the diagonal of the square is that it always has a ratio of the square root of 2 multiplied by the length of the side of the square.

Let's try calculating the area of a square from its diagonal based on this information.

Sample Question

What is the area of a square that has a diagonal of $2\sqrt{2}$?



In order to figure this one out, we'll need to calculate the length of the side of this square. You already know that in order to calculate the length of the diagonal from the length of the side of a square, you must multiply the side length by $\sqrt{2}$; thus, in order to work from the diagonal to the side, you have to divide the diagonal by $\sqrt{2}$.

$$\frac{2\sqrt{2}}{2}$$

In this case, that looks like this:

The numerator is the length of our diagonal; the denominator is our ratio of the side to the diagonal. We've put this in fraction form, but every fraction is really a division problem, so here we're dividing the numerator by the square root of 2.

It's an easy problem to solve: just take the parts that are alike in the numerator and the denominator, and cancel them out through division.

$$\sqrt{2} \div \sqrt{2} = 1$$

$$\frac{2 \times 1}{2}$$

Now our fraction looks like this: $\frac{2 \times 1}{2}$, which is equal to 2.

If our side length is 2, we know that the area of our square is 4, since $2 \times 2 = 4$. We also know that the perimeter of our square is 8.

You can also treat the diagonal of a square as the hypotenuse of a right isosceles triangle, since the diagonal divides the square into two such triangles.

Sample Question

The perimeter of a square is 48. What is the length of its diagonal?

$$\text{Perimeter} = \text{Side} \cdot 4$$

$$48 = \text{Side} \cdot 4$$

$$\text{Side} = 12$$

We can break up the square into two equal right triangles. The diagonal of the square is then the hypotenuse of these two triangles. Therefore, we can use the Pythagorean Theorem to solve for the diagonal:

$$\text{side}^2 + \text{side}^2 = \text{hypotenuse}^2$$

$$12^2 + 12^2 = \text{hypotenuse}^2$$

$$144 + 144 = \text{hypotenuse}^2$$

$$\sqrt{144 + 144} = \sqrt{\text{hypotenuse}^2}$$

$$\text{hypotenuse} = \sqrt{2 \cdot 144} = \sqrt{2 \cdot 12 \cdot 12} = 12\sqrt{2}$$

As you see, if you apply the Pythagorean Theorem, you actually come up with the same answer, which is to multiply the side (12) by the square root of 2, resulting in $12\sqrt{2}$.



Rectangles

Rectangles are only slightly more complex than squares. The same general set of rules applies, but this time your sides come in pairs: a rectangle will have two sets of equal sides, but each set will be different from the other. The diagonal is going to be different, too. Unlike a square, since a rectangle has no fixed relative dimensions, the diagonal has no set ratio that works for all rectangles, so you're not going to worry about $\sqrt{2}$. This is where the Pythagorean Theorem is going to be applied, because the diagonal of any rectangle is always the hypotenuse of a triangle. Let's start with the basics, however.

Rectangular Area

Area is not too difficult with rectangles, either. Just multiply the length by the width and you have your answer. A rectangle with side lengths of 10 units and 5 units has an area of 50 units squared, because $10 \times 5 = 50$. If you understand squares, you understand rectangles, at least to this point.

Rectangular Diagonals

Instead of using $\sqrt{2}$, you need to use the Pythagorean Theorem to solve for the diagonal of any rectangle. In other words, you need to recognize that the diagonal of a rectangle splits it into two equal triangles.

The Pythagorean Theorem is $a^2 + b^2 = c^2$

For our purposes, treat a as the length of the rectangle and b as the width, so that c will give you the length of the diagonal. The length and width act as the base and height of the triangle. It doesn't matter whether you assign the length to a and the width to b , or vice-versa; the only thing you have to keep straight is that c is the diagonal, no matter what.

A rectangle has a length of 12 and a width of 5; what is the length of its diagonal?

$$\begin{aligned}a &= 12 \\b &= 5\end{aligned}$$

For the Pythagorean Theorem, we need to square those values.

$$\begin{aligned}a^2 &= 144 \\b^2 &= 25\end{aligned}$$

Now we need to add those two values together, in order to find the value of c^2 .

$$\begin{aligned}144 + 25 &= 169 \\c^2 &= 169 \\c &= \sqrt{169} \\\sqrt{169} &= 13 \\c &= 13\end{aligned}$$

The length of the diagonal of this 5 by 12 rectangle is 13, which is, in fact, one of the classic triangles of plane geometry; (another is the 3-4-5 triangle—3 squared is 9; 4 squared is 16; c squared is 25; the square root of 25 is 5).



5-12-13 triangles, along with 3-4-5 triangles, are really useful to you, because they are very easy to remember. As soon as you see the elements of these triangles falling into place, you have your answer, so you can stop wasting valuable time calculating and use it somewhere else on the math section of the test.

Keep in mind that 3-4-5 and 5-12-13 triangles may be of different proportions, but the ratios remain the same. Here's an example:

A rectangle has a width of 9 and a diagonal of 15, what is its length?

That may seem like it will need a lot of complicated calculations, but it doesn't. If you know about 3-4-5 triangles, you can solve this in your head, answer, then move on. The key is recognizing the relationship of 9 and 15 as a ratio.

9 : 15 is also 3 : 5, since we can reduce each side of the ratio through division, in this case, by 3.

9 divided by 3 is 3, while 15 divided by 3 is 5. Thus, we have a rectangle with a 3 : 5 ratio between its width and its hypotenuse. Looks like a 3-4-5 triangle, but the answer isn't 4 in this case. Remember, the actual measurements of this rectangle are 9 for the width and 15 for the diagonal, so we need to take that 4 and multiply it by whatever number we divided 9 and 15 by to get our answer. We divided by 3, in order to get the simplest ratio of 9 : 15, so now we must multiply that 3 by 4 to get the answer, which is 12.

The length of the rectangle is 12.

The dimensions of its width, length, and diagonal are 9, 12, and 15, respectively. It's still a pair of classic 3-4-5 triangles, but the measurements are just larger; the proportion remains the same.

Let's try one that you'll actually have to calculate:

The sides of rectangle ABCD are 4 and 13.

How long is the diagonal of rectangle ABCD?

A diagonal of a rectangle cuts the rectangle into 2 right triangles with sides equal to the sides of the rectangle and with a hypotenuse that is the diagonal. All you need to do is use the Pythagorean Theorem:

$a^2 + b^2 = c^2$ where a and b are the sides of the rectangle and c is the length of the diagonal.

$$\sqrt{4^2 + 16^2} = \sqrt{185} = c$$



Triangles

We've already been doing triangles, but they've been not-so-well disguised as diagonals in squares and rectangles. Perimeter, area, and side-length are more difficult to discover in many triangles, and unlike squares or rectangles, triangles can have a variety of angles, not just 90 degrees. Triangles also have an important measurement, height, which may or may not correspond to the length of one of their legs. Let's consider some general rules about triangles:

- The sum of the measurement of the internal angles of a triangle must equal 180 degrees.
- The angles of a particular triangle may be 90, 45, and 45 degrees, while for another they are 178, 1, and 1.
- As long as the internal measurement of the three angles adds up to 180, you have a triangle.
- The sum of the length of any two sides must be greater than the length of the third side.

Hypotenuse

In dealing with squares and rectangles, we've already seen how to calculate the hypotenuse of a right triangle, so let's start there.

Sample Question

If the base of a right triangle is 5 units long and the height of the triangle is 7 units longer than the base, what is the length of the third side of the triangle?

Find the height of the triangle.

$$5 + 7 = 12$$

Use the Pythagorean Theorem to solve for the length of the third side, or hypotenuse.

$$\begin{aligned}c^2 &= a^2 + b^2 \\c^2 &= 5^2 + 12^2 \\c^2 &= 25 + 144 = 169 \\c &= 13\end{aligned}$$

There's that 5-12-13 right triangle again. You'll likely see it on the ACT.

Area of a Triangle

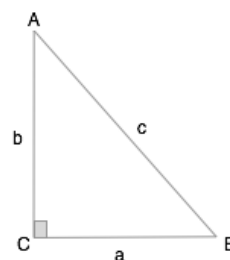
The area of any triangle can be found by multiplying one-half the base by the height. This is easiest to do in a right triangle, where the height corresponds with one of the legs.

The formula for the area of a triangle is expressed as $A = \frac{1}{2}b \cdot h$. Let's have a look at an actual problem.



Sample Question

In the triangle shown at the right, if $a = 7$ and $b = 12$ what is the area of the triangle?



Half the base is 3.5. 3.5 times 12 is 42. Note that you can choose to express this as 7×6 , or even as one half of 7×12 :

$A = \frac{1}{2}b \cdot h$, since the base is 7 and the height is 12, plugging in yields:

$$\frac{1}{2} \cdot 84 = 42$$

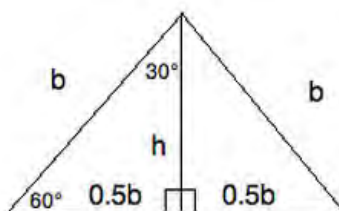
Equilateral Triangles

The above example is simple enough, especially since the height of a right triangle corresponds to one of its legs. What about those cases in which height does not correspond to one of the legs? Equilateral triangles, in which all legs are the same length, are one such case. Here is a specific example, which covers area, perimeter, and height:

Sample Question

What is the perimeter of an equilateral triangle with an area of $56.25\sqrt{3}$?

Recall that from any vertex of an equilateral triangle, you can drop a height that is a bisector of that vertex as well as a bisector of the correlative side. This gives you the following figure:



Notice that the small triangles within the larger triangle are both $30-60-90$ triangles. Therefore, you can create a ratio to help you find h .



The ratio of the small base to the height is the same as $1:\sqrt{3}$. Therefore, you can write the following equation:

$$\frac{0.5b}{h} = \frac{1}{\sqrt{3}}$$

This means that $0.5b\sqrt{3} = h$.

Now, the area of a triangle can be written $A = \frac{1}{2}bh$, and based on our data, we can replace h with $\frac{b\sqrt{3}}{2}$. This gives you:

$$56.25\sqrt{3} = \frac{1}{2} \cdot b \cdot \frac{b\sqrt{3}}{2} = \frac{b^2\sqrt{3}}{4}$$

Now, let's write that a bit more simply:

$$56.25\sqrt{3} = \frac{b^2\sqrt{3}}{4}$$

Solve for b . Begin by multiplying each side by 4:

$$225\sqrt{3} = b^2\sqrt{3}$$

Divide each side by $\sqrt{3}$:

$$b^2 = 225$$

Finally, take the square root of both sides. This gives you $b = 15$. Therefore, the perimeter is $15 \cdot 3 = 45$.

Dividing an equilateral triangle into two equal 30-60-90 triangles (30, 60, and 90 represent the angles, in degrees, of the vertices) gives us a pair of another of the classic triangles of geometry (along with the 3-4-5 and the 5-12-13): the $1:2:\sqrt{3}$. The first number is the ratio of the shortest leg, the second is the hypotenuse, and the third is the leg that forms a 90-degree angle with the short leg, here used as the height of an equilateral triangle.



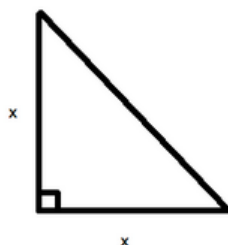
Isosceles Triangles

Another type of triangle you'll see on the ACT is the isosceles. While equilateral triangles have three sides of equal length, isosceles triangles have two sides of equal length. One of the most common you'll run across is the right isosceles triangle, which has angles at the vertices of 45, 45, and 90. Here is an example:

Sample Question

The area of an isosceles right triangle is 80in^2 . What is its height that is correlative and perpendicular to a side that is not the hypotenuse?

Recall that an isosceles right triangle is a 45 – 45 – 90 triangle. That means that it looks like this:



This makes calculating the area very easy! Recall, the area of a triangle is defined as $A = \frac{1}{2}bh$.

However, since for our triangle, $b = h = x$, we know:

$$A = \frac{1}{2}x^2$$

Now, we know that $A = 80\text{in}^2$. Therefore, we can write:

$$80 = \frac{1}{2}x^2$$

Solving for x , we get:

$$\begin{aligned} 160 &= x^2 \\ x &= \sqrt{160} = \sqrt{16} \cdot \sqrt{10} = 4\sqrt{10}\text{in} \end{aligned}$$

This is the length of the height of the triangle for the side that is not the hypotenuse.

One of the keys to the right isosceles triangle is recognizing that it always represents half of a square that has been divided along its diagonal; i.e. any two congruent right isosceles triangles can be combined into a single square. You can think of the problem above as solving for the side of a square when you are provided with half the area of that square. In other words, if you know how to solve for the perimeter of a square



Circles

based on knowing its area, you can solve the same problem for any right isosceles triangle.

The last basic form of plane geometry is the circle. Circles require you to work with π , which is roughly equal to 3.14. While that's a nice bit of information to have, it is often just better to think of π as π —i.e., as a number just as actual as 1 or 2. This can take some getting used to, but it's much more accurate to refer to π as π , not as “3.14,” and it'll actually save you time in most instances, whether you are taking the ACT or not, simply to calculate answers in terms of π .

The two most important things you can know about a circle are 1) the formula for calculating the area of a circle is $A = \pi r^2$, where r is the radius; 2) π is the ratio of the circumference of a circle to its diameter, but the formula for calculating the circumference of a circle is expressed in terms of radius as $2r\pi$.

Sample Question

A circle has a circumference of 16π . Given this information, find the area of the circle.

To find the area of a circle, we use the formula $A = \pi r^2$, where r is the radius.

However, the problem does not give us the circle's radius. In order to solve for the area we must find the radius using the circumference. Circumference of a circle follows the equation $C = 2\pi r$, so since we know the circumference, we can manipulate the equation and plug in our value to solve for radius.

$$r = \frac{C}{2\pi} = \frac{16\pi}{2\pi} = 8$$

Now that we know the radius, we simply plug it into the area formula to solve for our final answer.

$$A = \pi(8)^2 = 64\pi$$



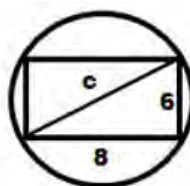
Bringing It All Together

Now let's have a look at some problems that give you the information you need, but do so in sneaky ways that require you to use everything you know about various geometrical forms. Here's an example:

Sample Question

A 6-by-8 rectangle is inscribed in a circle. What is the area of the circle?

The image below shows the rectangle inscribed in the circle.



Dividing the rectangle into two triangles allows us to find the diameter of the circle, which is equal to the length of the line we drew. Using $a^2 + b^2 = c^2$, we get $6^2 + 8^2 = c^2$. $c^2 = 100$, so $c = 10$. The area of a circle is $A = \pi r^2$. Radius is half of the diameter of the circle (which we know is 10), so $r = 5$.

Here, you need to realize that the rectangle doesn't give you all the information you need to solve for the area of the circle. If you just look at this problem, it seems immense. It seems like there's not enough information to solve it.

That's because there's not, at least if you just stare at the problem: This kind of problem wants you to **do something with it**. The Geometry questions on the math section of the ACT are very much about testing your ability to solve problems by **interacting** with geometrical forms. Interacting with something means that you're being active in your approach. The first question you should ask yourself when you see some geometric forms in a spatial relationship on the test is, "What can I add to this diagram that might give me more information?" In the above question, all you have to add is the diagonal of the rectangle. That diagonal also happens to be the diameter of the circle. Once you take that first step, everything else follows. Geometry questions reward engagement, so engage. If the question doesn't seem to give you enough information, see if you can pull some more information out of it.

Here's another example:

Sample Question

A circle with diameter of length d is inscribed in a square. Which of the following is equivalent to the area inside of the square, but outside of the circle?

In order to find the area that is inside the square but outside the circle, we will need to subtract the



area of the circle from the area of the square. The area of a circle is equal to πr^2 ; however, since we are given the length of the diameter, we will need to solve for the radius in terms of the diameter. Because the diameter of a circle is twice the length of its radius, we can write the following equation and solve for r :

$$d = 2r$$

Divide both sides by 2.

$$r = \frac{d}{2}$$

We will now substitute this into the formula for the area of the circle.

$$\text{Area of circle} = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi \frac{d}{2} \cdot \frac{d}{2} = \frac{\pi d^2}{4}$$

We next will need to find the area of the square. Because the circle is inscribed in the square, the diameter of the circle is equal to the length of the circle's side. In other words, the square has side lengths equal to d . The area of any square is equal to the square of its side length. Therefore, the area of the square is d^2 .

$$\text{Area of square} = d^2$$

Lastly, we will subtract the area of the circle from the area of the square.

$$\text{Difference in areas} = d^2 - \frac{\pi d^2}{4}$$

We will rewrite d^2 so that its denominator is 4.

$$\text{Difference in areas} = \frac{4d^2}{4} - \frac{\pi d^2}{4} = \frac{(4 - \pi)d^2}{4}$$

$$\text{The answer is } \frac{(4 - \pi)d^2}{4}.$$

This is another one that seems to ask you something almost impossible to solve in classical plane geometry: the area of the four crescent-moon shapes in the negative space of a circle inscribed in a square. The reality is, however, much simpler: the circle is inside the square, so it must be smaller in area than the square. We know the diameter of the circle, so we know the side-length of the square. Thus, we can calculate the respective areas of both the circle and the square. After that, all that's left is a simple subtraction problem. We can deduce that the answer to the subtraction problem (what is left over in the square) must be the area of the four crescent shapes.

When you see a problem like this, don't try to solve the complex features directly. Instead, look for the simpler parts of the problem that you do know how to solve. In this case, that's the area of a square and the area of a circle. Once you see the problem in terms of its simplest elements, see if you can deduce their relationship. In this case it's a matter of subtracting the area of one shape from another in order to see what is left over.



Complex Shapes

For more complex shapes, you want to think first about how you might transform them into aggregations of simpler shapes. A parallelogram can be divided into two triangles and a rectangle, such that the area of these different parts taken together represent the area of the entire figure, as in this example:

Sample Question

A parallelogram has a base of 12cm and its side is 5cm long. A line is drawn to connect the edge of the top base with the bottom base. The line is perpendicular to the bottom base, and the base of this triangle is one-fourth the length of the bottom base. Find the area of the parallelogram.

The formula for the area of a parallelogram is given by the equation $A = b \cdot h$, where b is the base and h is the height of the parallelogram.

The only given information is that the base is 12cm , the side is 5cm , and the base of the right triangle in the parallelogram (the triangle formed between the edge of the top base and the bottom base) is 3cm because $12\text{cm} \div 4 = 3\text{cm}$.

The last part of information that is required to fulfill the needs of the area formula is the parallelogram's height, h . The parallelogram's height is given by the mystery side of the right triangle described in the question. In order to solve for the triangle's third side, we can use the Pythagorean Theorem, $a^2 + b^2 = c^2$.

In this case, the unknown side is one of the legs of the triangle, so we will label it a . The given side of the triangle that is part of the base we will call b , and the side of the parallelogram is also the hypotenuse of the triangle, so in the Pythagorean Formula its length will be represented by c . At this point, we can substitute in these values and solve for a :

$$a^2 + 3^2 = 5^2$$

$$a^2 + 9 = 25$$

$$a^2 = 25 - 9$$

$$a^2 = 16$$

$$a = \sqrt{16}$$

$a \pm 4$, but because we're finding a length, the answer must be 4. The negative option can be negated.

Remembering that we temporarily called h " a " for the Pythagorean theorem, this means that $h = 4\text{cm}$.

Now all the necessary parts for the area of a parallelogram equation are available to be used:

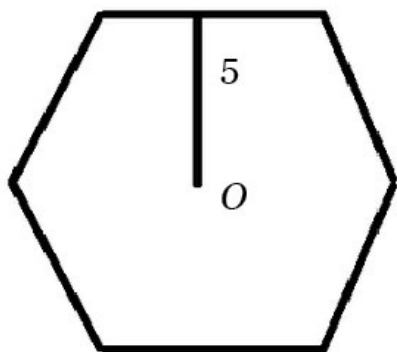
$$A = (12\text{cm})(4\text{cm})$$

$$A = 48\text{cm}^2$$



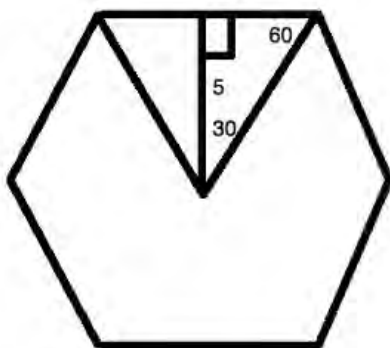
If a parallelogram is made up of two triangles and a rectangle (or, in some cases, a square), a regular hexagon can be considered a collection of equilateral triangles. It may also be considered a collection of twice as many 30-60-90 triangles, as in the following sample problem:

Sample Question



The figure to the left is a regular hexagon. O is the center of the figure. The line drawn is perpendicular to the side. What is the area of the figure above?

You can redraw the figure so it is easier to notice the equilateral triangle formed within the hexagon. Since a hexagon can have the 360 degrees of its internal rotation divided up evenly, the central angle is 60 degrees. The two angles formed with the sides also are 30 degrees. Thus, you could draw:



Now, the 5 is located on the side that is the same as $\sqrt{3}$ on your standard 30-60-90 triangle. The base of the little triangle formed here is 1 on the standard triangle. Let's call our unknown value x .

We know, then, that:

$$\frac{5}{\sqrt{3}} = \frac{x}{1}$$

Another way to write $\frac{5}{\sqrt{3}}$ is:



$$\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

Now, there are several ways you can proceed from here. Notice that there are 12 of those little triangles in the hexagon. Since you know that the area of a triangle is:

$$A = \frac{1}{2} \cdot b \cdot h$$

Substituting in the problem's specific values and rearranging to solve for b , you get the following:

$$b = \frac{5\sqrt{3}}{3}$$

The area of the whole figure is:

$$12 \cdot \left(\frac{1}{2} \cdot 5 \cdot \frac{5\sqrt{3}}{3} \right) = 50\sqrt{3} \text{ units}^2$$

If you can solve the area of one equilateral triangle, you can solve for six congruent equilateral triangles, which will give you the area of a regular hexagon.



Solid Geometry

While there isn't as much solid geometry on the ACT as there is plane geometry, it is still useful to know how to handle some of the more common shapes you will encounter, particularly cubes, cylinders, and spheres.

Cubes

A cube is just a square with a third dimension added. There are a few things you need to know:

You can calculate surface area, and a cube consists of six identical squares, which are called "sides." If you can find the area of one of these squares, you can find the surface area of the cube.

You can calculate the volume of a cube if you can figure out the length of one of its edges. Just multiply one edge by two other edges to get volume: edge $\times$ edge $\times$ edge.

You can calculate the diagonal of a cube by drawing a line through the two farthest corners of the cube. If the line goes through nearer corners, you're simply calculating the diagonal of a square. Remember how the diagonal of a square was in the ratio to the side of the square of $1 : \sqrt{2}$? The diagonal of a cube has a ratio of $1 : \sqrt{3}$, but it otherwise is calculated the same way.

If you need to calculate the total length of the edges of a cube, you can simply calculate the perimeter of the square that makes up one of its sides, then multiply the answer by 3. You might be thinking, "Why not 6, since there are 6 sides?" The reason it is only 3 is that each edge is shared between two sides, so you only need half as many edges as you'd think. This is a good tip when you're under time pressure on the test: don't double count your edge length. You can likely bet that one of the (incorrect) answers will give the doubled edge length in an attempt to draw you into an error.

Sample Question

A cube has a volume of 64cm^3 . What is the area of one side of the cube?

The cube has a volume of 64cm^3 , making the length of one edge 4cm ($4 \cdot 4 \cdot 4 = 64$).

So the area of one side is $4 \cdot 4 = 16\text{cm}^2$.

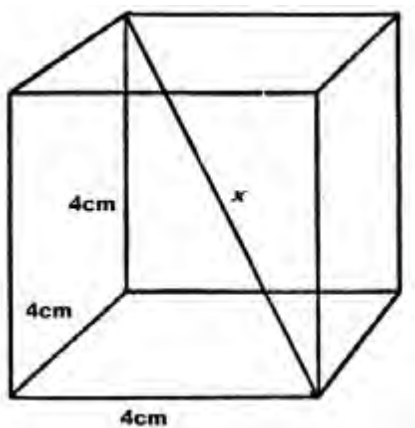
Here's how the diagonal works; it's all about $\sqrt{3}$:



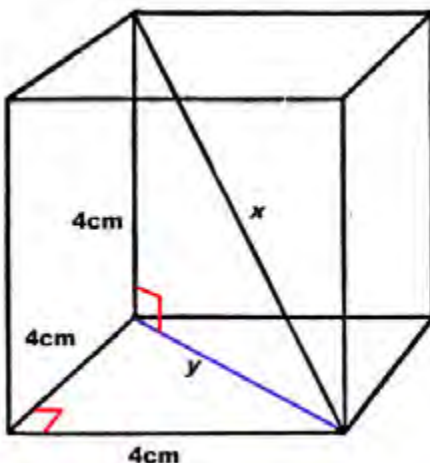
Sample Question

Find the length of the diagonal of a cube with side length of 4cm .

We begin with a picture, noting that the diagonal, labeled as x , is the length across the cube from one vertex to the opposite side's vertex.



However, the trick to solving the problem is to also draw in the diagonal of the bottom face of the cube, which we labeled y .



Note that this creates two right triangles. Though our end goal is to find x , we can begin by looking at the right triangle in the bottom face to find y . Using either the Pythagorean Theorem or the fact that we have a 45-45-90 right triangle, we can calculate the hypotenuse.

$$y = 4\sqrt{2}$$

Now that we know the value of y , we can turn to our second right triangle to find x using the Pythagorean Theorem.



$$\begin{aligned}(4)^2 + (4\sqrt{2})^2 &= x^2 \\ 16 + 32 &= x^2 \\ 48 &= x^2\end{aligned}$$

Taking the square root of both sides and simplifying gives the answer.

$$x = 4\sqrt{3}cm$$

While it is enlightening to see how that works, you really simply need to remember the ratio of the edge of a cube to its farthest diagonal is $\sqrt{3}$. If you *forget* that, now you know how to calculate it. More than that, however, this problem demonstrates again that when you have a geometry problem that seems at first too complex, start thinking about what you know what to do. You know how to find the hypotenuse of a triangle, so build on that. Sometimes that requires you to think to draw another line or two, but once you do this, things should become clearer.

Cylinders

At first glance, cylinders seem complex, but really, they are actually quite simple: take any circle and give it a third dimension: height. For cylinders, you'll be asked to calculate surface area or volume. Volume is simply the area of either circle at the ends of your cylinder multiplied by the height of the cylinder. If you can calculate the area of a circle, you'll be able to solve for the volume of a cylinder, as long as you know (or can figure out) the height.

Surface area works similarly, but is just slightly more complex. First, calculate the area of the circles at each end of the cylinder. (You really only calculate one area, then double it, since the circles are identical.) Next, calculate the circumference of the circle and multiply it by the height of the cylinder. Add this result to the area of both circles to find the surface area.

There's one other thing you'll want to know about cylinders: any number of cotangent spheres stacked one on top of the other in the length of the cylinder will **always** have a volume equivalent to $\frac{2}{3}$ of the volume of that cylinder. This can be helpful as a time-saver on the ACT test, when you don't have the time to calculate the answer to this classic problem. It can also help you if you forget how to calculate the volume of either a sphere or a cylinder, but cannot remember both.

Here's a problem that uses several elements of a cylinder together:

Sample Question

The volume of a cylinder is 81π . If the radius of the cylinder is three, what is the surface area of the cylinder?

The volume of a cylinder is equal to



$$V = \pi r^2 h$$

Use this formula and the given radius to solve for the height.

$$\begin{aligned} 81\pi &= \pi(3)^2 h \\ 81\pi &= 9\pi(h) \\ h &= 9 \end{aligned}$$

Now that we know the height, we can solve for the surface area. The surface area of a cylinder is equal to the area of the two bases plus the area of the outer surface. The outer surface can be “unwrapped” to form a rectangle with a height equal to the cylinder height and a base equal to the circumference of the cylinder base. Add the areas of the two bases and this rectangle to find the total area.

$$\begin{aligned} A &= 2A_{\text{base}} + A_{\text{rec}} \\ A &= 2\pi r^2 + (2\pi r)(h) \end{aligned}$$

Use the radius and height to solve.

$$\begin{aligned} A &= 2\pi(3)^2 + 2\pi(3)(9) \\ A &= 18\pi + 54\pi \\ A &= 72\pi \end{aligned}$$

Spheres

Spherical geometry requires you to learn a new formula or two. Here you are interested in calculating surface area and volume. For the volume of a sphere, you need to know this formula:

$$V = \frac{4}{3}\pi r^3$$

The formula for the surface area of a sphere is $A = 4\pi r^2$.

Here's an example of surface area being used to solve for volume:

Sample Question

Suppose a sphere has a surface area of $16\pi\text{cm}^2$. What is its volume?

The first step is to use the surface area formula to find the radius of the sphere.

$$\begin{aligned} A &= 4\pi r^2 \\ 16\pi &= 4\pi r^2 \\ 16 &= 4r^2 \\ 4 &= r^2 \\ 2 &= r \end{aligned}$$



The next step is to plug the value of the radius into the volume formula.

$$V = \frac{4}{3}\pi r^2$$

$$V = \frac{4}{3}\pi(2)^2$$

$$V = \frac{4}{3}\pi(8)$$

$$V = \frac{32}{3}\pi \text{ cm}^3$$

It doesn't seem easy at first, but you simply have to plug in the steps. What you need to find the volume of a sphere is its radius. If you know the surface area of a given sphere, you can work backwards from that "answer" to find what you really want: the radius. Once you've found the radius, you can plug it right in to the formula for volume.

You can always check your work by placing this sphere into a cylinder. The sphere will always have $\frac{2}{3}$ of the volume of the cylinder. For example:

We found that the radius of the sphere is 2, so the height of a cotangent cylinder must be 4 (the diameter of the sphere), so, once we find the area of the circle, we'll multiply that area by 4 to get the volume of the cylinder. $\frac{2}{3}$ of that total will equal the volume of the sphere.

$$\text{Area of a circle} = \pi r^2$$

$$\text{Area of this circle} = \pi 2^2$$

$$\text{Area of this circle} = 4\pi$$

$$\text{Volume of a cylinder} = V = \pi r^2 h$$

$$\text{Volume of this cylinder} = V = 4\pi \cdot 4$$

$$\text{Volume of this cylinder} = 16\pi$$

$$\text{Volume of the sphere} = \frac{32}{3}\pi \text{ (our answer from above)}$$

Now we need to make the denominators equal:



$$16\pi = \frac{48}{3}\pi$$

$$\frac{48}{3}\pi \cdot \frac{2}{3} = \frac{96}{9}\pi$$

$$\frac{96}{9}\pi = \frac{32}{3}\pi$$

$\frac{2}{3}$ of the volume of a cylinder is always equivalent to the volume of a cotangent sphere that it contains.

In fact, any number of such spheres will achieve the same answer, since the height of the cylinder will grow proportionally to the number of spheres it contains. When you see spheres in cylinders on the ACT test, you can shorten your time toward reaching the correct answer by remembering this relationship of volume of the two forms.

Angles

Straight lines are 180 degrees. This is useful information when you intersect such lines with other lines:

Sample Question

$\overline{AB}$ is a straight line. $\overline{CD}$ intersects $\overline{AB}$ at point E . If $\angle AEC$ measures 120 degrees, what must be the measure of $\angle BEC$?

$\angle AEC$ and $\angle BEC$ must add up to 180 degrees. So, if $\angle AEC$ is 120 degrees, $\angle BEC$ (the supplementary angle) must equal 60 degrees, for a total of 180.

Triangular Angles

Triangles have 180 degrees of internal angles, i.e., if you add up the total of all three internal angles in a triangle, they will always have the sum of 180.

Let's have a look at some examples:

Sample Question

Two interior angles in an obtuse triangle measure 123° and 11° . What is the measurement of the third angle?

Interior angles of a triangle always add up to 180 degrees, so the answer must be 46° , since $180 - 123 - 11 = 46$.

As long as you know that triangles need to have 180 degrees of total interior angles, questions like this one are simple subtraction problems.



Some triangular angle questions are actually algebra problems:

Sample Question

In an isosceles triangle the base angle is five less than twice the vertex angle. What is the sum of the vertex angle and the base angle?

Every triangle has 180 degrees. An isosceles triangle has one vertex angle and two congruent base angles.

Let x = the vertex angle

and $2x - 5$ = base angle

So the equation to solve becomes

$$x + (2x - 5) + (2x - 5) = 180$$

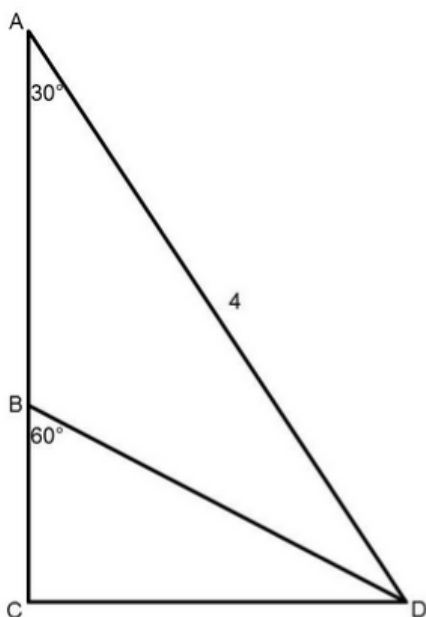
or

$$5x - 10 = 180$$

Thus, the vertex angle must be 38° and the base angle must be 71° , so their sum is 109° .

All of the questions on the ACT won't be quite so easy as this. Sometimes you're going to have to put together the information you need out of oblique data:

Sample Question



Points A, B, and C are collinear (they lie along the same line). The measure of angle CAD is 30° . The measure of angle CBD is 60° . The length of segment $\overline{AD}$ is 4. Find the measure of $\angle ADB$.



The measure of $\angle ADB$ is 30° . Since A , B , and C are collinear, and the measure of $\angle CBD$ is 60° , we know that the measure of $\angle ABD$ is 120° (since a line must be 180°).

Because the measures of the three angles in a triangle also must add up to 180° , and two of the angles in triangle are 30° and 120° , the third angle, $\angle ADB$, is 30° .

Circular Angles

Circular angles are often a bit more complex than triangular angles. You may be asked to calculate length of an arc in terms of degrees of a circle. Always remember that a circle has 360 degrees of arc.

Sample Question

What is the angle of a sector that has an arc length of $13.5in$ on a circle of diameter $12in$?

You cannot directly calculate the answer given this information using classical geometry, but you can start filling in the data that you need. When approaching geometry, always ask yourself the question, "What can I do with this information that will give me more information?" It's often up to you to figure out the steps you need to take. Practice helps you get better at thinking in this way, so don't be afraid to practice. It will hone your critical thinking and problem-solving skills.

The first thing to do for this problem is to compute the total circumference of the circle. Notice that you were given the diameter. The proper equation is therefore:

$$C = \pi d$$

For your data, this means,

$$C = 12\pi$$

Now, to compute the angle, note that you have a percentage of the total circumference based upon your arc length:

$$\frac{13.5}{12\pi} \cdot 360 = 128.9155039044336$$

Rounded to the nearest hundredth, this is 128.92° .



Clock Math

The ACT often asks you to do clock math. The test isn't asking if you know how to tell time, but whether you are able to figure out various angles and arcs of a circle. It's as simple as that. Again, recall that a circle is 360 degrees.

Sample Question

What is the measure, in degrees, of the acute angle formed by the hands of a 12-hour clock that reads exactly 3:10?

One hand is pointing at the 2, while the other is pointing at the 3, so they're $\frac{1}{12}$ of a 360-degree circle apart (there are 12 numbers on a typical clock), right? $\frac{360}{12}$ is 30, so the answer must be 30 degrees, right?

Not so fast.

The entire clock measures 360°

As the clock is divided into 12 sections, the distance between each number is equivalent to $30^\circ \left(\frac{360}{12} \right)$.

The distance between the 2 and the 3 on the clock is 30° . One has to account, however, for the 10 minutes that have passed. 10 minutes is $\frac{1}{6}$ of an hour, so the hour hand has also moved $\frac{1}{6}$ of the distance between the 3 and the 4, which adds 5° ($\frac{1}{6}$ of 30°).

The total measure of the angle, therefore, is 35° .

That's right—the hour hand is moving too, 5 degrees every ten minutes. Forgetting to account for the movement of the hour hand is what trips-up most students dealing with clock math, which is otherwise quite easy. Here's another for practice:

Sample Question

On a standard analog clock, what is the angle between the hands when the clock reads 11:20? Give the smaller of the two angles.

To find the degrees of a clock hand, first find the angle between each hour-long sections. Since there are evenly spaced sections, we find that each section has an angle of: $\frac{360^\circ}{12} = 30^\circ$. At 11:20 the hour hand has gone one-third of the way between the 11 and 12. Thus there are two-thirds of 30° between the hour hand and the 12. $\frac{2}{3} \cdot 30^\circ = 20^\circ$. There are 60° between 12 and 2, where the minute hands is. Thus there's a total of $20^\circ + 60^\circ = 80^\circ$ between the hands.



Coordinate Geometry

Coordinate geometry is the basis for all graphical study in mathematics. From the most fundamental skill of identifying a point on a coordinate plane, to graphing complex multi-axial figures the ability to visualize the significance of mathematical phenomena is a critical step in math education. The ACT has a strong focus on graphing and analyzing linear equations and their corresponding graphical representations, with a dabbling of parabolic and quadratic functions. To succeed on the exam, you will need to brush up on basic concepts like the distance formula and slope-intercept form, and delve into the more complex topics like circles and exponential functions.

Section Outline

- [X and Y Intercept](#)

- [Lines](#)

 - [Standard Form \(\$y = mx + b\$ \)](#)

 - [Distance Formula](#)

 - [Midpoint Formula](#)

 - [Parallel, Perpendicular, and Tangent Lines](#)

- [Graphing](#)

 - [Graphing Points](#)

 - [Graphing Lines](#)

 - [Graphing Exponential Functions](#)

 - [Graphing Quadratic Functions](#)

 - [Graphing Inequalities](#)



Coordinate Geometry

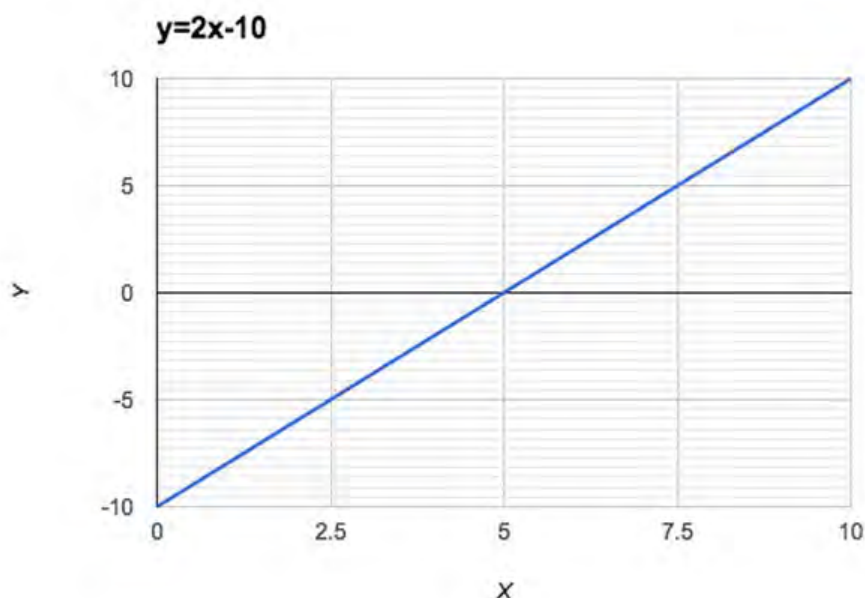
x- and y-Intercept

The concepts of the x - and y -intercepts are straightforward. The x -intercept is where a graph crosses the x -axis. The y -intercept is where a graph crosses the y -axis. As we can see, these concepts are simple; the real challenge is how to algebraically solve for the x -intercept and y -intercept.

When we are going to solve for the x -intercept and y -intercept, we have to recall what we mean by these terms. The x -intercept is where the graph crosses the x -axis and the y -intercept is where the graph crosses the y -axis. What does this mean algebraically? Algebraically, the x -intercept is the line $y = 0$, and the y -intercept is the line $x = 0$. Let's graphically see what the x -intercept and y -intercept look like and how to algebraically solve for them.

Sample Question

Solve for the x - and y -intercepts of the equation graphed below.



We can visually see from the above graph that the x -intercept is 5 and the y -intercept is -10.

Lets see how we solve for the x -intercept and y -intercept algebraically.

Let's first solve for the x -intercept. In order to solve for the x -intercept, we have to substitute 0 for y , and solve for x .

$$\begin{aligned} y &= 2x - 10 \\ 0 &= 2x - 10 \\ 10 &= 2x \\ x &= 5 \end{aligned}$$



According to our algebra, we found that the x -intercept is 5. If we look at the graph of $y = 2x - 10$, we can see that the x -intercept is indeed 5.

Now let's solve for the y -intercept. In order to solve for the y -intercept, we have to substitute 0 for x , and solve for y .

$$\begin{aligned}y &= 2x - 10 \\y &= 2(0) - 10 \\y &= 0 - 10 \\y &= -10\end{aligned}$$

We found that the y -intercept is -10. If we look at the graph for $y = 2x - 10$, we can see that the y -intercept is -10.

Let's go over a few more examples of how to solve for x - and y -intercepts.

Sample Question

Solve for the x - and y -intercepts of the following equation: $y = 10(x + 2)^2 - 2$

Let's solve for the x -intercept first. First, we need to set y equal to 0. Now our equation looks like this:

$$10(x + 2)^2 - 2 = 0$$

Now we can solve for x :

$$\begin{aligned}10(x + 2)^2 &= 2 \\(x + 2)^2 &= \frac{1}{5}\end{aligned}$$

$$x + 2 = \pm\sqrt{\frac{1}{5}}$$

$$x = \pm\frac{1}{\sqrt{5}} - 2$$

This means that we have two x -intercepts: one at $x = \frac{1}{\sqrt{5}} - 2$, and one at $x = -\frac{1}{\sqrt{5}} - 2$.

Now let's solve for the y -intercept. Substitute 0 into the equation for x and solve for y :

$$10(x + 2)^2 - 2 = 0$$



$$\begin{aligned}y &= 10(0 + 2)^2 - 2 \\y &= 10(2)^2 - 2 \\y &= 10(4) - 2 \\y &= 40 - 2 \\y &= 38\end{aligned}$$

So, our y -intercept is 38.

Sample Question

Solve for the x - and y -intercepts of the following equation: $y = x^2 - 2x + 1$

Let's solve for the y -intercept first. Set $x = 0$:

$$y = 0^2 - 2(0) + 1$$

We solve for y and get $y = 1$. The y -intercept is 1.

Now, let's solve for the x -intercept. Start by setting $y = 0$:

$$x^2 - 2x + 1 = 0$$

In order to solve for x , we need to use the quadratic formula. Remember that the quadratic formula is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this equation, a , b , and c correspond to the coefficients in the quadratic equation $ax^2 + bx + c = 0$. So, for our equation, $a = 1$, $b = -2$, and $c = 1$.

$$\begin{aligned}x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)} \\x &= \frac{2 \pm \sqrt{4 - 4}}{2} \\x &= \frac{2}{2} \\x &= 1\end{aligned}$$

We only have one x -intercept, which is 1.



Practice Problems

Find all x - and y -intercepts.

1. $y = x + 100$

2. $y = 10x + 4$

3. $y = x^2 - 6$

4. $y = x^2 - 6x + 10$

5. $y = 6x^2 - 4$

6. $y = x^3 - 2x^2 + x$

See Correct Answers and Explanations



$$0 = 10x + 4$$

$$-4 = 10x$$

$$-\frac{4}{10} = x$$

$$x = -\frac{2}{5}$$

Finding the y-intercept:

First we set $x = 0$:

$$y = 10(0) + 4$$

Now we solve for y :

$$y = 0 + 4$$

$$y = 4$$

So our x-intercept = $-\frac{2}{5}$ and y-intercept = 4.

3. $y = x^2 - 6$

Finding the x-intercept:

First we set $y = 0$:

$$0 = x^2 - 6$$

Now we solve for x :

$$0 = x^2 - 6$$

$$6 = x^2$$

$$x = \pm\sqrt{6}$$

This means that we have two x-intercepts: one at $x = \sqrt{6}$ and another at $x = -\sqrt{6}$.

Finding the y-intercept:

First we set $x = 0$:

$$y = (0)^2 - 6$$

Now we solve for y

$$y = (0)^2 - 6$$

$$y = 0 - 6$$

$$y = -6$$

So our x-intercept = $\pm\sqrt{6}$ and our y-intercept = -6.

4. $y = x^2 - 6x + 10$

Finding the x-intercept:

First we set $y = 0$:

$$0 = x^2 - 6x + 10$$



Now we solve for x . In order to solve for x , we are going to need to use the quadratic formula. Remember that the quadratic formula is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this equation, a , b , and c refer to the coefficients in the quadratic equation, $ax^2 + bx + c = 0$. In our particular case, $a = 1$, $b = -6$, and $c = 10$.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 40}}{2}$$

$$x = \frac{6 \pm \sqrt{-4}}{2}$$

$$x = 3 \pm 2i$$

Since the number under the square root is negative, we can conclude that the x -intercepts are not real answers.

Finding the y -intercept:

First, set $x = 0$:

$$y = (0)^2 - 6(0) + 10$$

Now, solve for y :

$$y = 0 - 0 + 10$$

$$y = 10$$

So our x -intercepts are not real answers, and the y -intercept = 10 .

5. $y = 6x^2 - 4$

Finding the x -intercept:

First we set $y = 0$:

$$0 = 6x^2 - 4$$

Now solve for x :



$$\begin{aligned}
 0 &= 6x^2 - 4 \\
 4 &= 6x^2 \\
 \frac{4}{6} &= x^2 \\
 \frac{2}{3} &= x^2 \\
 x &= \pm \sqrt{\frac{2}{3}}
 \end{aligned}$$

This means that we have two x -intercepts: one at $x = \frac{2}{3}$, and another at $x = -\frac{2}{3}$.

Finding the y -intercept:

First, we set $x = 0$:

$$y = 6(0)^2 - 4$$

Now, solve for y :

$$\begin{aligned}
 y &= 6(0)^2 - 4 \\
 y &= 0 - 4 \\
 y &= -4
 \end{aligned}$$

So our x -intercept $= \pm \sqrt{\frac{2}{3}}$ and our y -intercept $= -4$.

$$6. y = x^3 - 2x^2 + x$$

Finding the x -intercept:

First, we set $y = 0$:

$$0 = x^3 - 2x^2 + x$$

Now, factor an x on the right hand side:

$$\begin{aligned}
 0 &= x^3 - 2x^2 + x \\
 0 &= x(x^2 - 2x + 1)
 \end{aligned}$$

After factoring an x , we can already determine one of the x -intercepts, which is $x = 0$.

Now we solve for x within the parentheses. In order to solve for x within the parenthesis, we are going to need to use the quadratic formula. Remember that the quadratic formula is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this equation, a , b , and c refer to the coefficients in the quadratic equation, $ax^2 + bx + c = 0$. In our particular case $a = 1$, $b = -2$, and $c = 1$.



$$x = \frac{-(-2) \pm \sqrt{(2)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - 4}}{2}$$

$$x = \frac{2 \pm \sqrt{0}}{2}$$

$$x = \frac{2}{2}$$

$$x = 1$$

So we have two x -intercepts: one at $x = 0$, and one at $x = 1$.

Finding the y -intercept:

First, we set $x = 0$:

$$y = (0)^3 - 2(0)^2 + 0$$

Now, solve for y :

$$y = 0 - 0 + 0$$

$$y = 0$$

So our first x -intercept = 0, our second x -intercept = 1, and our y -intercept = 0.



Lines

Distance Formula

Have you been curious how far 2 points are from each other? Well there is a mathematical way to calculate the distance between 2 points. This is called the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In this formula, d is distance, (x_1, y_1) corresponds to the coordinates of point one, and (x_2, y_2) corresponds to the coordinates of point two.

Sample Question

Find the distance between points $(1, 2)$ and $(3, 4)$.

We just substitute $x_1 = 1$, $y_1 = 2$, $x_2 = 3$, and $y_2 = 4$ into the distance formula and solve for d .

$$d = \sqrt{(3-1)^2 + (4-2)^2}$$

$$d = \sqrt{(2)^2 + (2)^2}$$

$$d = \sqrt{4+4}$$

$$d = \sqrt{8}$$

$$d = \sqrt{4 \cdot 2}$$

$$d = 2\sqrt{2}$$

The distance between these two points is $2\sqrt{2}$ units.

A nice attribute of the distance formula is it doesn't matter which point you pick as "point 1" and "point 2." To show this, we will do the calculation for Example 1 again, but switch the points.

Substitute $x_1 = 3$, $y_1 = 4$, $x_2 = 1$, and $y_2 = 2$:

$$d = \sqrt{(1-3)^2 + (2-4)^2}$$

$$d = \sqrt{(-2)^2 + (-2)^2}$$

$$d = \sqrt{4+4}$$

$$d = \sqrt{8}$$

$$d = \sqrt{4 \cdot 2}$$

$$d = 2\sqrt{2}$$

As we can see, we got the exact same answer for d .



Sample Question

Find the distance between the points $(0,0)$ and $(-5,2)$.

Substitute $x_1 = 0$, $y_1 = 0$, $x_2 = -5$, and $y_2 = 2$ and solve for d .

$$d = \sqrt{(-5 - 0)^2 + (2 - 0)^2}$$

$$d = \sqrt{(-5)^2 + (2)^2}$$

$$d = \sqrt{25 + 4}$$

$$d = \sqrt{29}$$

Practice Problems

Find the distance between these points.

1. $(1,1)$, $(2,2)$
2. $(1,3)$, $(10, 10)$
3. $(0,-1)$, $(-1,-1)$
4. $(2,-3)$, $(1, -2)$
5. $(5, 6)$, $(-10, 0)$
6. $(1,-8)$, $(2, 4)$



Practice Problems Explanations and Answers

1. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (1,1) and point 2 = (2,2). Substitute $x_1 = 1$, $y_1 = 1$, $x_2 = 2$, and $y_2 = 2$ into the distance formula and solve:

$$d = \sqrt{(2-1)^2 + (2-1)^2}$$

$$d = \sqrt{(1)^2 + (1)^2}$$

$$d = \sqrt{2}$$

2. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (1,3) and point 2 = (10,10). Substitute $x_1 = 1$, $y_1 = 3$, $x_2 = 10$, and $y_2 = 10$ into the distance formula and solve:

$$d = \sqrt{(10-1)^2 + (10-3)^2}$$

$$d = \sqrt{(9)^2 + (7)^2}$$

$$d = \sqrt{81+49}$$

$$d = \sqrt{130}$$

3. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (0,-1), and point 2 = (-1,-1). Substitute $x_1 = 0$, $y_1 = -1$, $x_2 = -1$, and $y_2 = -1$ into the distance formula and solve:

$$d = \sqrt{(-1-0)^2 + (-1-(-1))^2}$$

$$d = \sqrt{(-1)^2 + (0)^2}$$

$$d = \sqrt{1+0}$$

$$d = 1$$

4. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (2,-3), and point 2 = (1,-2). Substitute $x_1 = 2$, $y_1 = -3$, $x_2 = 1$, and $y_2 = -2$ into the distance equation and solve:

$$d = \sqrt{(1-2)^2 + (-2-(-3))^2}$$

$$d = \sqrt{(-1)^2 + (1)^2}$$

$$d = \sqrt{1+1}$$

$$d = \sqrt{2}$$



5. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (5,6) and point 2 = (-10,0). Substitute $x_1 = 5$, $y_1 = 6$, $x_2 = -10$, and $y_2 = 0$ into the distance formula and solve:

$$d = \sqrt{(-10 - 5)^2 + (0 - 6)^2}$$

$$d = \sqrt{(-15)^2 + (-6)^2}$$

$$d = \sqrt{225 + 36}$$

$$d = \sqrt{261} = 3\sqrt{29}$$

6. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (1,-8) and point 2 = (2,4). Substitute $x_1 = 1$, $y_1 = -8$, $x_2 = 2$, and $y_2 = 4$ into the distance formula and solve:

$$d = \sqrt{(2 - 1)^2 + (4 - (-8))^2}$$

$$d = \sqrt{(1)^2 + (12)^2}$$

$$d = \sqrt{1 + 144}$$

$$d = \sqrt{145}$$

Midpoint Formula

The next formula finds the midpoint between two points:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

In this formula, (x_1, y_1) corresponds to the coordinates of point one and (x_2, y_2) corresponds to the coordinates of point two.

Sample Question

Find the midpoint between (1,2) and (3,4).

We just substitute $x_1 = 1$, $y_1 = 2$, $x_2 = 3$, and $y_2 = 4$ into the midpoint formula.

$$\left(\frac{1+3}{2}, \frac{2+4}{2} \right)$$

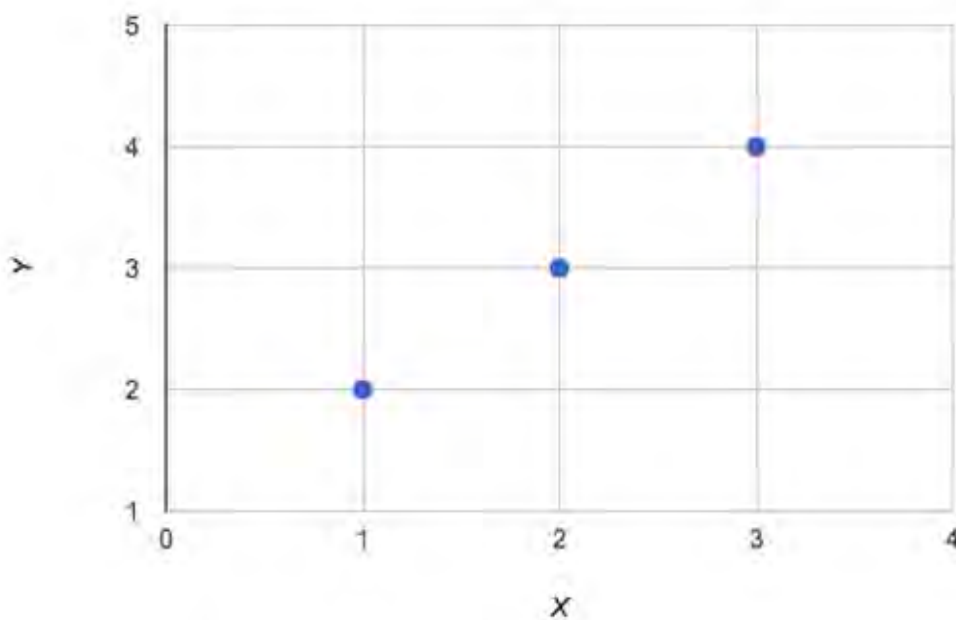
$$\left(\frac{4}{2}, \frac{6}{2} \right)$$

$$(2,3)$$



Thus, the midpoint between $(1,2)$ and $(3,4)$ is $(2,3)$.

Below, we can see the midpoint between $(1,2)$ and $(3,4)$ graphically.



We can see that $(2,3)$ is indeed the midpoint.

Sample Question

Find the midpoint between $(0,25)$ and $(3,46)$.

We just substitute $x_1 = 0$, $y_1 = 25$, $x_2 = 3$, and $y_2 = 46$ into the midpoint formula.

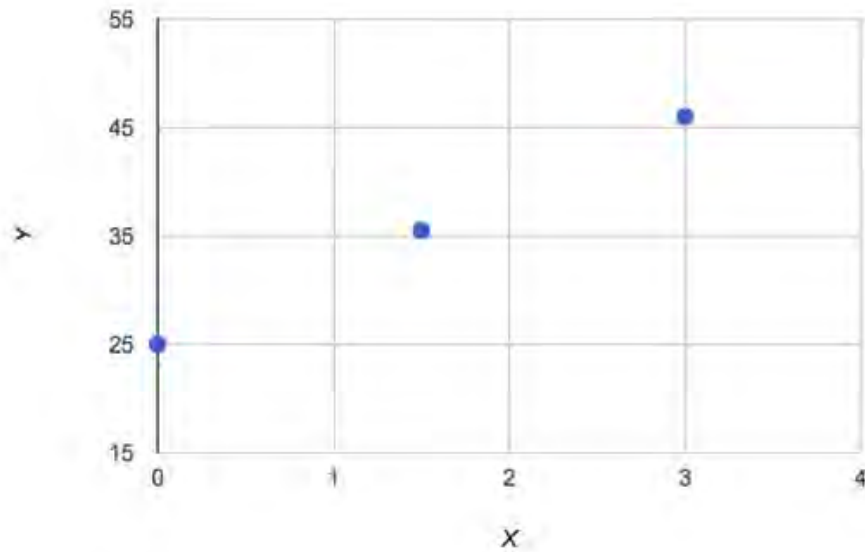
$$\frac{0+3}{2}, \frac{25+46}{2}$$

$$\frac{3}{2}, \frac{71}{2}$$

$$(1.5, 35.5)$$

Below, we can see the midpoint between $(0,25)$ and $(3,46)$.





We can see that $(1.5, 35.5)$ is the midpoint.

Practice Problems

Find the midpoint between these points:

1. $(1, 1)$, $(2, 2)$
2. $(1, 3)$, $(10, 10)$
3. $(0, -1)$, $(-1, -1)$
4. $(2, -3)$, $(1, -2)$
5. $(5, 6)$, $(-10, 0)$
6. $(1, -8)$, $(2, 4)$



Practice Problem Answers and Explanations

1. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (1,1) and point 2 = (2,2). Substitute $x_1 = 1$, $y_1 = 1$, $x_2 = 2$, and $y_2 = 2$ into the midpoint formula and solve:

$$\left(\frac{1+2}{2}, \frac{1+2}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$$

The midpoint is therefore $\left(\frac{3}{2}, \frac{3}{2}\right)$.

2. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (1,3) and point 2 = (10,10). Substitute $x_1 = 1$, $y_1 = 3$, $x_2 = 10$, and $y_2 = 10$ into the midpoint formula and solve:

$$\left(\frac{1+10}{2}, \frac{3+10}{2}\right) = \left(\frac{11}{2}, \frac{13}{2}\right)$$

The midpoint is therefore $\left(\frac{11}{2}, \frac{13}{2}\right)$.

3. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (0,-1) and point 2 = (-1,-1). Substitute $x_1 = 0$, $y_1 = -1$, $x_2 = -1$, and $y_2 = -1$ into the midpoint formula and solve:

$$\left(\frac{0+(-1)}{2}, \frac{-1+(-1)}{2}\right) = \left(-\frac{1}{2}, -1\right)$$

The midpoint is therefore $\left(-\frac{1}{2}, -1\right)$.

4. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (2,-3) and point 2 = (1,-2). Substitute $x_1 = 2$, $y_1 = -3$, $x_2 = 1$, and $y_2 = -2$ into the midpoint formula and solve:

$$\left(\frac{2+1}{2}, \frac{-2+(-3)}{2}\right) = \left(\frac{3}{2}, -\frac{5}{2}\right)$$

The midpoint is therefore $\left(\frac{3}{2}, -\frac{5}{2}\right)$.



5. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (5,6) and point 2 = (-10,0). Substitute $x_1 = 5$, $y_1 = 6$, $x_2 = -10$, and $y_2 = 0$ into the midpoint formula and solve:

$$\left(\frac{5+(-10)}{2}, \frac{6+0}{2} \right) = \left(-\frac{5}{2}, 3 \right)$$

The midpoint is therefore $\left(-\frac{5}{2}, 3 \right)$.

6. Pick one of the points to be “point 1” and the other to be “point 2.” Let point 1 = (1,-8) and point 2 = (2,4). Substitute $x_1 = 1$, $y_1 = -8$, $x_2 = 2$, and $y_2 = 4$ into the midpoint formula and solve:

$$\left(\frac{1+2}{2}, \frac{-8+4}{2} \right) = \left(\frac{3}{2}, -2 \right)$$

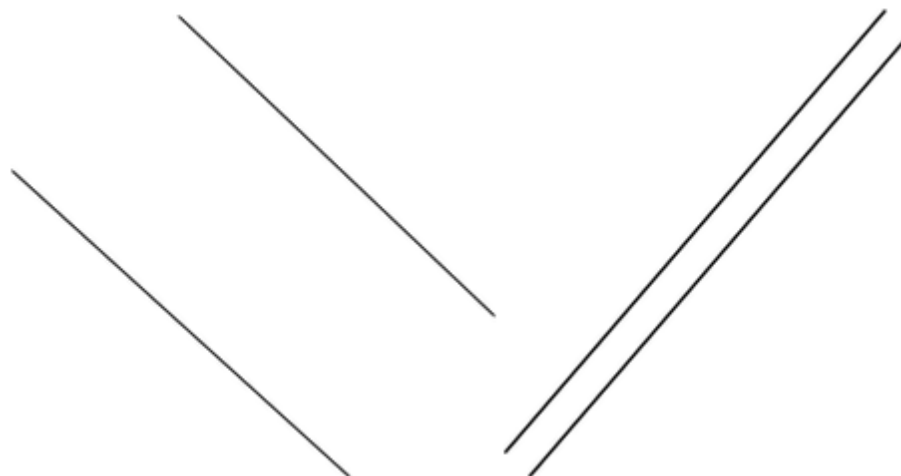
The midpoint is therefore $\left(\frac{3}{2}, -2 \right)$.

Parallel, Perpendicular, and Tangent Lines

There are 3 types of lines that we should be aware of: parallel lines, perpendicular lines, and tangent lines. Each of these is unique.

Let's first discuss parallel lines. Parallel lines are lines that will never cross or touch each other and are the same distance apart.

Here are examples of parallel lines:



We can see that they don't touch or cross at any point, and they are the same distance apart.

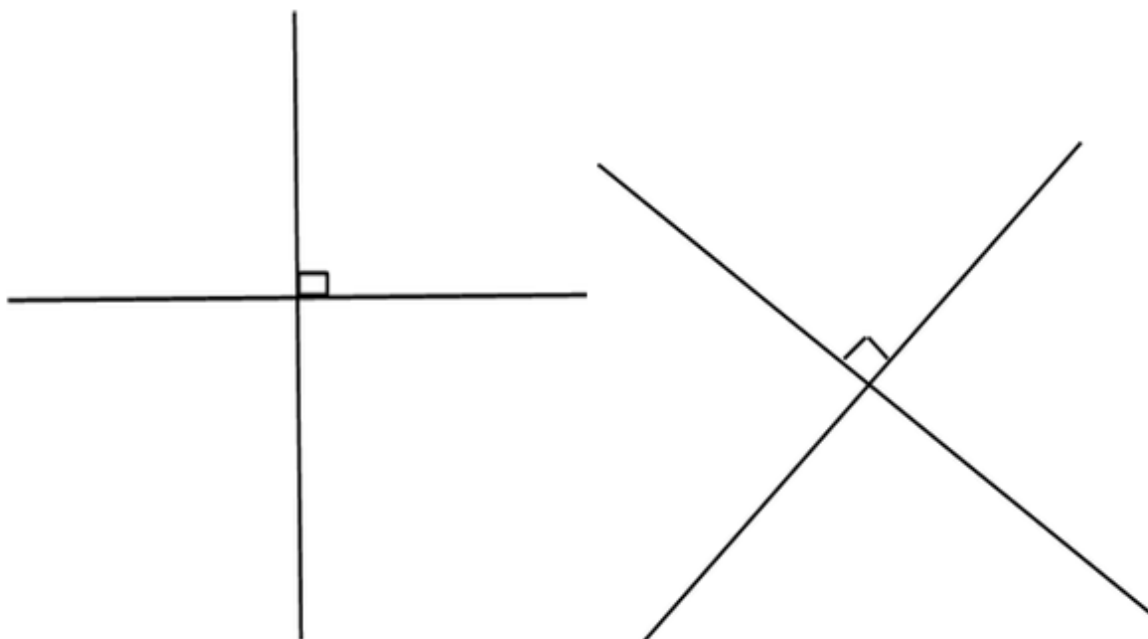
Equations of parallel lines have the same slopes.

Example: $y = \frac{1}{2}x + 3$

$$y = \frac{1}{2}x + 7$$

Perpendicular lines are a different story than parallel lines. Perpendicular lines are lines that intersect each other at a 90° angle.

Here are examples of perpendicular lines.



We can see that these lines intersect each other at a 90° angle.

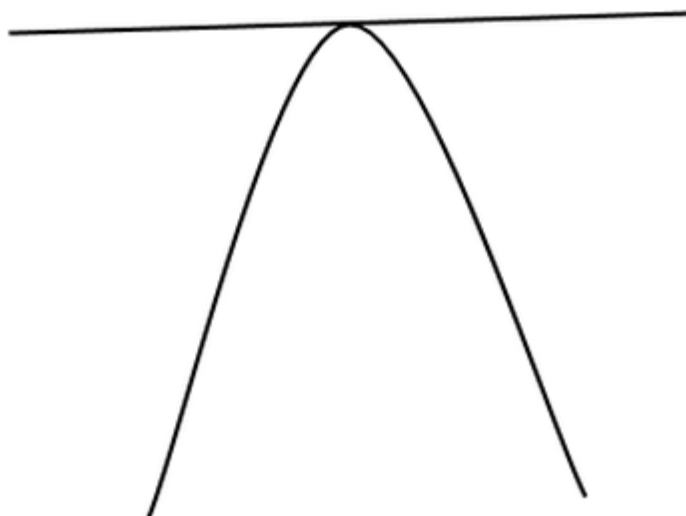
As equations, these lines will have opposite reciprocal slopes.

Example: $y = \frac{1}{4}x + 8$

$$y = -4x + 2$$

Tangent lines are lines that touch a curve at one particular point.



Graphing

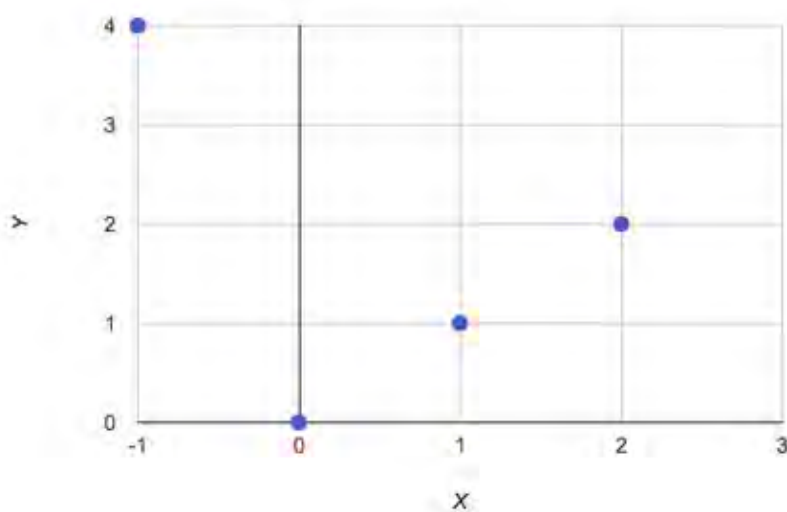
We can see that the line touches the curve at one point.

Graphing Points

Graphing points is straightforward. We look at the x - and y -coordinate of a given point, which correspond to the x and y values on a graph.

Sample Question

Plot the points $(0,0)$, $(1,1)$, $(2,2)$, and $(-1, 4)$.



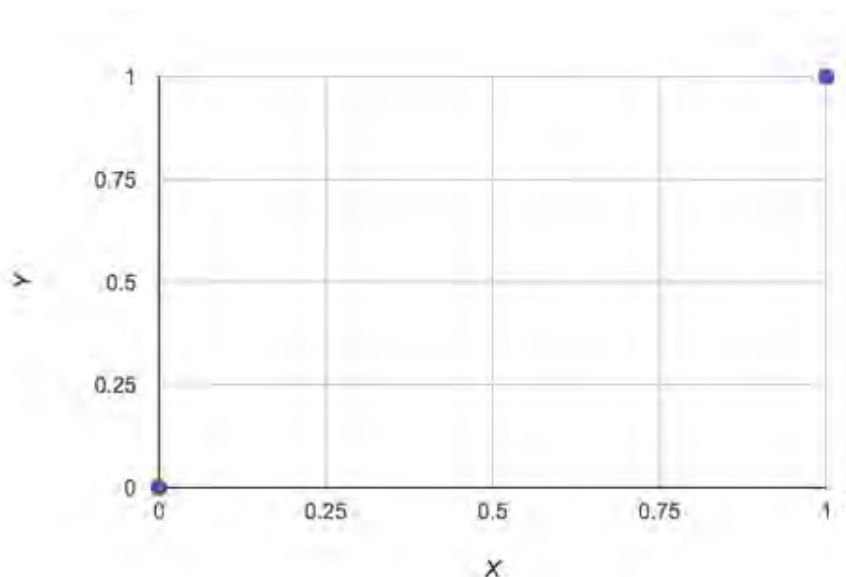
If we look at the above graph, we can see that each point is plotted. To plot a point like $(-1, 4)$, we go to -1 on the x -axis, and then go to 4 on the y -axis. Then we proceed to do the same process with the other points.

Practice Problems

Plot these points:

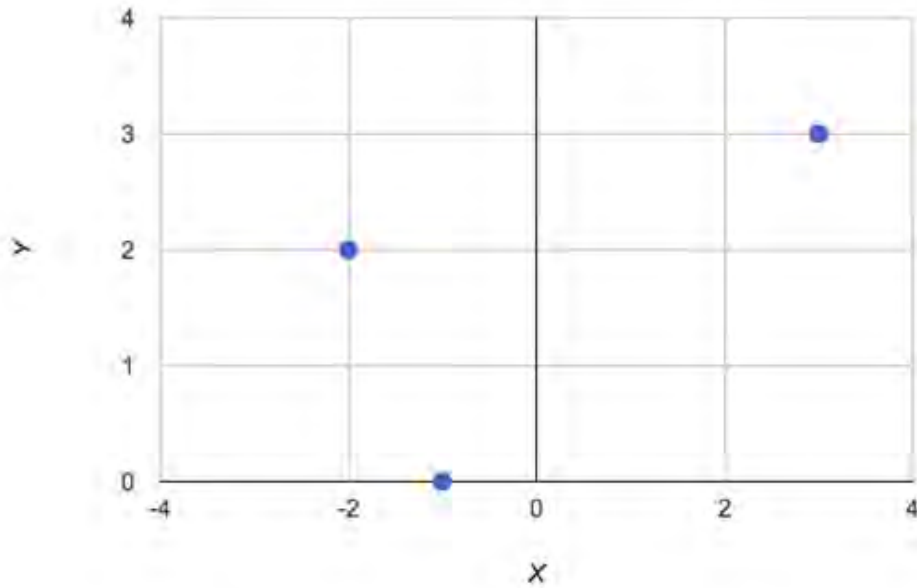
1. $(0,0)$, $(1,1)$
2. $(-2,2)$, $(3,3)$, $(-1,0)$
3. $(0,2)$, $(2,0)$, $(-2,0)$, $(0,-2)$
4. $(6,6)$, $(10,3)$, $(4,4)$, $(0,0)$

1. First plot $(0,0)$, which is located at $x=0$, $y=0$, then plot $(1,1)$, which is located at $x=1$, $y=1$. The plotting can be seen below.

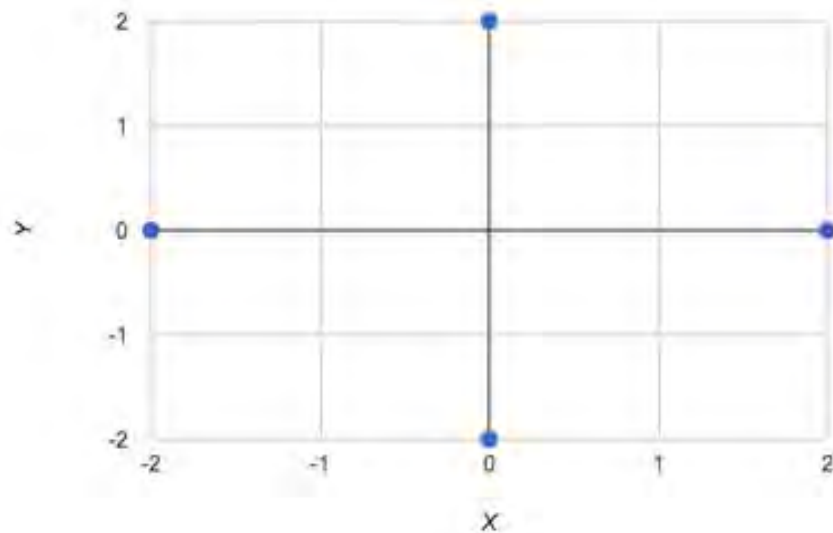


2. First plot $(-2,2)$, which is located at $x=-2$, $y=2$, then plot $(3,3)$, which is located at $x=3$, $y=3$, then plot $(-1,0)$, which is located at $x=-1$, and $y=0$. The plotting can be seen below.



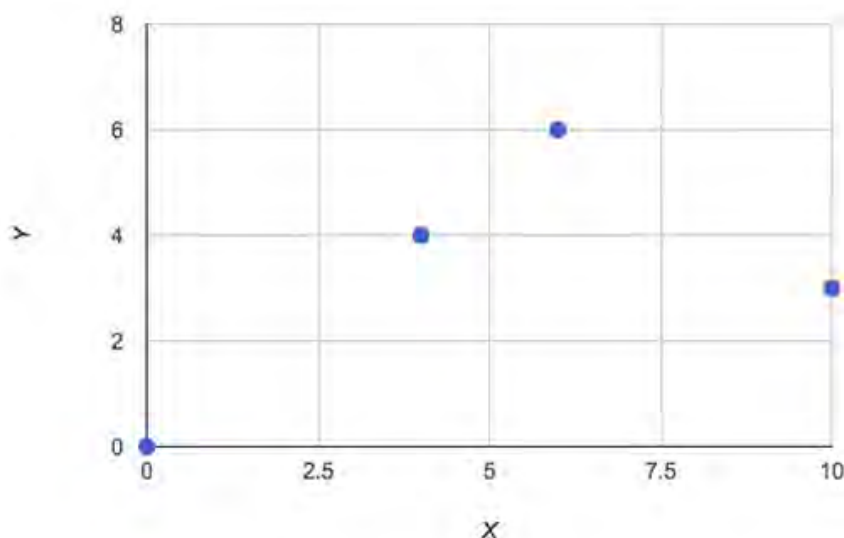


3. First plot $(0,2)$, which is located at $x=0$, $y=2$, then plot $(2,0)$, which is located at $x=2$, $y=0$, then plot $(-2,0)$, which is located at $x=-2$, $y=0$, and then lastly plot $(0,-2)$, which is located at $x=0$, $y=-2$. The plotting can be seen below.



4. First plot $(6,6)$, which is located at $x=6$, $y=6$, then plot $(10,3)$, which is located at $x=10$, $y=3$, then plot $(4,4)$, which is located at $x=4$, $y=4$, and then lastly plot $(0,0)$, which is located at $x=0$, $y=0$. The plotting can be seen below.





Graphing Lines

Graphing lines is bit different because typically you will be given an equation and you have to come up with your own points which go with the equation.

Sample Question

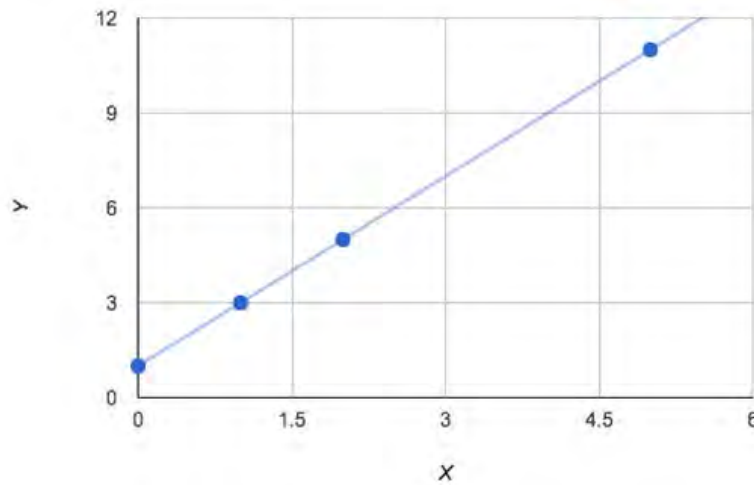
Plot $y = 2x + 1$.

As we can see, we have an equation and no points to plot on a graph. In order to plot this equation, we have to choose x -values and plug them into our equation. We will create a table of values for this example.

X-Value	Y-Value	(X,Y)
0	$2(0) + 1 = 1$	(0,1)
1	$2(1) + 1 = 3$	(1,3)
2	$2(2) + 1 = 5$	(2,5)
5	$2(5) + 1 = 11$	(5,11)

Now we can plot the points that are on the line on a graph.





After we plot the points, we can draw a line through the points we plotted.

Sample Question

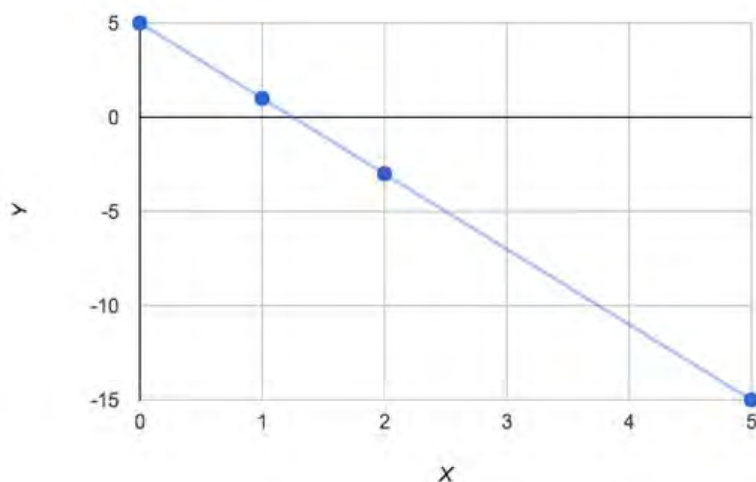
Plot $y = -4x + 5$.

In order to plot this equation, we have to choose x -values and plug them into our equation. We will create a table of values for this example.

X-Value	Y-Value	(X,Y)
0	$-4(0) + 5 = 5$	(0,5)
1	$-4(1) + 5 = 1$	(1,1)
2	$-4(2) + 5 = -3$	(2,-3)
5	$-4(5) + 5 = -15$	(5,-15)

Now we can plot the points that are on the line on a graph.





After we plot the points, we can draw a line through the points we plotted.

Practice Problems

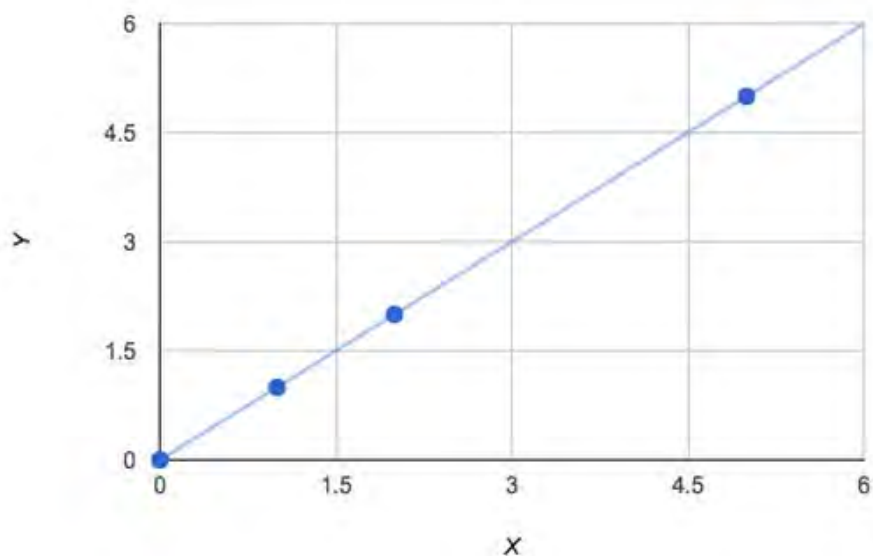
Plot these lines:

1. $y = x$
2. $y = 3x + 2$
3. $y = 5x - 10$
4. $y = 50x - 2$
5. $y = x - 1$
6. $y = -x + 5$

1. In order to plot $y = x$, we need to create a table of coordinates, and then plot the coordinates, and lastly put a straight line through them.

X-Value	Y-Value	(X,Y)
0	0	(0,0)
1	1	(1,1)
2	2	(2,2)
5	5	(5,5)

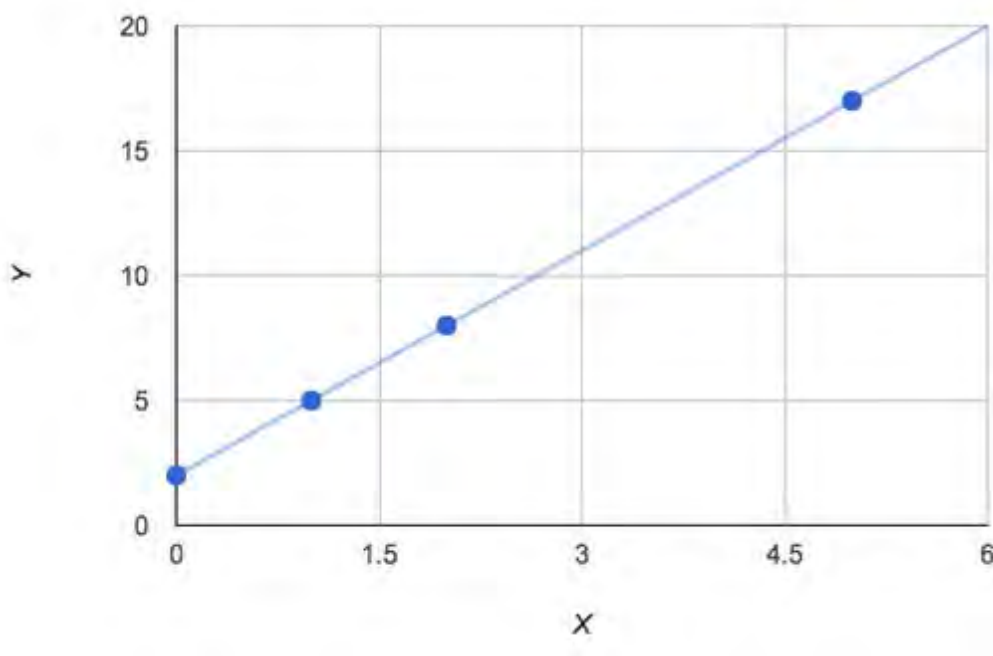




2. In order to plot $y = 3x + 2$, we need to create a table of coordinates, and then plot the coordinates, and lastly put a straight line through them.

X-Value	Y-Value	(X,Y)
0	$3(0) + 2 = 2$	(0,2)
1	$3(1) + 2 = 5$	(1,5)
2	$3(2) + 2 = 8$	(2,8)
5	$3(5) + 2 = 17$	(5,17)

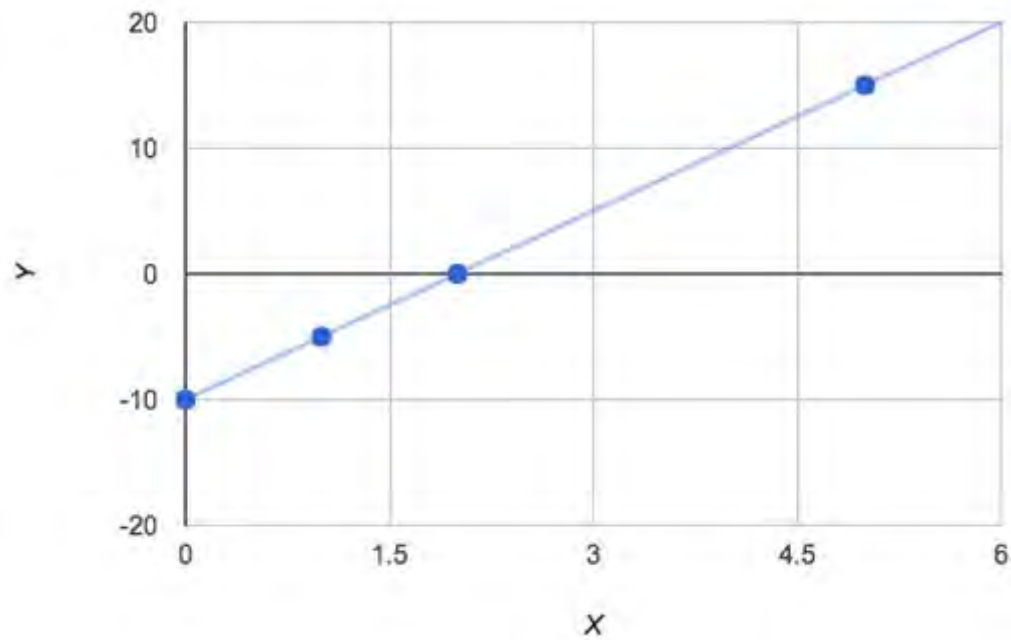




3. In order to plot $y = 5x - 10$, we need to create a table of coordinates, and then plot the coordinates, and lastly put a straight line through them.

X-Value	Y-Value	(X,Y)
0	$5(0) - 10 = -10$	(0, -10)
1	$5(1) - 10 = -5$	(1, -5)
2	$5(2) - 10 = 0$	(2,0)
5	$5(5) - 10 = 15$	(5,15)

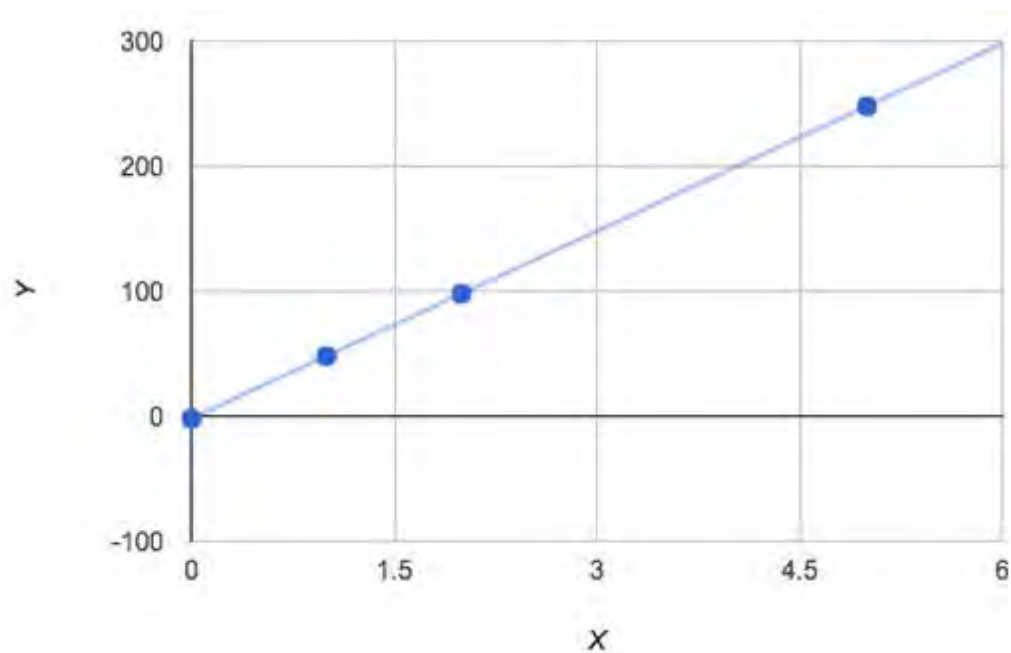




4. In order to plot $y = 50x - 2$, we need to create a table of coordinates, and then plot the coordinates, and lastly put a straight line through them.

X-Value	Y-Value	(X,Y)
0	$50(0) - 2 = -2$	(0, -2)
1	$50(1) - 2 = 48$	(1, 48)
2	$50(2) - 2 = 98$	(2, 98)
5	$50(5) - 2 = 248$	(5, 248)

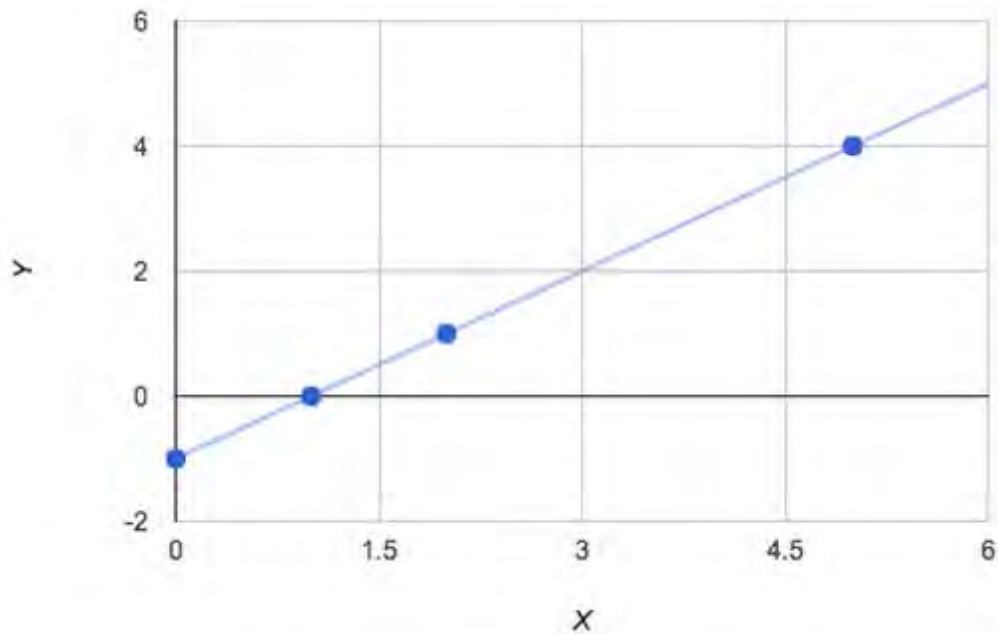




5. In order to plot $y = x - 1$, we need to create a table of coordinates, and then plot the coordinates, and lastly put a straight line through them.

X-Value	Y-Value	(X,Y)
0	$(0) - 1 = -1$	(0, -1)
1	$(1) - 1 = 0$	(1, 0)
2	$(2) - 1 = 1$	(2, 1)
5	$(5) - 1 = 4$	(5, 4)

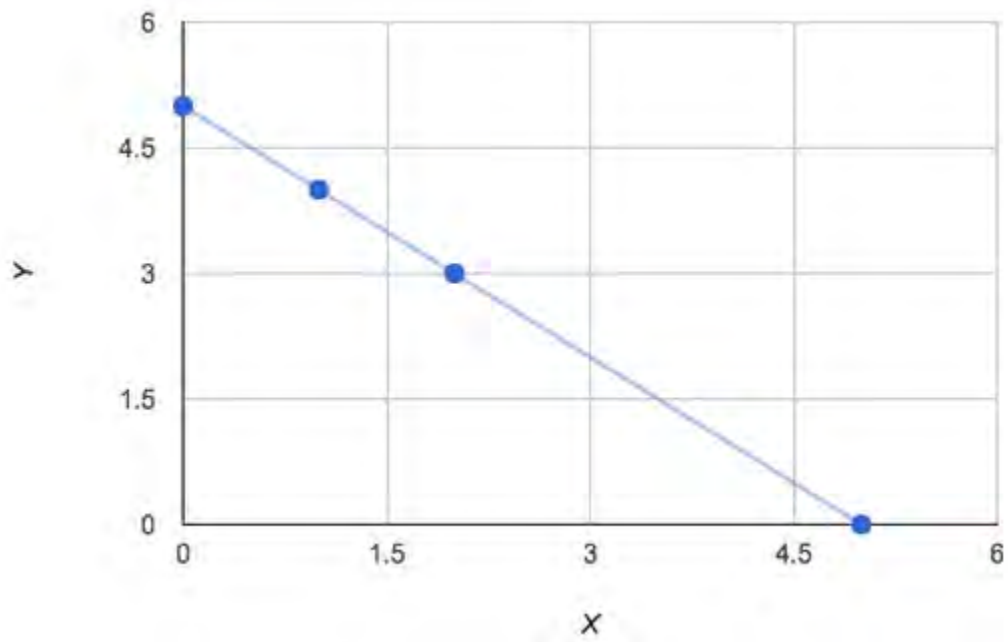




6. In order to plot $y = -x + 5$, we need to create a table of coordinates, and then plot the coordinates, and lastly put a straight line through them.

X-Value	Y-Value	(X,Y)
0	$-0 + 5 = 5$	(0, 5)
1	$(-1) + 5 = 4$	(1, 4)
2	$(-2) + 5 = 3$	(2, 3)
5	$(-5) + 5 = 0$	(5, 0)





Graphing Quadratic Functions

Graphing quadratic functions is the same process as graphing lines. Rather than having a line in our graph, we will have a curved line.

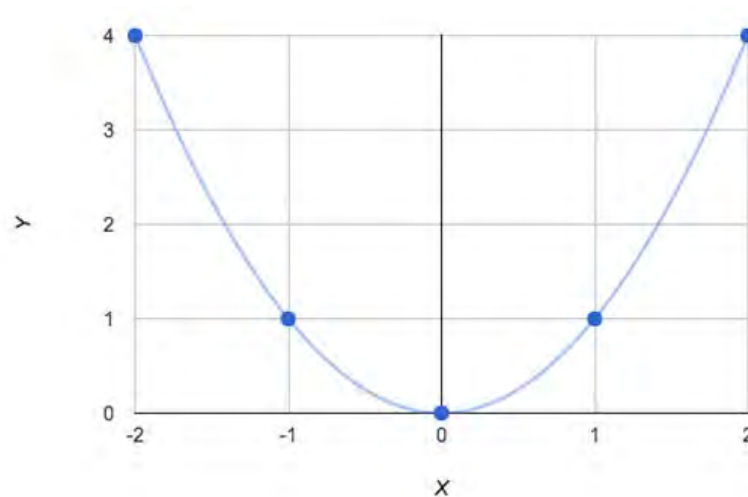
Sample Question

Plot $y = x^2$.

In order to plot this equation, we have to choose x -values and plug them into our equation. We will create a table of values for this example.

X-Value	Y-Value	(X,Y)
-2	$(-2)^2 = 4$	$(-2, 4)$
-1	$(-1)^2 = 1$	$(-1, 1)$
0	$0^2 = 0$	$(0, 0)$
1	$1^2 = 1$	$(1, 1)$
2	$2^2 = 4$	$(2, 4)$

After we plot the points, we can draw a curved line through the points we plotted.

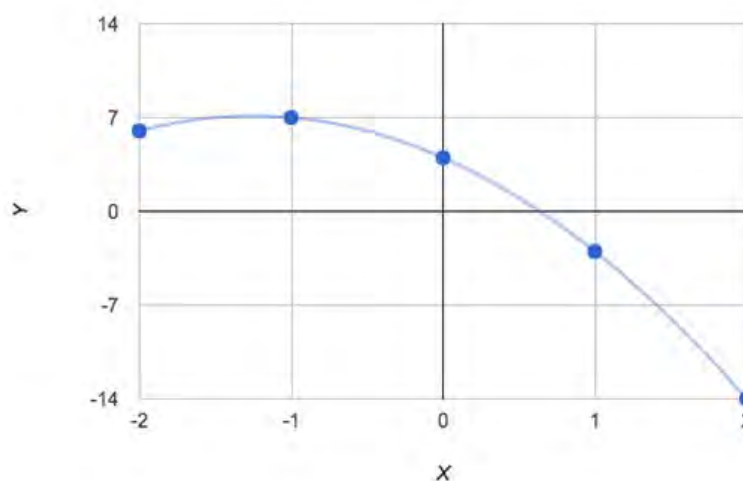


Sample Question

$$\text{Plot } y = -2x^2 - 5x + 4.$$

X-Value	Y-Value	(X,Y)
-2	$-2(-2)^2 - 5(-2) + 4 = 6$	$(-2, 6)$
-1	$-2(-1)^2 - 5(-1) + 4 = 7$	$(-1, 7)$
0	$-2(0)^2 - 5(0) + 4 = 4$	$(0, 4)$
1	$-2(1)^2 - 5(1) + 4 = -3$	$(1, -3)$
2	$-2(2)^2 - 5(2) + 4 = -14$	$(2, -14)$

After we plot the points, we can draw a curved line through the points we plotted.



There is one more point that we should know how to calculate, the vertex. The vertex is where the quadratic function changes direction (upwards to downwards or vice versa). Most of the time in order to find the vertex, you have to solve algebraically; sometimes you can spot it on the graph of the function. We will now go into how to find the vertex algebraically.

The vertex is described as a point (h, k) , where $h = \frac{-b}{2a}$, and a and b refer to the coefficients in the quadratic equation $ax^2 + bx + c = 0$. k is found by plugging in h into the quadratic function. Let's go step-by-step from our previous examples and find their vertices.



Vertex for Example 1:

Our equation is $y = x^2$. In order to find h , we need to determine what a and b are. In this particular case, $a = 1$ and $b = 0$. Thus $h = -\frac{0}{2(1)} = 0$. Now we can plug in h into $y = x^2$ in order to get k . $y = 0^2 = 0$. Now we have done all the calculations for the vertex, so our vertex is therefore $(0, 0)$.

Vertex for Example 2:

Our equation is $y = -2x^2 - 5x + 4$. In order to find h , we need to determine what a and b are. In this particular case, $a = -2$ and $b = -5$. Thus $h = \frac{-(-5)}{2(-2)} = \frac{5}{4}$. Now we can plug in h into $y = -2x^2 - 5x + 4$ in order to get k . $y = -2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + 4 = -\frac{43}{8}$. Now we have done all the calculations for the vertex, so our vertex is therefore $\left(\frac{5}{4}, -\frac{43}{8}\right)$.

Practice Problems

Plot these quadratic functions and find their vertices:

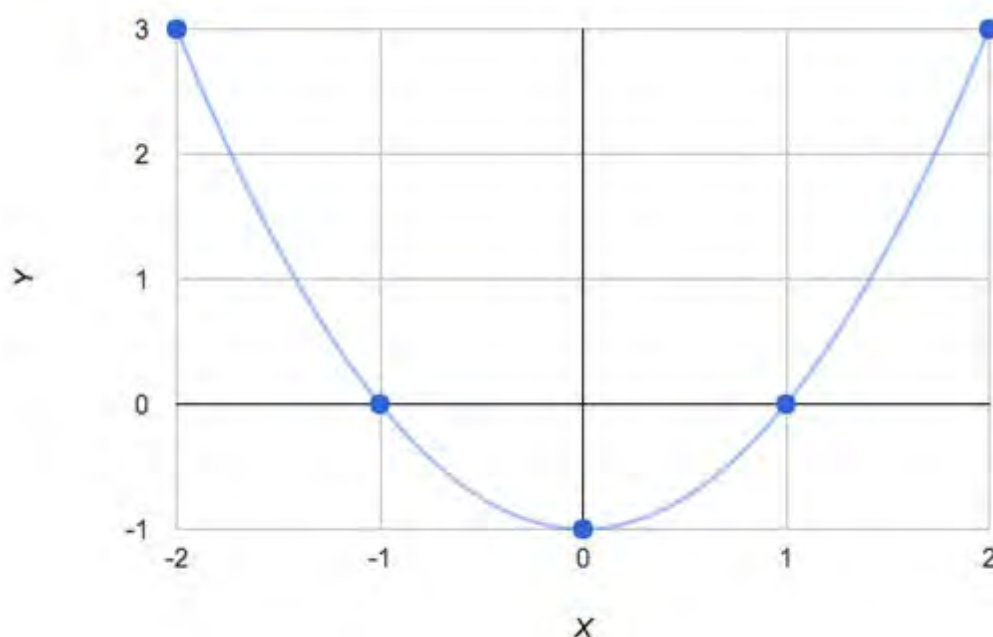
1. $y = x^2 - 1$
2. $y = 10x^2 - 10x + 10$
3. $y = -x^2 + 5x$
4. $y = 8x^2 - 5$
5. $y = -5x^2 + 5x - 3$
6. $y = 3x^2 + 10x - 30$



Practice Problem Answers and Explanations

1. In order to plot $y = x^2 - 1$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
-2	$(-2)^2 - 1 = 3$	$(-2, 3)$
-1	$(-1)^2 - 1 = 0$	$(-1, 0)$
0	$(0)^2 - 1 = -1$	$(0, -1)$
1	$(1)^2 - 1 = 0$	$(1, 0)$
2	$(2)^2 - 1 = 3$	$(2, 3)$



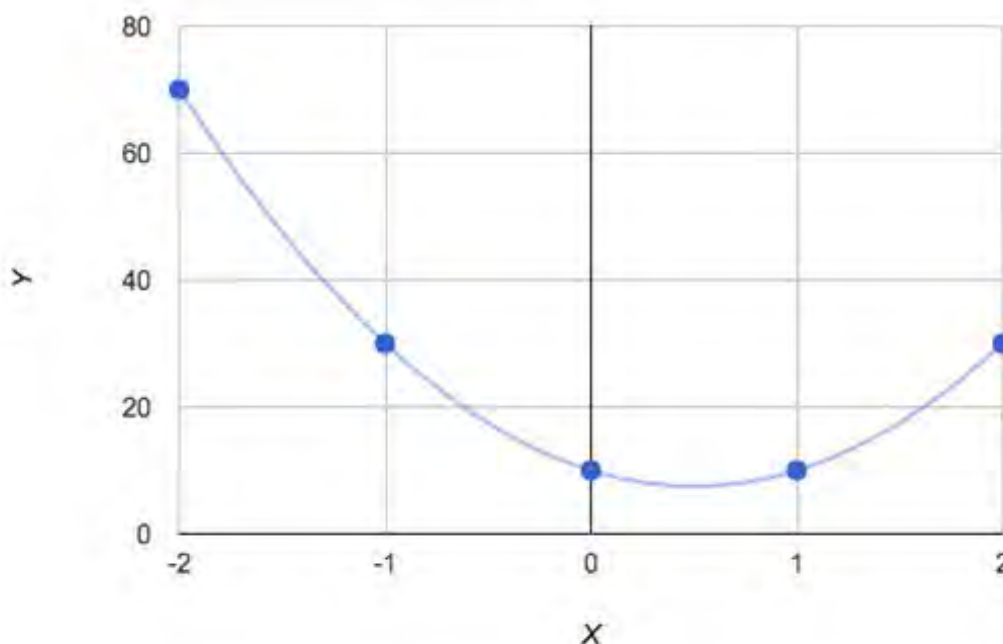
Now we need to calculate the vertex.

In this case, $a = 1$ and $b = 0$, so $h = \frac{-0}{2(1)} = 0$. Next, we plug h into our original equation: $y = (0)^2 - 1 = -1$. Thus, our vertex is at $(0, -1)$.



2. In order to plot $y = 10x^2 - 10x + 10$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
-2	$10(-2)^2 - 10(-2) + 10 = 70$	$(-2, 70)$
-1	$10(-1)^2 - 10(-1) + 10 = 30$	$(-1, 30)$
0	$10(0)^2 - 10(0) + 10 = 10$	$(0, 10)$
1	$10(1)^2 - 10(1) + 10 = 10$	$(1, 10)$
2	$10(2)^2 - 10(2) + 10 = 30$	$(2, 30)$



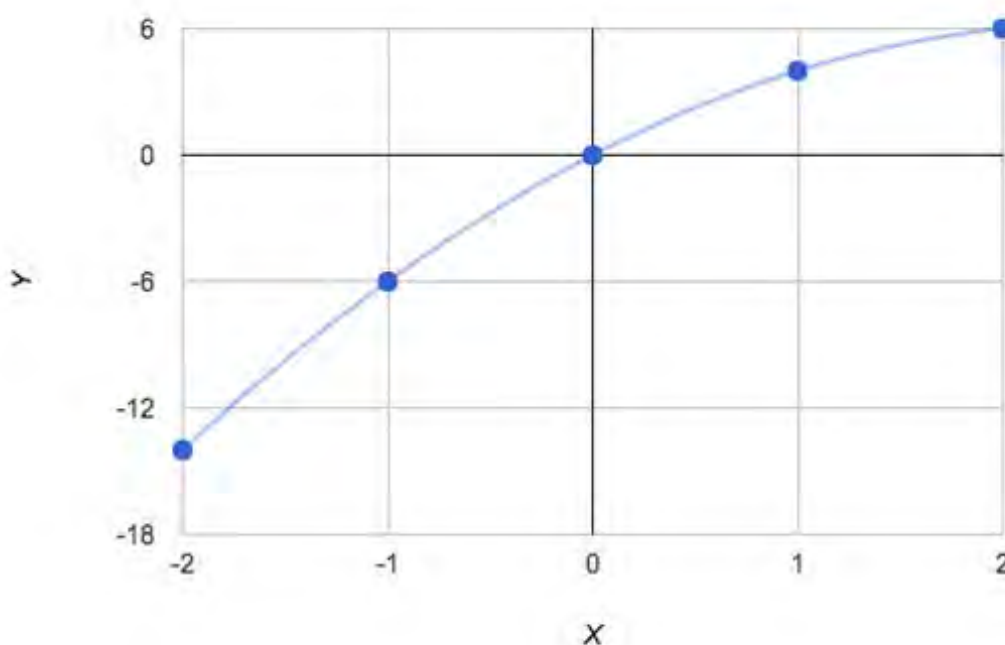
Now we need to calculate the vertex.

In this case, $a = 10$ and $b = -10$, so $h = \frac{10}{2(10)} = \frac{1}{2}$. Next, we plug h into our original equation:
 $y = 10\left(\frac{1}{2}\right)^2 - 10\left(\frac{1}{2}\right) + 10 = 7.5$. Thus, our vertex is at $\left(\frac{1}{2}, 7.5\right)$.



3. In order to plot $y = -x^2 + 5x$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
-2	$-(-2)^2 + 5(-2) = -14$	$(-2, -14)$
-1	$-(-1)^2 + 5(-1) = -6$	$(-1, -6)$
0	$-(0)^2 + 5(0) = 0$	$(0, 0)$
1	$-(1)^2 + 5(1) = 4$	$(1, 4)$
2	$-(2)^2 + 5(2) = 6$	$(2, 6)$



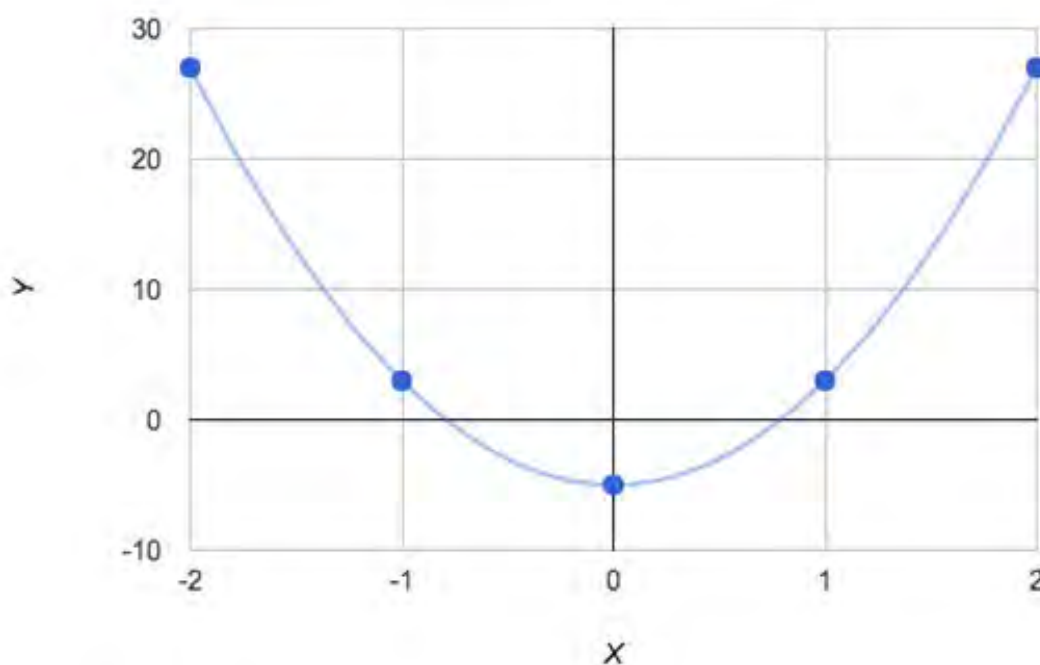
Now we need to calculate the vertex.

In this case, $a = -1$ and $b = 5$, so $h = \frac{-5}{2(-1)} = \frac{5}{2}$. Next, we plug h into our original equation:
 $y = -\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) = 6.25$. Thus, our vertex is at $\left(\frac{5}{2}, 6.25\right)$.



4. In order to plot $y = 8x^2 - 5$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
-2	$8(-2)^2 - 5 = 27$	$(-2, 27)$
-1	$8(-1)^2 - 5 = 3$	$(-1, 3)$
0	$8(0)^2 - 5 = -5$	$(0, -5)$
1	$8(1)^2 - 5 = 3$	$(1, 3)$
2	$8(2)^2 - 5 = 27$	$(2, 27)$



Now we need to calculate the vertex.

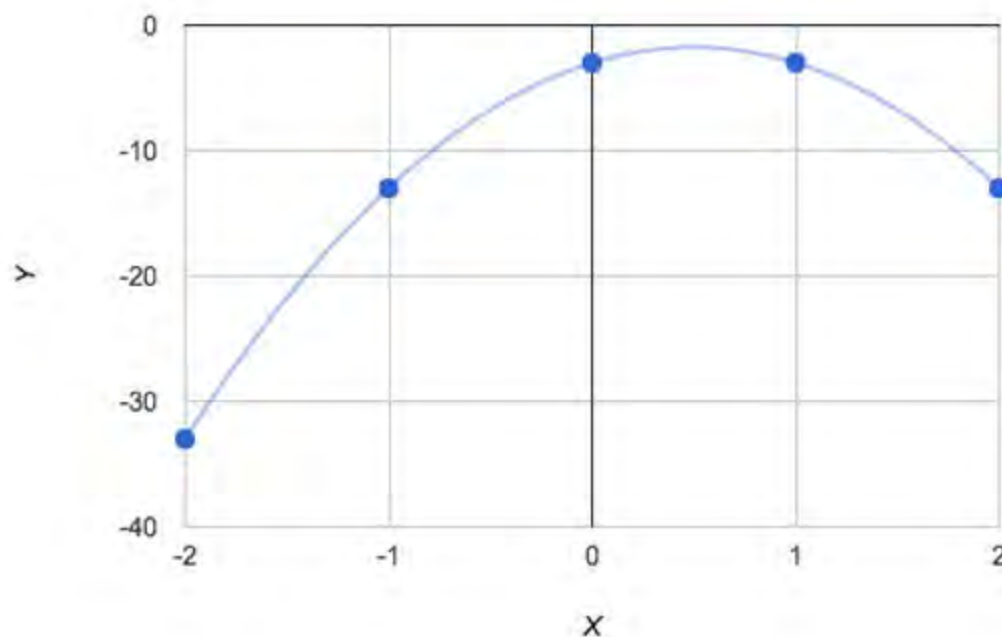
In this case, $a = 8$ and $b = 0$, so $h = \frac{-0}{2(8)} = 0$. Next, we plug h into our original equation:

$y = 8(0)^2 - 5 = -5$. Thus, our vertex is at $(0, -5)$.



5. In order to plot $y = -5x^2 + 5x - 3$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
-2	$-5(-2)^2 + 5(-2) - 3 = -33$	$(-2, -33)$
-1	$-5(-1)^2 + 5(-2) - 3 = -13$	$(-1, -13)$
0	$-5(0)^2 + 5(0) - 3 = -3$	$(0, -3)$
1	$-5(1) + 5(1) - 3 = -3$	$(1, -3)$
2	$-5(2)^2 + 5(2) - 3 = -13$	$(2, -13)$



Now we need to calculate the vertex.

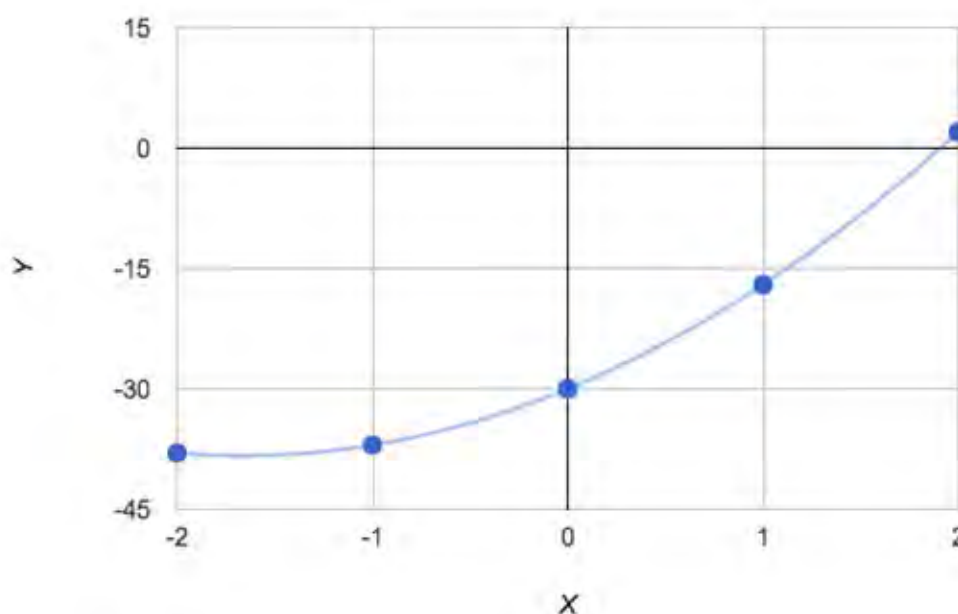
In this case, $a = -5$ and $b = 5$, $h = \left(\frac{-5}{2(-5)}\right) = \frac{1}{2}$. Next, we plug h into our original equation:

$$y = -5\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) - 3 = -1.75. \text{ Thus, our vertex is at } \left(\frac{1}{2}, -1.75\right).$$



6. In order to plot $y = 3x^2 + 10x - 30$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
-2	$3(-2)^2 + 10(-2) - 30 = -38$	$(-2, -38)$
-1	$3(-1)^2 + 10(-1) - 30 = -37$	$(-1, -37)$
0	$3(0)^2 + 10(0) - 30 = -30$	$(0, -30)$
1	$3(1)^2 + 10(1) - 30 = -17$	$(1, -17)$
2	$3(2)^2 + 10(2) - 30 = 2$	$(2, 2)$



Now we need to calculate the vertex.

In this case, $a = 3$ and $b = 10$, $h = \frac{-10}{2(3)} = -\frac{5}{3}$. Now we plug h into our original equation.

$$y = 3\left(-\frac{5}{3}\right)^2 + 10\left(-\frac{5}{3}\right) - 30 = -38.33. \text{ Thus our vertex is at } \left(-\frac{5}{3}, -38.33\right).$$



Graphing Exponential Functions

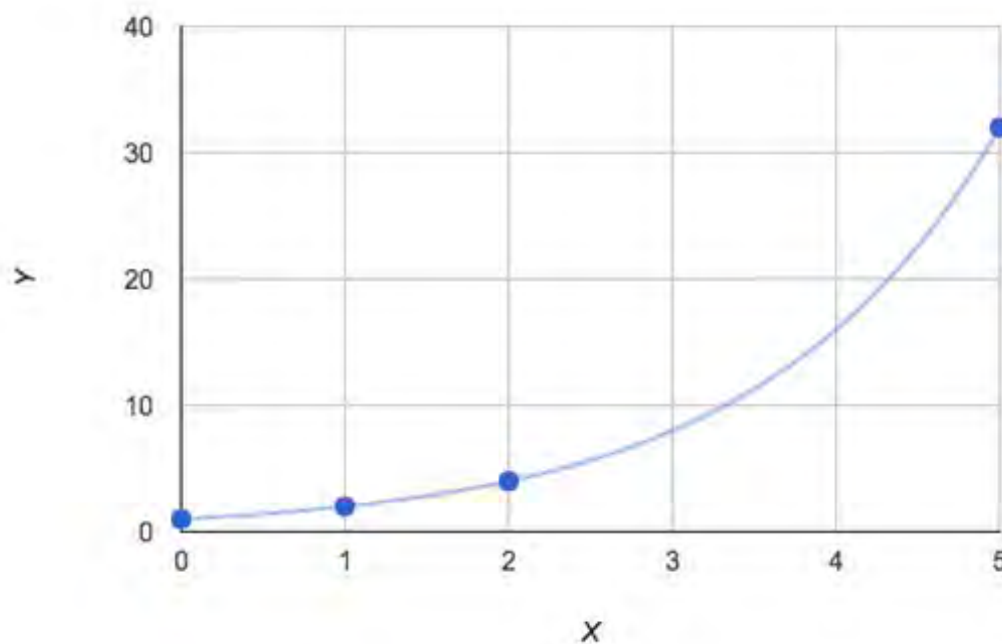
Graphing exponential functions is the same process as graphing lines. Rather than having a line in our graph, we will have a curved line.

Sample Question

Plot $y = 2^x$.

In order to plot this equation, we have to choose x values and plug them into our equation. We will create a table of values for this example.

X-Value	Y-Value	(X,Y)
0	$2^0 = 1$	(0, 1)
1	$2^1 = 2$	(1, 2)
2	$2^2 = 4$	(2, 4)
5	$2^5 = 32$	(5, 32)



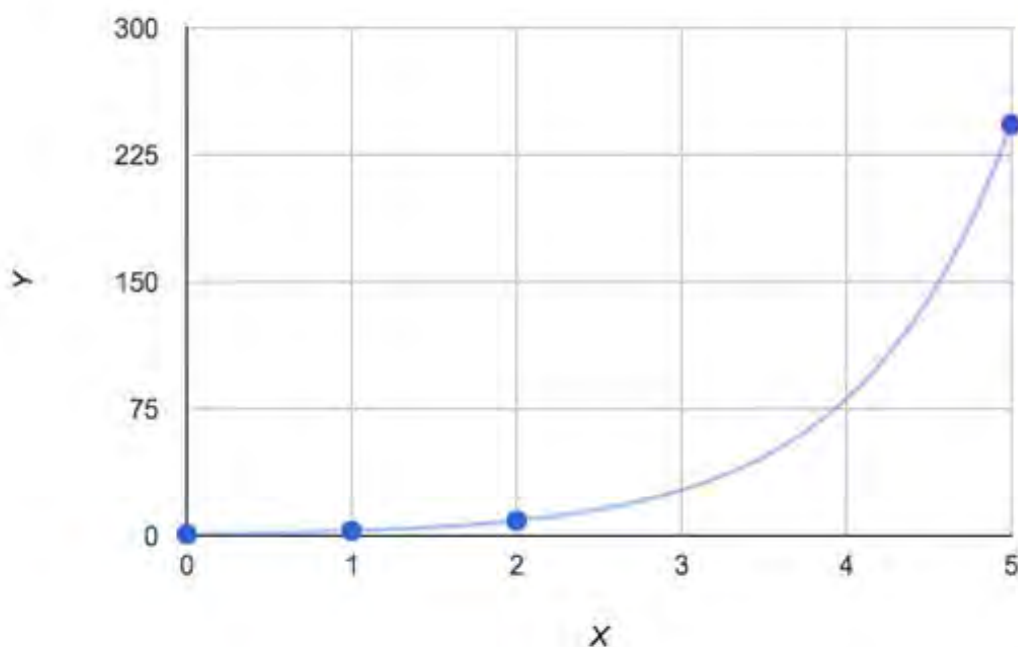
After we plot the points, we can draw a curved line through the points we plotted.



Sample Question

Plot $y = 3^x$.

X-Value	Y-Value	(X,Y)
0	$3^0 = 1$	(0, 1)
1	$3^1 = 3$	(1, 3)
2	$3^2 = 9$	(2, 9)
5	$3^5 = 243$	(5, 243)



After we plot the points, we can draw a curved line through the points we plotted.

Practice Problems

Plot these exponential functions:

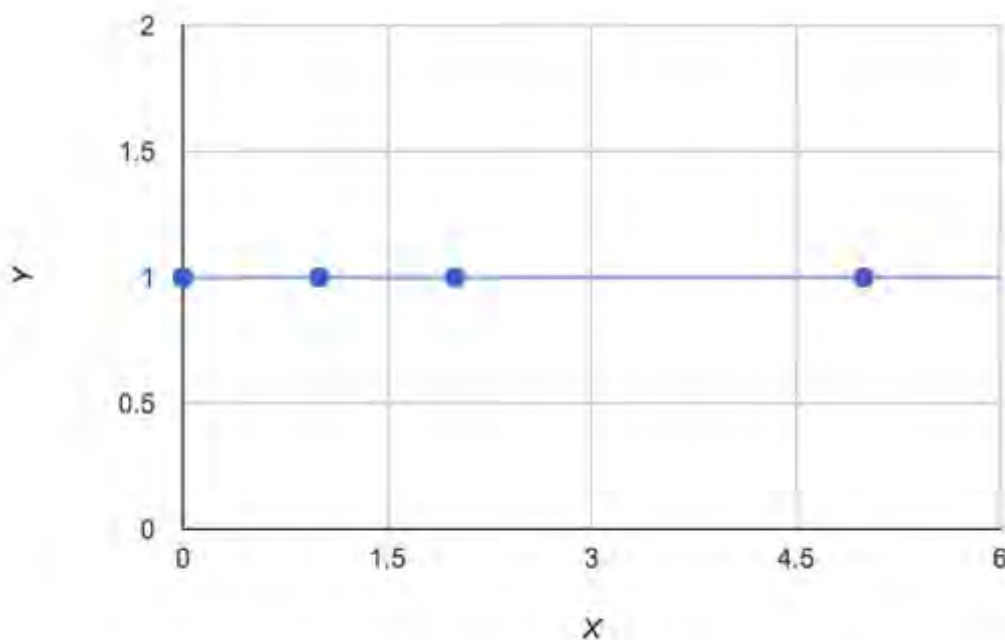
1. $y = 1^x$
2. $y = 4^x$
3. $y = (2.5)^x$
4. $y = 10^x$
5. $y = \left(\frac{5}{4}\right)^x$
6. $y = 3.6^x$



Practice Problem Answers and Explanations

1. In order to plot $y = 1^x$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
0	$1^0 = 1$	(0,1)
1	$1^1 = 1$	(1,1)
2	$1^2 = 1$	(2,1)
5	$1^5 = 1$	(5,1)

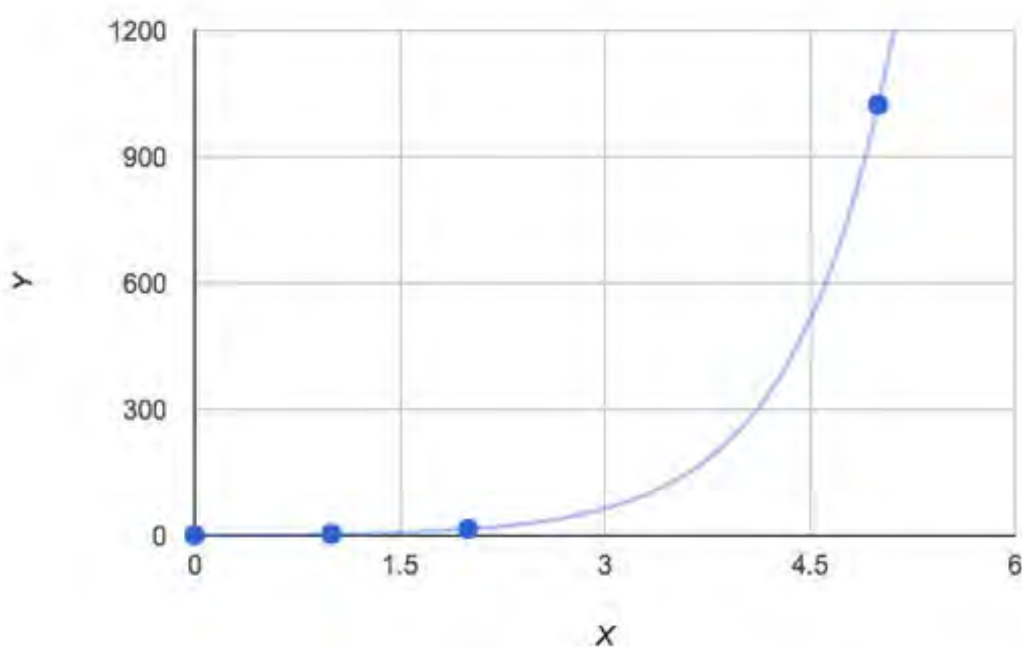


Since $y = 1^x$ is a straight line, our plot of it is going to be a straight line rather than a curvy line.



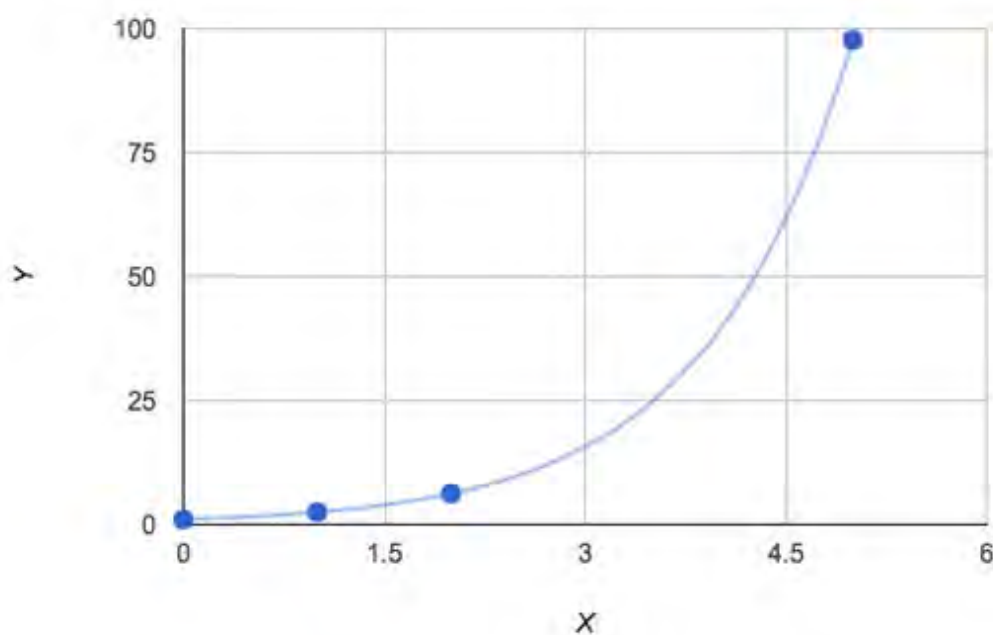
2. In order to plot Graphing inequalities is more complex $y=4^x$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
0	$4^0 = 1$	(0,1)
1	$4^1 = 4$	(1,4)
2	$4^2 = 16$	(2,16)
5	$4^5 = 1024$	(5,1024)



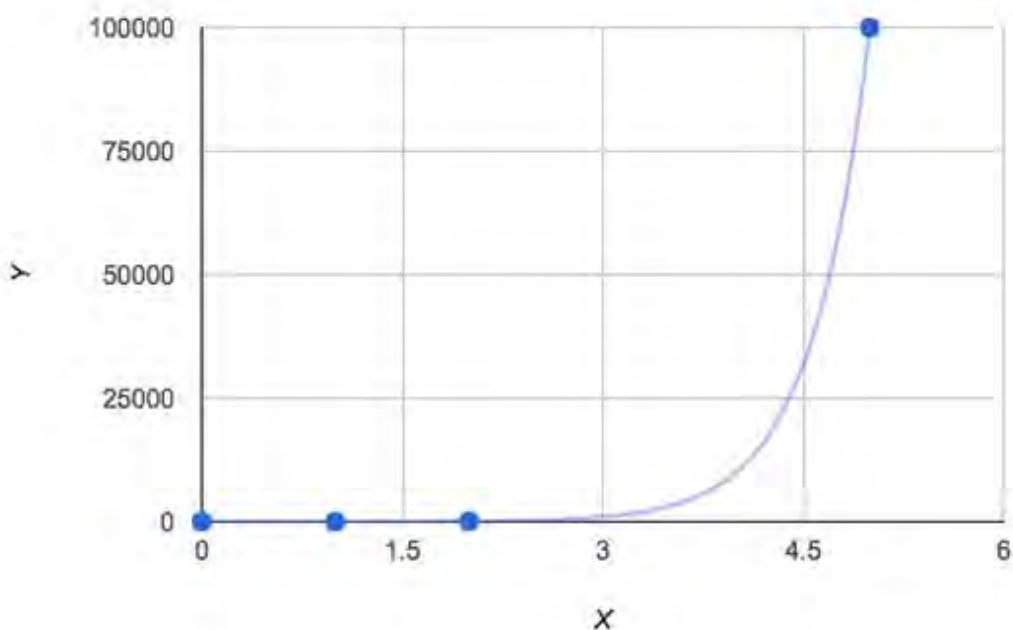
3. In order to plot $y = (2.5)^x$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
0	$(2.5)^0 = 1$	(0, 1)
1	$(2.5)^1 = 2.5$	(1, 2.5)
2	$(2.5)^2 = 6.25$	(2, 6.25)
5	$(2.5)^5 = 97.65625$	(5, 97.65625)



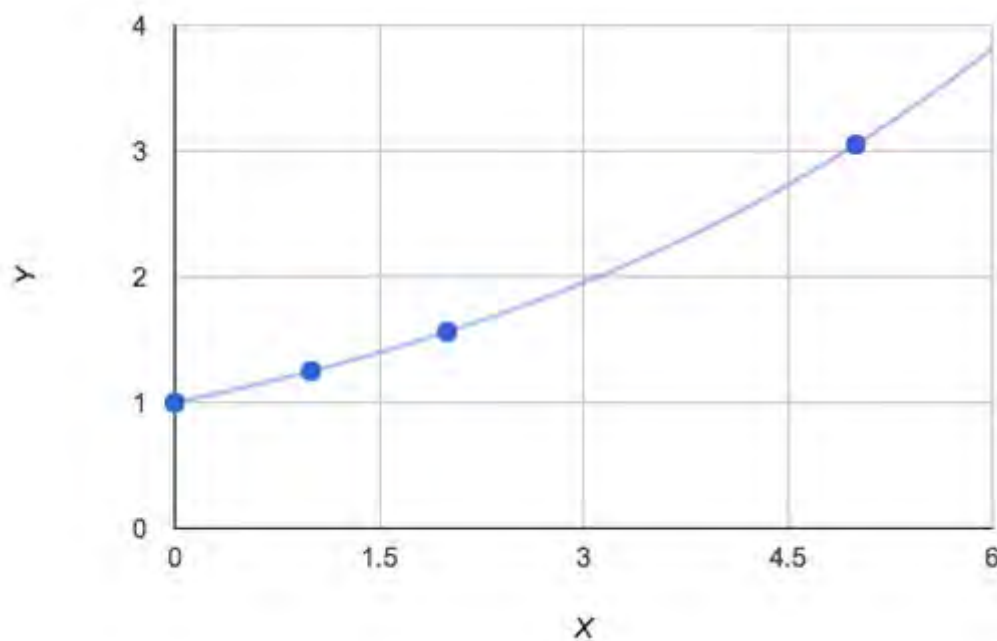
4. In order to plot $y = 10^x$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
0	$(10)^0 = 1$	(0, 1)
1	$(10)^1 = 10$	(1, 10)
2	$(10)^2 = 100$	(2, 100)
5	$(10)^5 = 100000$	(5, 100000)



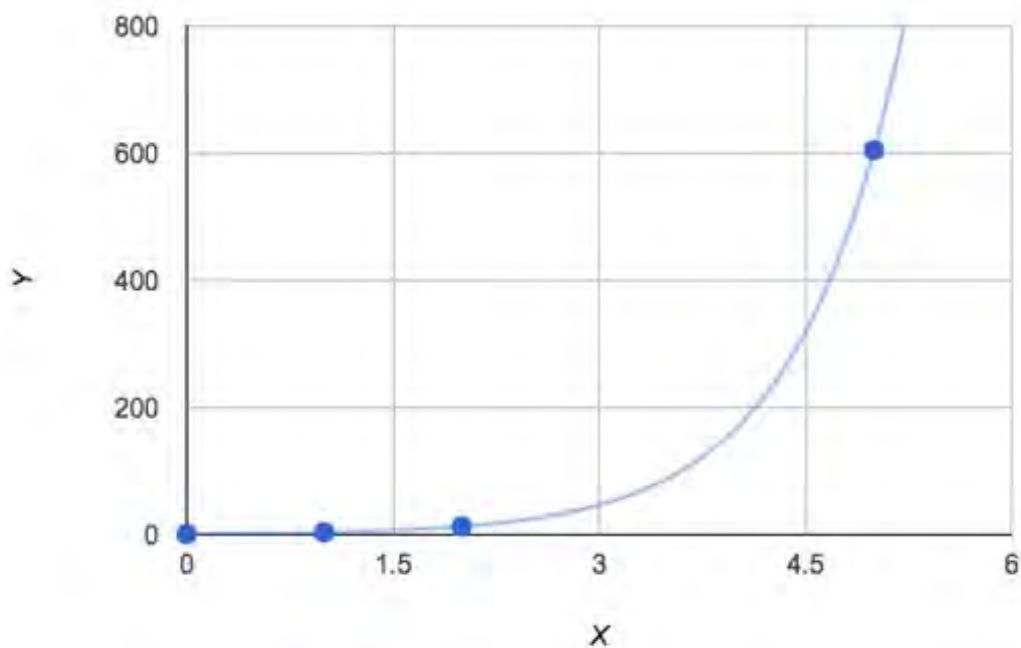
5. In order to plot $y = \left(\frac{5}{4}\right)^x$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
0	$\left(\frac{5}{4}\right)^0 = 1$	(0, 1)
1	$\left(\frac{5}{4}\right)^1 = \frac{5}{4}$	$\left(1, \frac{5}{4}\right)$
2	$\left(\frac{5}{4}\right)^2 = \frac{25}{16}$	$\left(2, \frac{25}{16}\right)$
5	$\left(\frac{5}{4}\right)^5 = \frac{3125}{1024}$	$\left(5, \frac{3125}{1024}\right)$



6. In order to plot $y = 3.6^x$, we need to create a table of coordinates, then plot the coordinates, and lastly put a curved line through them.

X-Value	Y-Value	(X,Y)
0	$(3.6)^0 = 1$	(0,1)
1	$(3.6)^1 = 3.6$	(1,3.6)
2	$(3.6)^2 = 12.96$	(2,12.96)
5	$(3.6)^5 = 604.66176$	(5,604.66176)



Graphing Inequalities

Graphing inequalities is more complicated than graphing points, lines, exponentials, and quadratic functions because we are describing a region.

Sample Question

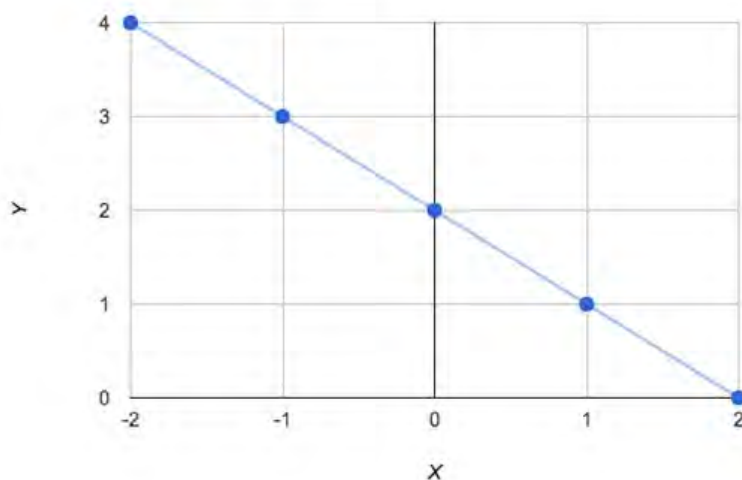
Graph the inequality $y + x \leq 2$.

First, let's simplify this inequality.

$$y + x \leq 2$$

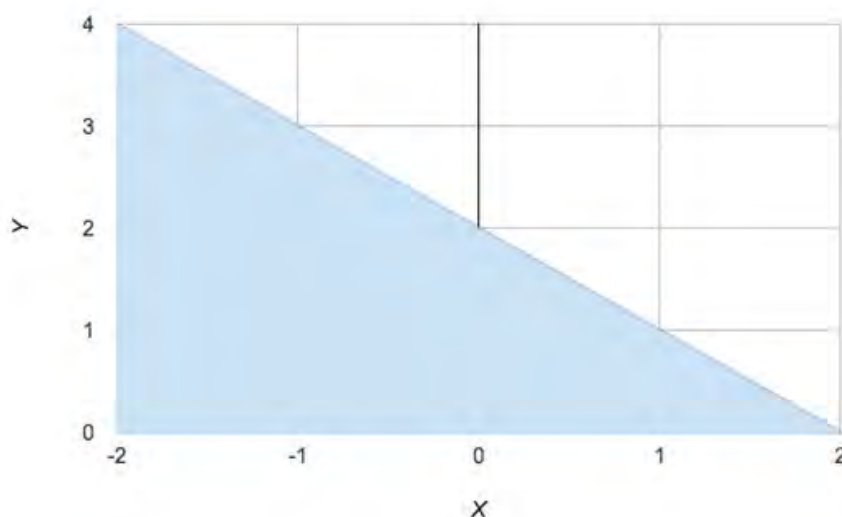
$$y \leq 2 - x$$

Now how do we interpret $y \leq 2 - x$? This is saying that the y -values are less than or equal to the line $2 - x$. The first step is to graph the line $2 - x$.



Since $y \leq 2 - x$, we are going to shade everything below and including the line $2 - x$. We will represent $2 - x$ with a solid line; we are including the line since it's strictly less than or equal to y (see below).





We shade everything up to and including the line because our inequality includes the line $2 - x$.

Sample Question

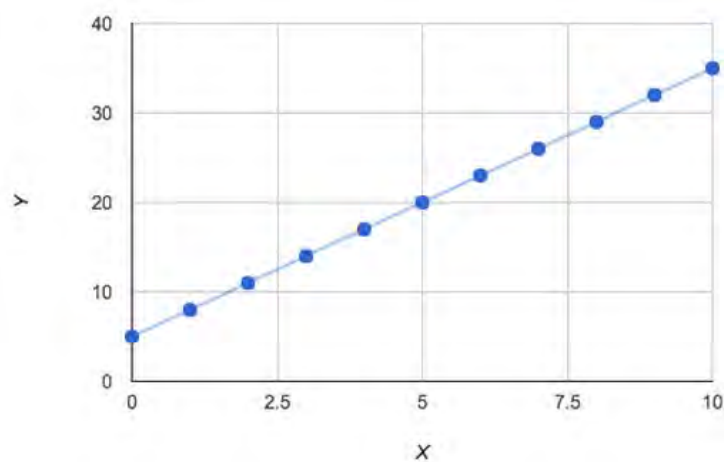
Graph the inequality $y - 3x + 5 > 10$.

First, let's simplify this inequality.

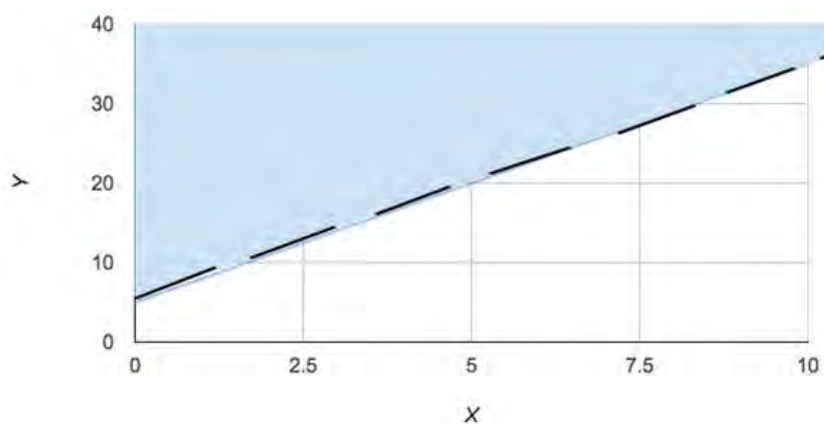
$$\begin{aligned}y - 3x + 5 &> 10 \\y - 3x &> 10 - 5 \\y - 3x &> 5 \\y &> 5 + 3x\end{aligned}$$

Now how do we interpret $y > 5 + 3x$? This is saying that the y -values are strictly greater than the line $5 + 3x$. The first step is to graph the line $5 + 3x$.





Since $y > 5 + 3x$, we are going to shade everything above the line $5 + 3x$. We will represent $5 + 3x$ with a dashed line because we aren't including the line since it's strictly greater than y (see below).



Sample Question

Graph the inequality $-2y - x \leq 10$.

First, let's simplify this inequality.

$$-2y - x \leq 10$$

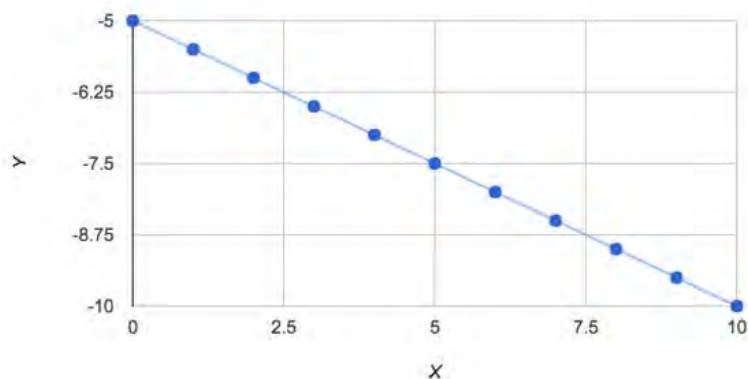
$$-2y \leq 10 + x$$

$$y \geq \frac{10}{-2} + \frac{x}{-2}$$

$$y \geq -\frac{x}{2} - 5$$

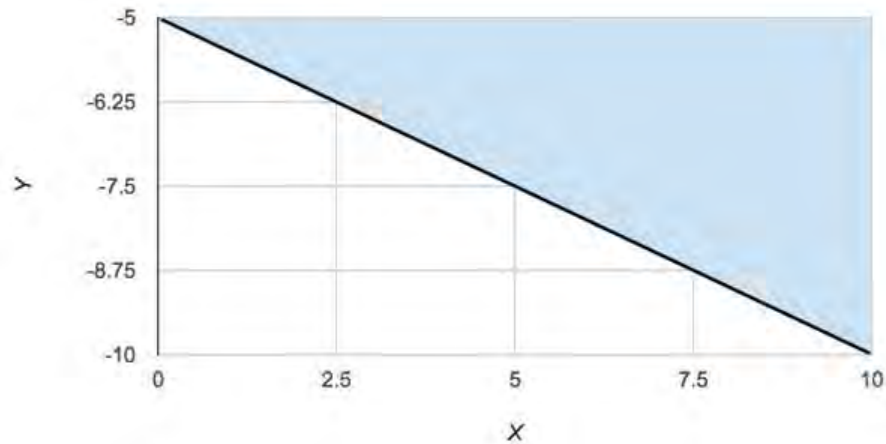
Remember: whenever you divide by a negative number, the inequality flips.

Now how do we interpret $y \geq -\frac{x}{2} - 5$? This is saying that the y -values are strictly greater than or equal to the line $-\frac{x}{2} - 5$. The first step is to graph the line $-\frac{x}{2} - 5$.



Since $y \geq -\frac{x}{2} - 5$, we are going to shade everything above and including the line $-\frac{x}{2} - 5$. We will represent $-\frac{x}{2} - 5$ with a solid line because we are including the line since it's strictly greater than or equal to y (see below).



**Practice Problems**

Plot these inequalities:

1. $y + 2x \leq 5$
2. $y - 5x \geq 10$
3. $-y - 5x + 10 \leq 0$
4. $-2y + x - 3 > 2$
5. $23y - 23x + 23 < 0$
6. $-2y + 10x > 5$



Answers and Explanations for Practice Problems

1. $y + 2x \leq 5$

Step 1: Use algebraic manipulation to get the inequality into slope-intercept form.

$$y \leq -2x + 5$$

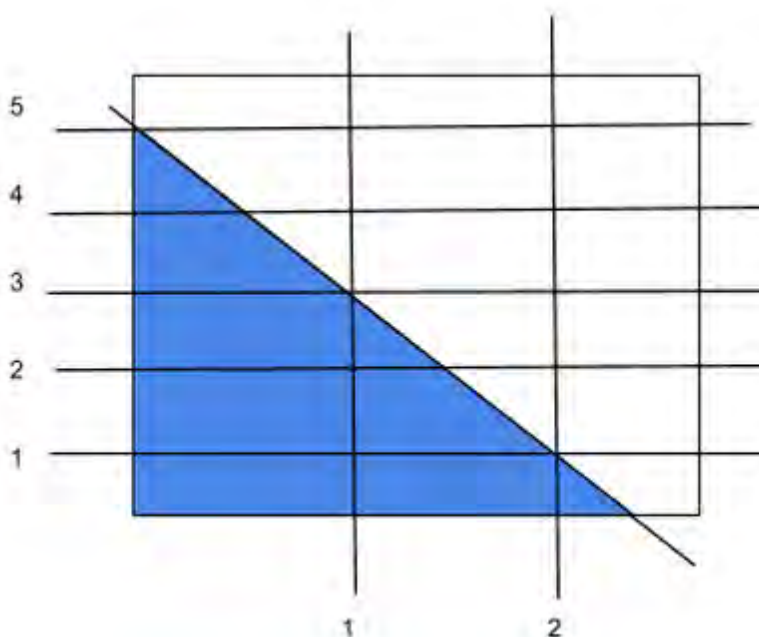
Step 2: Identify what type of inequality exists.

For this particular problem, there is a less than or equal to sign, which means that the line that is created will be a solid line and the shaded region will be below the line.

Step 3: Identify the slope and y-intercept of the graph.

$$m = -2, y\text{-int} = 5$$

Step 4: Create the graph by plotting the y-intercept and shading the area underneath the line.



2. $y - 5x \geq 10$

Step 1: Use algebraic manipulation to get the inequality into slope-intercept form.

$$y \geq 5x + 10$$

Step 2: Identify what type of inequality exists.

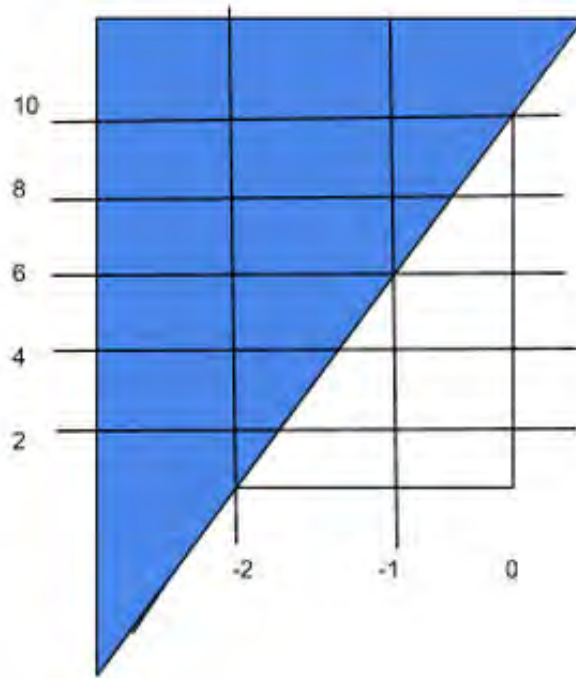
For this particular problem there is a greater than or equal to sign, which means that the line that is created will be a solid line and the shaded region will be above the line.

Step 3: Identify the slope and y-intercept of the graph.

$$m = 5, y\text{-int} = 10$$

Step 4: Create the graph by plotting the y-intercept and shade the area above the line.





3. $-y - 5x + 10 \leq 0$

Step 1: Use algebraic manipulation to get the inequality into slope-intercept form.

$$y \geq -5x + 10$$

Step 2: Identify what type of inequality exists.

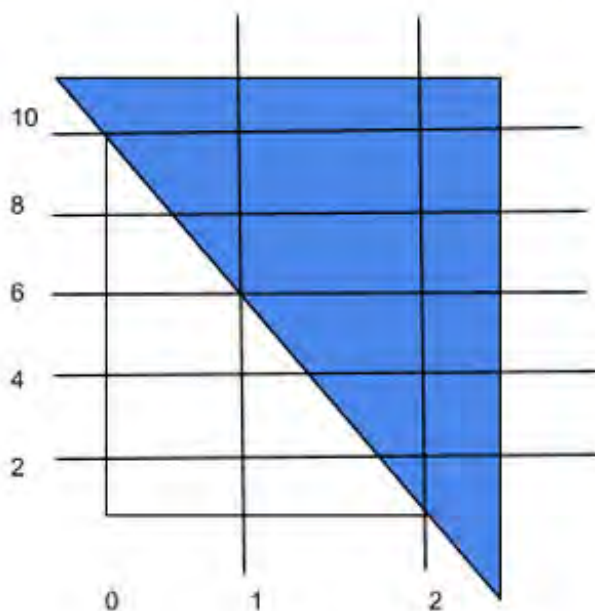
For this particular problem, there is a greater than or equal to sign which, means that the line that is created will be a solid line and the shaded region will be above the line.

Step 3: Identify the slope and y-intercept of the graph.

$$m = -5, y\text{-int} = 10$$

Step 4: Create the graph by plotting the y-intercept and shading the area above the line.





4. $-2y + x - 3 > 2$

Step 1: Use algebraic manipulation to get the inequality into slope-intercept form.

$$x - 3 > 2y + 2$$

$$x - 5 > 2y$$

$$y < \frac{1}{2}x - \frac{5}{2}$$

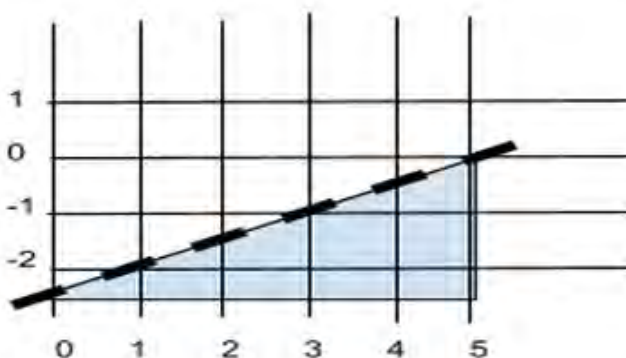
Step 2: Identify what type of inequality exists.

For this particular problem, there is a less than sign, which means that the line that is created will be a dotted line and the shaded region will be below the line.

Step 3: Identify the slope and y-intercept of the graph.

$$m = \frac{1}{2}, \text{ y-int} = -\frac{5}{2}$$

Step 4: Create the graph by plotting the y-intercept and shade the area below the line.



5. $23y - 23x + 23 < 0$

Step 1: Use algebraic manipulation to get the inequality into slope-intercept form.



$$23y - 23x + 23 < 0$$

$$23y < 23x - 23$$

$$y < x - 1$$

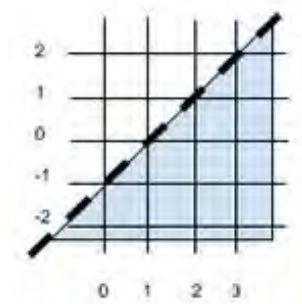
Step 2: Identify what type of inequality exists.

For this particular problem there is a less than sign, which means that the line that is created will be a dotted line and the shaded region will be below the line.

Step 3: Identify the slope and y-intercept of the graph.

$$m = 1, y\text{-int} = -1$$

Step 4: Create the graph by plotting the y-intercept and shading the area below the line.



6. $-2y + 10x > 5$

Step 1: Use algebraic manipulation to get the inequality into slope-intercept form.

$$-2y + 10x > 5$$

$$2y < 10x - 5$$

$$y < 5x - \frac{5}{2}$$

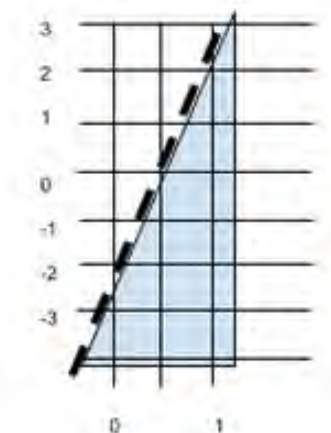
Step 2: Identify what type of inequality exists.

For this particular problem, there is a less than sign, which means that the line that is created will be a dotted line and the shaded region will be below the line.

Step 3: Identify the slope and y-intercept of the graph.

$$m = 5, y\text{-int} = -\frac{5}{2}$$

Step 4: Create the graph by plotting the y-intercept and shading the area below the line.



Trigonometry

The fundamentals of trigonometry are grounded in the analytics of right triangles. By definition, right triangles must have a right angle and two acute angles. Trigonometric functions take advantage of these restrictive parameters to analyze the qualities of right triangles. The primary functions—sine, cosine, and tangent—can be applied to two given sides or to a given acute angle. Doing so provides additional information about the figure at hand, allowing you to determine missing measurements. Each function can be depicted mathematically as an equation or as a graph, and provides a whole new realm of content for higher level mathematical applications. Beyond coursework in high school math, trigonometry is an essential element of physics, engineering, and calculus.

Section Outline

[SOHCAHTOA](#)

[Trigonometric Identities](#)

[Reciprocal Identities \(Trigonometric Ratios\)](#)

[Quotient Identities](#)

[Pythagorean Identities](#)

[Conversions Between Radians and Degrees](#)

[Unit Circle](#)

[Trigonometric Function Graphs](#)

[Sine Graphs](#)

[Cosine Graphs](#)

[Tangent Graphs](#)

[Amplitude, Period, Frequency, and Phase Shifts](#)

[Solutions for Triangles](#)

[Practice Problems: Trigonometry](#)



Trigonometry**SOHCAHTOA**

Trigonometry is based upon three primary relationships within a triangle. The mnemonic SOHCAHTOA has been commonly used to help students memorize the associations between the sine, cosine, and tangent in a right triangle. These trigonometric identities are used to find missing components of a right triangle.

SOH: Sine of θ equals **O**pposite side divided by the **H**ypotenuse

$$\sin \theta = \frac{opp}{hyp}$$

CAH: Cosine of θ equals **A**djacent side divided by the **H**ypotenuse

$$\cos \theta = \frac{adj}{hyp}$$

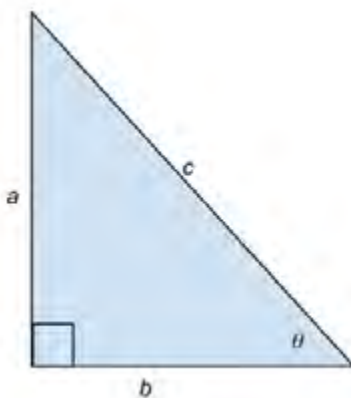
TOA: Tangent of θ equals **O**pposite side divided by the **A**djacent side

$$\tan \theta = \frac{opp}{adj}$$

These exercises may seem trivial and their purpose not entirely clear at this point; however, they are crucial in understanding topics such as physical tension, magnitude and direction of forces, and geometric proofs. In physics these three identities can be used to find missing angles and sides based upon information given in the problem. The basic skills presented in this exercise, as well as the ACT, will help you build a foundation in trigonometry and mathematics that is necessary for any career in medicine, engineering, and the sciences. Try to master your skills with the practice exercise presented below.

Practice Exercise

Find the sine, cosine, and tangent of the pictured triangle.



Find the sine of θ :

Remember SOHCAHTOA.

SOH: Sine of θ equals **O**pposite side divided by the **H**ypotenuse

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

In the figure, the side opposite of θ is side a and the hypotenuse is side c . In order to obtain the correct answer, substitute these variables into the above sine equation.

$$\sin \theta = \frac{a}{c}$$

Find the cosine of θ :

Remember SOHCAHTOA.

CAH: Cosine of θ equals **A**djacent side divided by the **H**ypotenuse

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

In the figure, the side adjacent to θ is side b and the hypotenuse is side c . In order to obtain the correct answer, substitute these variables into the above cosine equation.

$$\cos \theta = \frac{b}{c}$$

Find the tangent of θ :

Remember SOHCAHTOA.

TOA: Tangent of θ equals **O**pposite side divided by the **A**djacent side

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

In the figure, the side opposite of θ is side a and the side adjacent to θ is side b . In order to obtain the correct answer, substitute these variables into the above tangent equation.

$$\tan \theta = \frac{a}{b}$$



Trigonometric Identities

In mathematics, trigonometry refers to the relationships between the angles and sides of triangles. As seen in the previous section, there are special relationships which are known as sine, cosine, and tangent. These are the foundational trigonometric relationships or identities, which all other identities are built from.

Reciprocal Identities (Trigonometric Ratios)

Since there are three foundational identities, there are also three reciprocal identities. Reciprocal identities are also known as trigonometric ratios because by definition, a reciprocal is a ratio where the numerator and denominator are switched. The three foundational identities along with their reciprocal identities are as follows. Notice that the reciprocal of sine is cosecant, the reciprocal of cosine is secant, and the reciprocal of tangent is cotangent.

$$\begin{aligned}\sin(x) &= \frac{\text{opp}}{\text{hyp}} & \csc(x) &= \frac{1}{\sin(x)} = \frac{\text{hyp}}{\text{opp}} \\ \cos(x) &= \frac{\text{adj}}{\text{hyp}} & \sec(x) &= \frac{1}{\cos(x)} = \frac{\text{hyp}}{\text{adj}} \\ \tan(x) &= \frac{\text{opp}}{\text{adj}} & \cot(x) &= \frac{1}{\tan(x)} = \frac{\text{adj}}{\text{opp}}\end{aligned}$$

Sample Question

Plot $y = -2x^2 - 5x + 4$.

Step 1: Write the cotangent and cosecant in terms of sines and cosines.

$$\sin(x) \frac{\cos(x)}{\sin(x)} \frac{1}{\sin(x)}$$

Step 2: Reduce fractions that have common factors.

In this particular case, sine multiplied by cosecant will reduce to one.

$$\frac{\cos(x)}{\sin(x)}$$

Step 4: Rewrite term using the quotient identity.

$$\frac{\cos(x)}{\sin(x)} = \cot(x)$$



Sample Question

$$\text{Simplify: } \frac{1}{\csc(x)} \cos(x) \tan^2(x)$$

Step 1: Rewrite all terms in terms of sine and cosine.

$$\sin(x) \cos(x) \frac{\sin^2 x}{\cos^2 x}$$

Step 2: Reduce fractions that have common factors.

In this particular case, the cosine in the numerator and one in the denominator will cancel.

$$\sin(x) \frac{\sin^2 x}{\cos(x)}$$

Step 3: Multiply numerators together.

$$\frac{\sin^3 x}{\cos(x)}$$

Step 4: Simplify terms using the quotient identity.

$$\tan(x) \sin^2(x)$$



Quotient Identities

Quotient identities are those in which one trigonometric function can be written as the quotient of two other trigonometric functions. There are only two functions that fit this criteria: tangent and cotangent.

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \quad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

Pythagorean Identities

Pythagorean identities are directly derived from the Pythagorean Theorem. There are three Pythagorean identities; however, knowing one of these identities allows one to find the other two through algebraic manipulation and the use of trigonometric ratios. The foundational Pythagorean identity uses the relationship of sine and cosine.

$$\sin^2 x + \cos^2 x = 1$$

Manipulating this equation can result in the other two Pythagorean identities, which use tangent, secant, cotangent, and cosecant.

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Let's take a closer look at the algebraic manipulations performed on the sine-cosine identity that result in the other two Pythagorean identities:

1. $\boxed{\sin^2 x + \cos^2 x = 1}$

Step 1: First, multiply each term by $\frac{1}{\cos^2 x}$:

$$\frac{1}{\cos^2 x} \sin^2 x + \frac{1}{\cos^2 x} \cos^2 x = 1 \frac{1}{\cos^2 x}$$

Step 2: Reduce fractions and ratios.

Recall that $\tan(x) = \frac{\sin(x)}{\cos(x)}$ and $\sec(x) = \frac{1}{\cos(x)}$. Therefore, the equation becomes:

$$\tan^2 x + 1 = \sec^2 x$$

Step 3: Rewrite the equation to look like the Pythagorean identity:

$$1 + \tan^2 x = \sec^2 x$$

2. $\boxed{\sin^2 x + \cos^2 x = 1}$

Step 1: Multiply each term by $\frac{1}{\sin^2 x}$:

$$\frac{1}{\sin^2 x} \sin^2 x + \frac{1}{\sin^2 x} \cos^2 x = 1 \frac{1}{\sin^2 x}$$

Step 2: Reduce fractions and ratios.

Recall that $\cot(x) = \frac{\cos(x)}{\sin(x)}$ and $\csc(x) = \frac{1}{\sin(x)}$. Therefore, the equation becomes:

$$1 + \cot^2 x = \csc^2 x$$



Conversions Between Radians and Degrees

When dealing with trigonometric functions, it is important to know how and when to convert between the two units of measurement, radians and degrees. Radians deal in terms of π while degrees are numerical values. The key to converting between these two measurements is knowing that $180^\circ = \pi$. Therefore, to convert a measurement from radians to degrees, simply multiply the measurement by 180 and then divide by π . To convert from degrees to radians, multiply the measurement by π and then divide by 180. For example, convert 45° into radians. First, multiply $45 \cdot \pi = 45\pi$. Now divide by 180. This results in the following expression:

$$\frac{45\pi}{180} = \frac{45\pi}{45 \cdot 4}$$

Since there is a common factor of 45 in the numerator and denominator, they cancel each other out, and the simplified radian form is $\frac{\pi}{4}$.

Now convert $\frac{2\pi}{3}$ into degrees. First, multiply the numerator by 180:

$$2\pi \cdot 180 = 360\pi$$

Now divide by the denominator times π .

$$\frac{360\pi}{3\pi} = \frac{3 \cdot 120\pi}{3\pi} = 120$$

Notice that the numerator can be factored, and that there is a common factor of 3π in both the numerator and denominator. These cancel, and the simplified degree measurement is 120° .

Sample Question

Convert 210° to radians.

Step 1: Multiply the measurement by π .

$$210^\circ \cdot \pi = 210\pi$$

Step 2: Divide by 180.

Remember to factor the numerator and denominator in order to reduce and simplify the fraction.

In this particular case, there is a 30 in the numerator and in the denominator; thus, they reduce to one.

$$\frac{210\pi}{180} = \frac{30 \cdot 7\pi}{30 \cdot 6} = \frac{7\pi}{6}$$



Sample Question

Convert $\frac{15\pi}{8}$ to degrees.

Step 1: Multiply the numerator by 180.

$$\frac{15\pi \cdot 180}{8} = \frac{2700\pi}{8}$$

Step 2: Divide by the denominator times π .

In this particular case, 4 is a common factor in both the numerator and denominator and thus cancels out.

$$\frac{2700\pi}{8\pi} = \frac{4 \cdot 675\pi}{4 \cdot 2\pi} = \frac{675}{2} = 337.5^\circ$$



Unit Circle

The unit circle is an extremely helpful tool in solving trigonometric problems. For the purpose of trigonometry, the unit circle is located at the origin and has a radius of one unit. This is because right triangles can be formed using the x -axis as one leg of the triangle and the line from the origin to a point on the circle as the hypotenuse with a measurement of one. Therefore, the point on the circle has coordinates of $(\cos(x), \sin(x))$ for the angle that is created by the x -axis and the hypotenuse. It is important to know that $(x, y) = (\cos(x), \sin(x))$ and can be found using the triangle that is created on the unit circle. The unit circle is primarily used for angles with multiples of 30° , 45° , 60° , and 90° . It is also important to recall the signs for the different trigonometric functions in each quadrant. In quadrant one, all functions are positive. In quadrant two, sine and cosecant are positive while the rest of the trigonometric functions are negative. In quadrant three, tangent and cotangent are positive and all other trigonometric functions are negative, and lastly, in quadrant four, cosine and secant are positive while the rest of the functions are negative. The (x, y) measurements in quadrant one will be the same in the other quadrants with the same basic angle, but with different signs based on the quadrant in which they are located.

The figure below has four different colored lines: black, red, blue, and yellow. The red lines are multiples of 30° or $\frac{\pi}{6}$, the blue lines are multiples of 45° or $\frac{\pi}{4}$, the yellow lines are multiples of 60° or $\frac{\pi}{3}$, and the black lines are multiples of 90° or $\frac{\pi}{2}$.

The $(x, y) = (\cos(x), \sin(x))$ measurements for these specific angles are as follows.

Quadrant II:

Red: $\angle \frac{5\pi}{6} = (-\frac{\sqrt{3}}{2}, \frac{1}{2})$

Blue: $\angle \frac{3\pi}{4} = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

Yellow: $\angle \frac{2\pi}{3} = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$

Black: $\angle \pi = (-1, 0)$

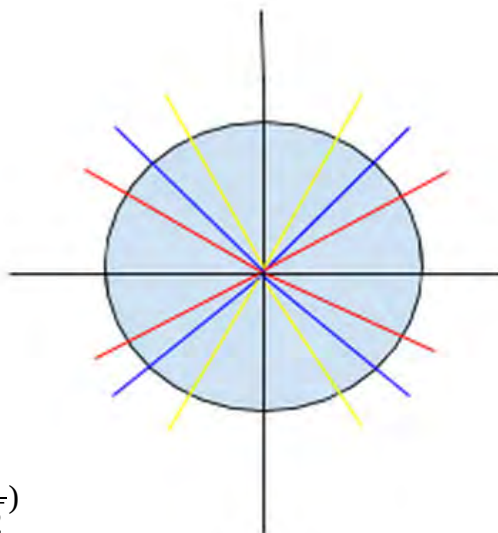
Quadrant III:

Red: $\angle \frac{7\pi}{6} = (-\frac{\sqrt{3}}{2}, -\frac{1}{2})$

Blue: $\angle \frac{5\pi}{4} = (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

Yellow: $\angle \frac{4\pi}{3} = (-\frac{1}{2}, -\frac{\sqrt{3}}{2})$

Black: $\angle \frac{3\pi}{2} = (0, -1)$



Quadrant I:

Red: $\angle \frac{\pi}{6} = (\frac{\sqrt{3}}{2}, \frac{1}{2})$

Blue: $\angle \frac{\pi}{4} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

Yellow: $\angle \frac{\pi}{3} = (\frac{1}{2}, \frac{\sqrt{3}}{2})$

Black: $\angle \frac{\pi}{2} = (0, 1)$

Quadrant IV:

Red: $\angle \frac{11\pi}{6} = (\frac{\sqrt{3}}{2}, -\frac{1}{2})$

Blue: $\angle \frac{7\pi}{4} = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

Yellow: $\angle \frac{5\pi}{3} = (\frac{1}{2}, -\frac{\sqrt{3}}{2})$

Black: $\angle 2\pi = (1, 0)$



Sample Question

Using the sum and difference formula for cosine, which states $\cos(u + v) = \cos(u)\cos(v) - \sin(u)\sin(v)$, what is $\cos(\frac{5\pi}{6})$?

Step 1: Break down the angle into two angles of addition that are used on the unit circle:

$$\text{Let } u = \frac{\pi}{3} \text{ and } v = \frac{\pi}{2}.$$

Step 2: Set up the sum formula for cosine:

$$\cos\left(\frac{5\pi}{6}\right) = \cos\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{2}\right)$$

Step 3: Plug in the specific values for the cosine and sine functions:

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{2}\right) = 0$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{2}\right) = 1$$

$$\cos\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{2}\right)$$

$$\cos\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = \frac{1}{2}(0) - \frac{\sqrt{3}}{2}(1)$$

$$\cos\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = 0 - \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = -\frac{\sqrt{3}}{2}$$



Sample Question

What is $\tan(\frac{5\pi}{3})$?

Step 1: Rewrite the trigonometric function using the quotient identity:

$$\tan(\frac{5\pi}{3}) = \frac{\sin(\frac{5\pi}{3})}{\cos(\frac{5\pi}{3})}$$

Step 2: Identify the $(x, y) = (\cos(x), \sin(x))$ for the specific angle:

$$(\cos(\frac{5\pi}{3}), \sin(\frac{5\pi}{3})) = (\frac{1}{2}, -\frac{\sqrt{3}}{2})$$

Step 3: Plug in the exact values and solve for tangent.

$$\frac{-\sqrt{3}}{\frac{1}{2}}$$

Since there is a fraction being divided by another fraction, multiply the top fraction by the reciprocal of the bottom fraction.

$$\frac{-\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$$

In this particular case, the 2 in the numerator and the 2 in the denominator cancel each other out, and we are left with a final answer of:

$$\tan(\frac{5\pi}{3}) = -\sqrt{3}$$

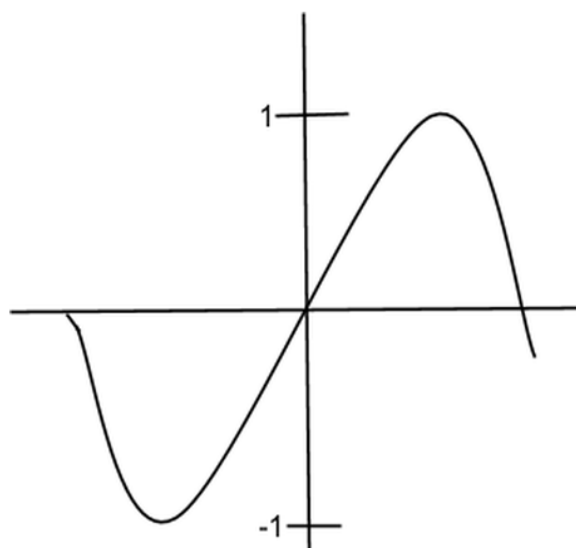
Trigonometric Function Graphs

All trigonometric functions have algebraic and graphical representations. When dealing with the graphical representation of trigonometric functions, remember to work in radians.

Sine Graphs

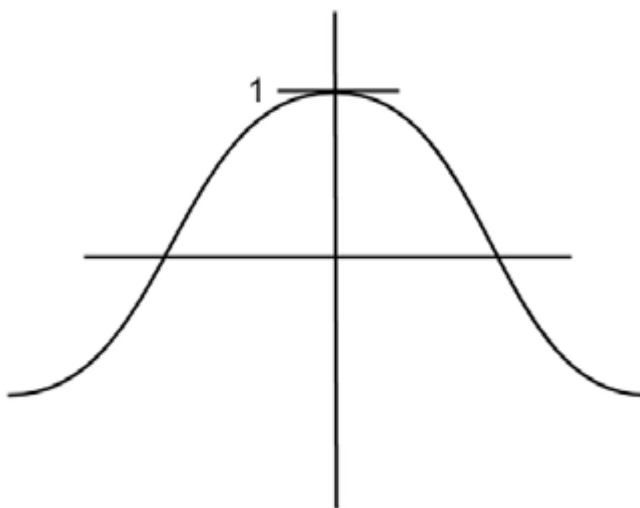
The graph of sine has a domain of all real numbers and a range from negative one to positive one. The period or time cycle of sine, also seen as the interval before the wave repeats, is 2π . The sine wave has an amplitude of one and passes through the origin. The basic sine function wave is depicted below. Recall the x-axis goes from $-\pi$ to π , and the maximum and minimum of the wave occurs at $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.





Cosine Graphs

The graph of cosine also has a domain of all real numbers and a range from negative one to positive one with a period of 2π and an amplitude of one. The cosine wave peaks at zero with a value of one. Cosine is the sine graph shifted to the right $\frac{\pi}{2}$. The basic cosine function wave is depicted below. Recall the x-axis goes from $-\pi$ to π , and the maximum and minimum of the wave occur at 0 and $-\pi, \pi$, respectively.

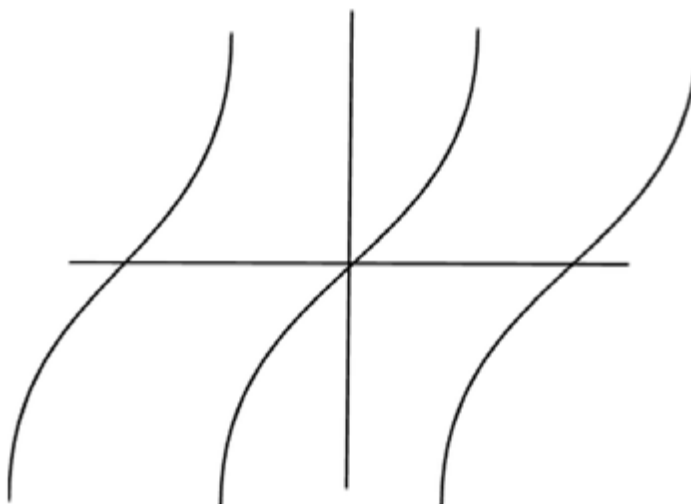


Tangent Graphs

The graph of tangent is different than the graphs of cosine and sine in a few ways. First and foremost, the graph of tangent has numerous asymptotes. This occurs at every $\frac{k\pi}{2}$, where k represents an odd integer. The reason behind these asymptotes stems from quotient identity of tangent, which states tangent is equal



to sine over cosine. Thus, there lies an asymptote everywhere cosine is zero.



Amplitude, Period, Frequency, and Phase Shifts

Translations of trigonometric function result in a number of modifications to the basic function. These modifications can be seen both algebraically as well as graphically. The general algebraic form that trigonometric functions take is as follows.

$$F(x) = Af(Bx - C) + D$$

A is the amplitude of the function, which indicates the magnitude or maximum height of the function.

B is used to find the period $\frac{2\pi}{|B|}$, which indicates the length of one cycle of the function before it begins to repeat. C is used to find the horizontal phase shift $\frac{C}{B}$, D represents the vertical shift, and the $f(x)$ is the basic form of the trigonometric function.

Sample Question

What is the equation of a sine function that has an amplitude of 3 and a period of 4π ?

Step 1: Write the general equation for a transformed sine function:

$$F(x) = A\sin(Bx - C) + D$$

Step 2: Identify the values for the variables:

$$A = 3, 4\pi = \frac{2\pi}{|B|} = \frac{1}{2}, C = 0, D = 0$$



Step 3: Plug the variable values into the transformed sine function:

$$F(x) = 3\sin\left(\frac{1}{2}x\right)$$

Sample Question

What is the phase and vertical shift of the following function?

$$F(x) = 4\tan(2x - \pi) + 2$$

Step 1: Write the general equation for a transformed tangent function:

$$F(x) = A\tan(Bx - C) + D$$

Step 2: Identify the values for the variables:

$$A = 4, \pi = \frac{2\pi}{|2|}, C = \pi, D = 2$$

Step 3: Recognize which variables represent the phase and vertical shift.

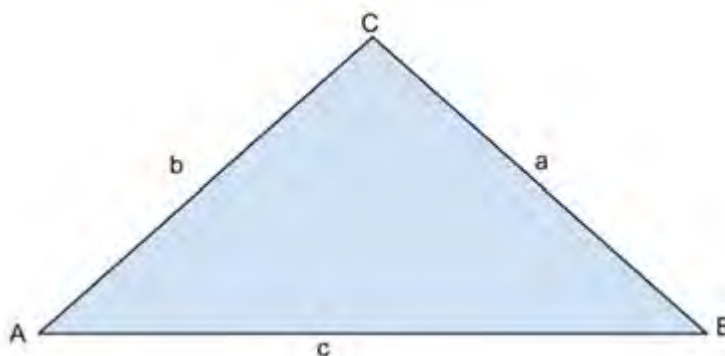
$C = \pi$ is the phase shift

$D = 2$ is the vertical shift

Solutions for Triangles

Several theories have contributed to our understanding of triangles and their attributes. The Law of Sines and the Law of Cosines are used to find relationships between the side lengths and angles of triangles. These laws are fundamental in trigonometry and will be useful on test day to help you find the solutions for various triangles. Master the given techniques and formulas. Familiarity with these mathematical relationships will be an asset on the ACT.

Consider the following triangle:



If a problem refers to only two sides and two angles of a triangle, use the Law of Sines to find the third. The Law of Sines states:



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

If a problem refers to three sides of the triangle and only one angle, use the Law of Cosines to find the missing angles. The Law of Cosines states the following:

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$b^2 = a^2 + c^2 - 2ac \cos(B)$$

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

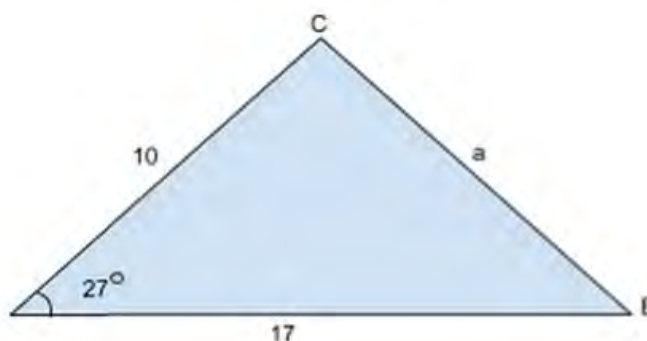
In this equation, s is known as the semiperimeter and equals the following:

$$s = \frac{a+b+c}{2}$$

Use these formulas to solve the following questions.

Sample Question

What is the measure of side a in the following triangle?



Step 1: Identify which Law of Cosine equation to use:

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

Step 2: Identify the values for the variables to be used in the formula:

$$a = ?, b = 10, c = 17, \angle A = 27^\circ$$

Step 3: Plug values into formula:



$$a^2 = 10^2 + 17^2 - 2(10)(17)\cos(27^\circ)$$

Step 4: Solve for a :

$$a^2 = 100 + 289 - 340\cos(27^\circ)$$

$$a^2 = 389 - 340(0.891)$$

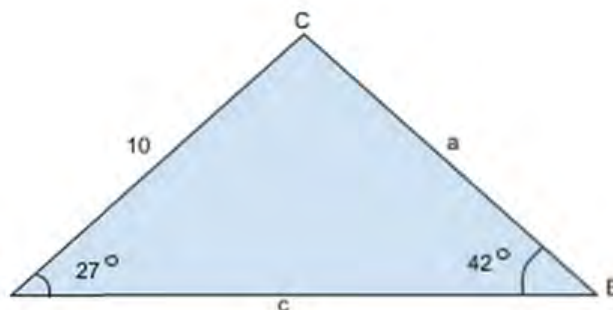
$$a^2 = 86.06$$

$$a = \sqrt{86.06}$$

$$a = 9.28$$

Sample Question

What is the measure of side a in the following triangle?



Step 1: Identify which Law of Sines equation to use:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Step 2: Identify the values for the variables to be used in the formula:

$$a = ?, b = 10, \angle A = 27^\circ, \angle B = 42^\circ$$

Step 3: Plug values into formula:

$$\frac{\sin(27)}{a} = \frac{\sin(42)}{10}$$

Step 4: Solve for a :

$$\frac{0.4540}{a} = \frac{0.6691}{10}$$

$$\frac{0.4540}{a} = 0.0669$$

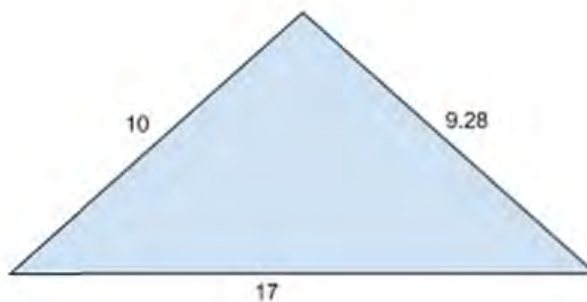
$$0.0669a = 0.4540$$

$$a = 6.79$$



Sample Question

What is the area of the following non-right triangle?



Step 1: Identify which method will be used to solve for the area:

In this particular case, since the triangle is a non-right triangle and all three sides are known, Heron's Formula will be used.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

In this equation, s is known as the semiperimeter and equals the following:

$$s = \frac{a+b+c}{2}$$

Step 2: Identify the variable values:

$$a = 9.28, b = 10, c = 17$$

$$s = \frac{9.28 + 10 + 17}{2} = \frac{36.28}{2} = 18.14$$

Step 3: Plug values into Heron's Formula and solve for the area:

$$A = \sqrt{18.14(18.14 - 9.28)(18.14 - 10)(18.14 - 17)}$$

$$A = \sqrt{18.14(8.86)(8.14)(1.14)}$$

$$A = \sqrt{1491.421}$$

$$A = 38.62$$



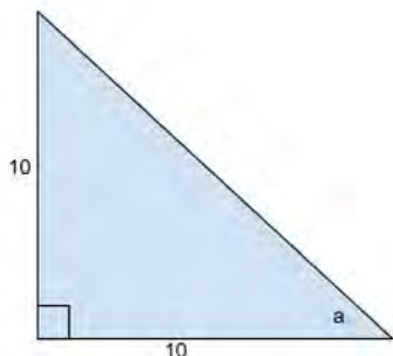
Practice Problems: Trigonometry

1. What is the sine of a in the following triangle?



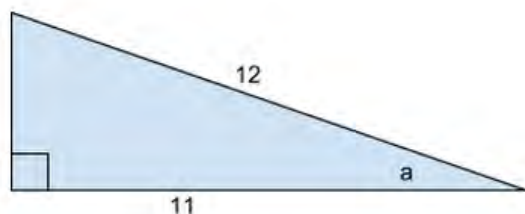
- A. 0.5
- B. 1
- C. 2
- D. None of the other answers

2. What is the tangent of a in the following triangle?



- A. 10
- B. 1
- C. 0.5
- D. None of the other answers

3. What is the cosine of a in the following triangle?



- A. 12
- B. $\frac{11}{12}$
- C. $\frac{5}{6}$
- D. None of the other answers

4. Simply the following trigonometric expression:

$$\frac{\sin(x)\tan(x)\csc^2(x)\cos(x)}{\cos^2(x)}$$

- A. $\tan(x)\sec(x)$
- B. $\sec^2(x)$
- C. $\tan(x)$
- D. $\sin(x)\cos(x)$

5. Simplify the following trigonometric expression:

$$2\sin^2(x) + \tan(x) + 2\cos^2(x)$$

- A. $2 + \tan(x)$
- B. $1 + \tan(x)$
- C. $\sin^2(x) + 1$
- D. 1

6. Write the following trigonometric expression in terms of sine and cosine:

$$\csc^2(x)\tan(x)$$

- A. $\frac{\sin(x)}{\cos(x)}$
- B. $\cos^2(x)$
- C. $\frac{1}{\sin(x)\cos(x)}$
- D. None of the other answers

7. Using the unit circle, what is the exact value of $\cos\frac{9\pi}{6}$?

- A. -1
- B. $\frac{\sqrt{3}}{2}$
- C. $-\frac{1}{2}$
- D. 0



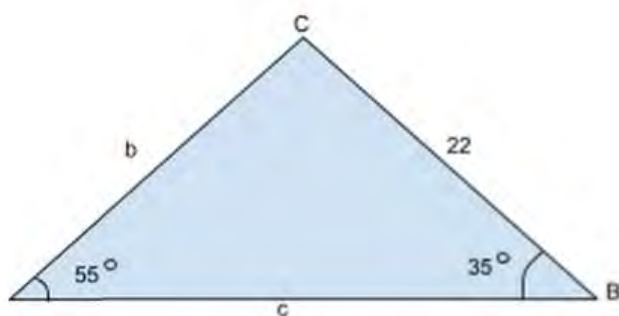
8. What is the equation for a cosine graph with an amplitude of five and a vertical shift of three?

A. $5 \cos(x) + 3$
 B. $3 \cos(x) + 5$
 C. $5 \cos(3x)$
 D. $3 \cos(x + 5)$

9. Using the double angle identity for sine, which states $\sin(2u) = 2 \sin(u) \cos(u)$, solve for the exact value of $\sin(2 \cdot \frac{5\pi}{6})$.

A. $-\frac{\sqrt{3}}{2}$
 B. $\frac{\sqrt{3}}{2}$
 C. $\frac{1}{2}$
 D. $-\frac{1}{2}$

10. Use the Law of Sines ($\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$) to find the measure of side b in the following triangle.



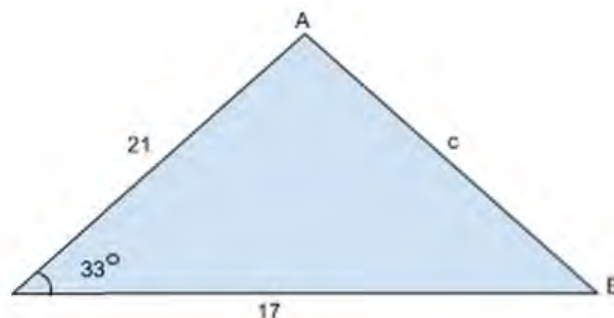
A. 14.52
 B. 15.52
 C. 25.52
 D. None of the other answers

11. Use the Law of Cosines, written out below, to find the measure of side c in the following triangle.

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

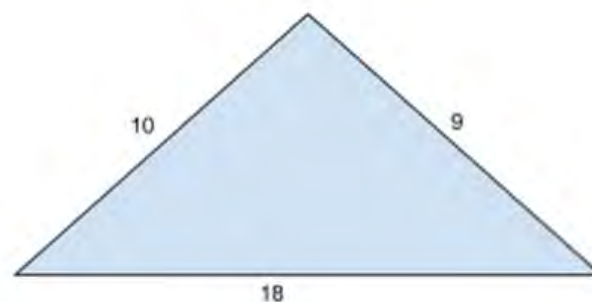
$$b^2 = a^2 + c^2 - 2ac \cos(B)$$

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$



A. 41.51
 B. 14.15
 C. 11.45
 D. None of the other answers

12. Find the area of the following non-right triangle.



A. 27.33
 B. 23.73
 C. 27.37
 D. 17.33



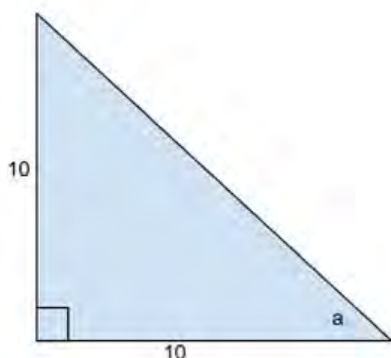
Practice Problems: Trigonometry: Answer Key

1. What is the sine of a in the following triangle?



- A. 0.5
 B. 1
 C. 2
 D. None of the other answers

2. What is the tangent of a in the following triangle?



- A. 10
 → B. 1
 C. 0.5
 D. None of the other answers

Remember **SOHCAHTOA**.

SOH: Sine of a equals **O**pposite side divided by the **H**ypotenuse

$$\sin(a) = \frac{\text{opp}}{\text{hyp}}$$

In the figure, the side opposite of a is 0.5 and the hypotenuse is 1. In order to obtain the correct answer, substitute these variables into the above sine equation.

$$\sin(a) = \frac{0.5}{1}$$

Reduce and simplify.

$$\sin(a) = 0.5$$

Remember **SOHCAHTOA**.

TOA: Tangent of a equals **O**pposite side divided by the **A**djacent side

$$\tan(a) = \frac{\text{opp}}{\text{adj}}$$

In the figure, the side opposite of a is 10 and the side adjacent to a is 10. In order to obtain the correct answer, substitute these variables into the above tangent equation.

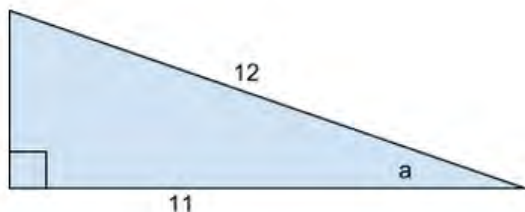
$$\tan(a) = \frac{1}{1}$$

Reduce and simplify.

$$\tan(a) = 1$$



3. What is the cosine of a in the following triangle?



- A. 12
 → B. $\frac{11}{12}$
 C. $\frac{5}{6}$
 D. None of the other answers

4. Simply the following trigonometric expression:

$$\frac{\sin(x)\tan(x)\csc^2(x)\cos(x)}{\cos^2(x)}$$

- A. $\tan(x)\sec(x)$
 → B. $\sec^2(x)$
 C. $\tan(x)$
 D. $\sin(x)\cos(x)$

Remember **SOHCAHTOA**.

CAH: Cosine of a equals **A**djacent side divided by the **H**ypotenuse

$$\cos(a) = \frac{adj}{hyp}$$

In the figure, the side adjacent to a is 11 and the hypotenuse is 12. In order to obtain the correct answer, substitute these variables into the above cosine equation.

$$\cos(a) = \frac{11}{12}$$

This result cannot be reduced and is the correct answer.

Step 1: Write all terms in terms of sine and cosine.

Recall that by the quotient identity $\tan(x) = \frac{\sin(x)}{\cos(x)}$ and by the reciprocal identity $\csc^2(x) = \frac{1}{\sin^2(x)}$,

the expression can be written as follows:

$$\frac{\sin(x) \frac{\sin(x)}{\cos(x)} \frac{1}{\sin^2(x)} \cos(x)}{\cos^2(x)}$$

Step 2: Rewrite the numerator to be one fraction.

$$\frac{\frac{\sin^2(x)\cos(x)}{\cos(x)\sin^2(x)}}{\cos^2(x)}$$

Step 3: Reduce the common terms in the fractional numerator.

In this particular case, the sine squared terms cancel, as do the cosine terms. Therefore the expression becomes:

$$\frac{1}{\cos^2(x)}$$

Step 4: Use the reciprocal identity to simplify the expression.

$$\frac{1}{\cos^2(x)} = \sec^2(x)$$



5. Simplify the following trigonometric expression:

$$2\sin^2(x) + \tan(x) + 2\cos^2(x)$$

- **A.** $2 + \tan(x)$
B. $1 + \tan(x)$
C. $\sin^2(x) + 1$
D. 1

6. Write the following trigonometric expression in terms of sine and cosine:

$$\csc^2(x)\tan(x)$$

- A.** $\frac{\sin(x)}{\cos(x)}$
B. $\cos^2(x)$
 → **C.** $\frac{1}{\sin(x)\cos(x)}$
D. None of the other answers

Step 1: Identify the Pythagorean identity that is being used.

$$\sin^2(x) + \cos^2(x) = 1, \text{ therefore,}$$

$$2\sin^2(x) + 2\cos^2(x) = 2(\sin^2(x) + \cos^2(x)) = 2(1) = 2$$

Step 2: Plug new value for identity back into original expression:

$$2 + \tan(x)$$

This is the simplified form of this expression.

Writing the following trigonometric expression in terms of sine and cosine:

$$\csc^2(x)\tan(x)$$

Step 1: Use the reciprocal identity to write cosecant in terms of sine:

$$\frac{1}{\sin^2(x)} = \csc^2(x)$$

Step 2: Use the quotient identity to write tangent in terms of sine and cosine:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

Step 3: Replace these new versions into the original expression:

$$\frac{1}{\sin^2(x)} \cdot \frac{\sin(x)}{\cos(x)}$$

Step 4: Simplify fraction by canceling common factors that are in both the numerator and denominator:

In this particular case, one sine in the denominator will cancel the sine in the numerator.

$$\frac{1}{\sin(x)\cos(x)}$$



7. Using the unit circle, what is the exact value of $\cos \frac{9\pi}{6}$?

- A. -1
 B. $\frac{\sqrt{3}}{2}$
 C. $-\frac{1}{2}$

→ D. 0

8. What is the equation for a cosine graph with an amplitude of five and a vertical shift of three?

- A. $5 \cos(x) + 3$
 B. $3 \cos(x) + 5$
 C. $5 \cos(3x)$
 D. $3 \cos(x + 5)$

9. Using the double angle identity for sine, which states $\sin(2u) = 2 \sin(u) \cos(u)$, solve for the exact value of $\sin\left(2 \cdot \frac{5\pi}{6}\right)$.

- A. $-\frac{\sqrt{3}}{2}$
 B. $\frac{\sqrt{3}}{2}$
 C. $\frac{1}{2}$
 D. $-\frac{1}{2}$

Step 1: Reduce the angle.

$$\frac{9\pi}{6} = \frac{3 \cdot 3\pi}{3 \cdot 2} = \frac{3\pi}{2}$$

Step 2: Identify the quadrant in which the angle is found:

$\frac{3\pi}{2}$ falls on the y -axis between quadrants III and IV.

This means that the cosine will be zero and the sine will be negative one since the point on that particular line is $(0, -1) = (x, y) = (\cos(\theta), \sin(\theta))$.

Step 1: Write the general form for the trigonometric transformation equation:

$$F(x) = A \cos(Bx - C) + D$$

Step 2: Identify the values for the variables:

$$A = 5, B = 1, C = 0, D = 3$$

Step 3: Plug variable values into the transformation equation:

$$F(x) = 5 \cos(x) + 3$$

Step 1: Identify u :

$$u = \frac{5\pi}{6}$$

Step 2: Find the $(x, y) = (\cos(\theta), \sin(\theta))$ of the angle using the unit circle:

$$(x, y) = (\cos(\theta), \sin(\theta)) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

Step 3: Plug the exact values into the double angle formula:

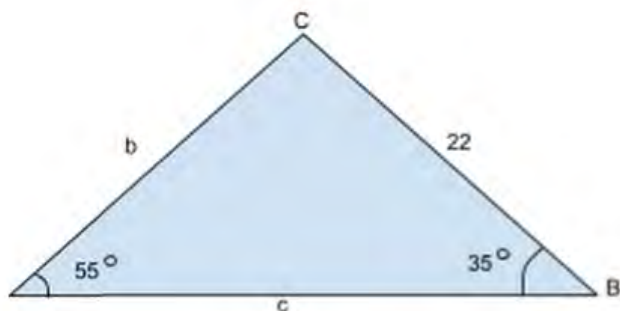
$$\begin{aligned} \sin\left(2 \cdot \frac{5\pi}{6}\right) &= 2 \sin\left(\frac{5\pi}{6}\right) \cos\left(\frac{5\pi}{6}\right) \\ \sin\left(2 \cdot \frac{5\pi}{6}\right) &= 2 \left(\frac{1}{2}\right) \left(-\frac{\sqrt{3}}{2}\right) \end{aligned}$$

Step 4: Simplify expression by canceling common factors in both the numerator and denominator. In this particular case, the two in the numerator and denominator cancel out and we are left with the final answer of:

$$\sin\left(2 \cdot \frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$



10. Use the Law of Sines ($\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$) to find the measure of side b in the following triangle.



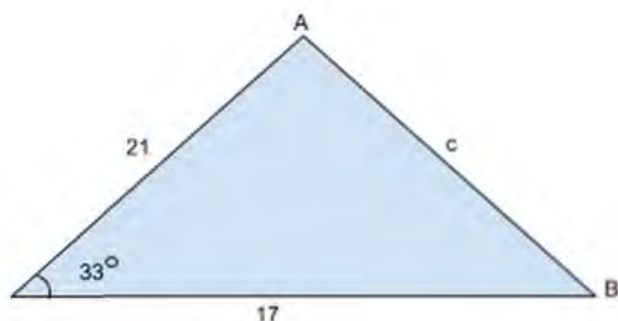
- A. 14.52
 → B. 15.42
 C. 25.52
 D. None of the other answers

11. Use the Law of Cosines, written out below, to find the measure of side c in the following triangle.

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$b^2 = a^2 + c^2 - 2ac \cos(B)$$

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$



- A. 41.51
 B. 14.15
 → C. 11.45
 D. None of the other answers

Step 1: Identify which Law of Sines equation to use:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

Step 2: Identify the values for the variables to be used in the formula:

$$a = 22, b = ?, \angle A = 55^\circ, \angle B = 35^\circ$$

Step 3: Plug values into formula:

$$\frac{\sin(55)}{22} = \frac{\sin(35)}{b}$$

Step 4: Solve for b :

$$\frac{0.8192}{22} = \frac{0.5736}{b}$$

$$\frac{0.5736}{b} = 0.0372$$

$$0.0372b = 0.5736$$

$$b = 15.42$$

Step 1: Identify which Law of Cosine equation to use:

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

Step 2: Identify the values for the variables to be used in the formula:

$$c = ?, b = 21, a = 17, \angle C = 33^\circ$$

Step 3: Plug values into formula:

$$c^2 = 17^2 + 21^2 - 2(17)(21)\cos(33^\circ)$$

Step 4: Solve for c :

$$c^2 = 289 + 441 - 714 \cos(33^\circ)$$

$$c^2 = 730 - 714(0.8387)$$

$$c^2 = 730 - 598.81$$

$$c^2 = 131.19$$

$$c^2 = \sqrt{131.19}$$

$$c = 11.45$$



12. Find the area of the following non-right triangle.



- A. 27.33
B. 23.73
C. 27.37
D. 17.33

Step 1: Identify which method will be used to solve for the area:

In this particular case, since the triangle is a non-right triangle and all three sides are known, the Heron's formula will be used.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Where s is known as the semiperimeter and equals the following:

$$s = \frac{a+b+c}{2}$$

Step 2: Identify the variable values:

$$a = 9, b = 10, c = 18$$

$$s = \frac{9+10+18}{2} = \frac{37}{2} = 18.5$$

Step 3: Plug values into the Heron's Formula and solve for the area:

$$A = \sqrt{18.5(18.5-9)(18.5-10)(18.5-18)}$$

$$A = \sqrt{18.5(9.5)(8.5)(0.5)}$$

$$A = \sqrt{746.9375}$$

$$A = 27.33$$



Full-Length ACT Diagnostic Test

Directions

The following pages contain tests in English, mathematics, reading, and science. These tests measure skills and abilities highly related to high school course work and success in college. **Calculators may be used on the mathematics test only.**

The questions in each test are numbered, and the suggested answers for each question are lettered. For each question, decide which answer is best. Choose only one answer to each question.

We recommend taking each diagnostic test sequentially in a timed environment to simulate the actual exam. If you do this and have extra time for a test, you should use the time remaining to reconsider questions you are unsure about in that test. You may **not** look back to a test you have already finished, and you may **not** go ahead to another test.

Good luck!



ENGLISH TEST

45 Minutes—75 Questions

DIRECTIONS: In the five passages that follow, certain words and phrases are underlined and numbered. In the right-hand column, you will find alternatives for the underlined part. In most cases, you are to choose the one that best expresses the idea, makes the statement appropriate for standard written English, or is worded most consistently with the style and tone of the passage as a whole. If you think the original version is best, choose "NO CHANGE." In some cases, you will find in the right-hand column a question about the underlined part. You are to choose the best answer to the question.

You will also find questions about a section of the passage, or about the passage as a whole. These questions do not refer to an underlined portion of the passage, but rather are identified by a number or numbers in a box.

For each question, choose the alternative you consider best. Read each passage through once before you begin to answer the questions that accompany it. For many of the questions, you must read several sentences beyond the question to determine the answer. Be sure that you have read far enough ahead each time you choose an alternative.

PASSAGE I

During the final months of 2007, the prices of basic grains nearly doubled in Northern Africa, Latin America, and much of Asia, the¹ high prices caused a global food crisis. The catastrophe sparked and incited² an international debate regarding the licensing of new technologies to developing nations. One economist warned that because of the risk of unforeseen price shocks, officials should proceed very cautiously³. The construction of private farms pose⁴ a serious financial threat to farmers in the United States; nevertheless, of the five most industrialized nations, the United States exports more crops⁵.

How do some countries cope with food crises better than others⁶. It is technology that accounts

1. **A. NO CHANGE**
B. of Asia. The
C. of Asia, the quite
D. for Asia. The
2. **E. NO CHANGE**
F. that incited
G. sparked, initiated, and incited
H. incited
3. **A. NO CHANGE**
B. cautious
C. caution
D. and cautiously
4. **E. NO CHANGE**
F. have posed
G. poses
H. are posing
5. **A. NO CHANGE**
B. the most crops
C. a greater number of
D. that great number
6. **E. NO CHANGE**
F. others;
G. others?
H. others,

GO ON TO THE NEXT PAGE.



for the majority of the difference. The rate at which countries adopt innovations depends₇ significantly on environmental factors. These environmental factors include climate, soil and₈ elevation₈. The variability in environment inhibits new technologies from gaining worldwide₉ popularity that are suited for one particular₉ region over another₉. For example₁₀, the pesticides used in Europe are much more acidic than North₁₁ America₁₁. Without the different levels of acidity, pests would prevent the crops to grow₁₂.

7. **A.** NO CHANGE
B. depend
C. are depending
D. have been depended
8. **E.** NO CHANGE
F. soil, and last but not least, elevation
G. soil, and elevation
H. soil, and, of course, elevation
9. **A.** NO CHANGE
B. new technologies that are suited for one particular region over another from gaining worldwide popularity
C. new technologies that is suited for one particular region over another from gaining worldwide popularity
D. new technologies from gaining worldwide popularity that have been suited for one particular region over another
10. **E.** NO CHANGE
F. Therefore
G. However
H. As a result
11. **A.** NO CHANGE
B. that used in North America
C. those used in North America's
D. those used in North America
12. **E.** NO CHANGE
F. to be growing
G. from growing
H. of growth

GO ON TO THE NEXT PAGE.



PASSAGE II

Adapted from *The Autobiography of John Adams*
(ed. 1856)

Not long after this, the three greatest measures of all were carried. Three committees were
13
appointed, one for preparing a declaration of
13
independence, another for reporting a plan of a treaty to be proposed to France, and a third to digest a system of articles of confederation to be proposed to the States. I was appointed on the committee of independence and on that for preparing the form of a treaty with France. On the
14
committee of confederation Mr. Samuel Adams
14
was appointed. The committee of independence
14 15
were Thomas Jefferson, John Adams, Benjamin
15
Franklin, Roger Sherman, and Robert R. Livingston. Mr. Jefferson had been now about a
16
year a member of Congress, but had attended his
16
duty in the house a very small part of the time,
16
and, when there, had never spoken in public.
16
During the whole time I sat with him in Congress,

I never hear him utter three sentences together.
17

13. A. NO CHANGE

- B. Three committees were appointed
- C. Three committees were appointed;
- D. Three committees were appointed:

14. E. NO CHANGE

- F. Mr. Samuel Adams was appointed to the committee of confederation.
- G. Mr. Samuel Adams was appointed upon the committee of confederation.
- H. On the committee of confederation was Mr. Samuel Adams appointed.

15. A. NO CHANGE

- B. The committee of independence had been
- C. The committee of independence was comprised of
- D. The committee of independence included

16. E. NO CHANGE

- F. Mr. Jefferson had been now about a year a member of Congress, but had attended his duty in the house a very small part of the time. When there, had never spoken in public.
- G. Mr. Jefferson had been now about a year a member of Congress; however, he had attended his duty in the house a very small part of the time and, when there, had never spoken in public.
- H. Mr. Jefferson had been now about a year a member of Congress but had attended his duty in the house a very small part of the time, and, when there, had never spoken in public.

17. A. NO CHANGE

- B. I never heard him utter three sentences together.
- C. I never would hear him utter three sentences together.
- D. I never could hear him utter three sentences together.

GO ON TO THE NEXT PAGE.



It will naturally be inquired how it happened that he was appointed on a committee of such importance. There were more reasons than one.

Mr. Jefferson had the reputation of a masterly
18
pen; he had been chosen a delegate in Virginia,
18
in consequence of a very handsome public paper
18
which he had written for the House of Burgesses,
18
which had given him the character of a fine writer.
18

Another reason was, that Mr. Richard Henry Lee
19
was not beloved by the most of his colleagues from
19
Virginia, and Mr. Jefferson was set up to rival
19
and supplant him. This could be done only by the
20
pen, for Mr. Jefferson could stand no competition
21
with him or any one else in elocution and public debate.

18. E. NO CHANGE

- F. Mr. Jefferson had the reputation of a masterly pen for he had been chosen a delegate in Virginia in consequence of a very handsome public paper that he had written for the House of Burgesses that had given him the character of a fine writer.
- G. Mr. Jefferson had the reputation of a masterly pen, for he had been chosen a delegate in Virginia in consequence of a very handsome public paper which he had written for the House of Burgesses that had given him the character of a fine writer.
- H. Mr. Jefferson had the reputation of a masterly pen; he had been chosen a delegate in Virginia in consequence of a very handsome public paper which he had written for the House of Burgesses and which had given him the character of a fine writer.

19. A. NO CHANGE

- B. Another reason was that Mr. Richard Henry Lee was not beloved by most of his colleagues from Virginia
- C. Additionally, Mr. Richard Henry Lee was not beloved by the most of his colleagues from Virginia
- D. Therefore Mr. Richard Henry Lee was not beloved by most of his colleagues from Virginia

20. To what does “this” refer?

- E. The potential supplanting of Lee by Jefferson
- F. The rivalry between Lee and his colleagues
- G. The overt hatred of Lee by his colleagues
- H. Jefferson’s excellent writing abilities

21. If you wanted to split this paragraph into two separate paragraphs, which sentence would be the second paragraph?

- A. “Mr. Jefferson had been now about a year . . .”
- B. “The committee of independence . . .”
- C. “Mr. Jefferson had the reputation of . . .”
- D. “During the whole time that I . . .”

22. Which of the following words would best replace the word “stand” in the last sentence?

- E. NO CHANGE
- F. position
- G. endure
- H. argue

GO ON TO THE NEXT PAGE.



PASSAGE III

Adapted from “The Nose Tree” in *German Fairy Tales and Popular Stories* by Jacob Grimm and Wilhelm Grimm (trans. Taylor, ed. 1864)

Then the king made known to all his kingdom,
23

that whomever would heal her of this dreadful
23 24

disease should be richly rewarded. Many tried,
25

but the princess got no relief. Now the old soldier

dressed himself up very sprucely as a doctor, and
26

said he could cure her. Therefore, he chopped
26

up some of the apple, and, to punish her a little
27

more, gave her a dose, saying he would call to-
27

morrow and see her again. The morrow came,

and, of course, instead of being better, the nose

had been growing on all night as before; and the
28 29

poor princess was in a dreadful fright. So the
29

doctor then chopped up a very little of the pear

and gave it to her. He said that he was sure that it
30

would help, and he would call again the next day.
30

23. A. NO CHANGE

- B. known to all his kingdom that
- C. known to all his kingdom: that
- D. known to all his kingdom; that

24. E. NO CHANGE

- F. he whom healed her of this dreadful disease
- G. whomever could heal her of this dreadful disease
- H. whoever would heal her of this dreadful disease

25. A. NO CHANGE

- B. disease would be richly rewarded
- C. disease will be richly rewarded
- D. disease could be richly rewarded

26. E. NO CHANGE

- F. sprucely, as a doctor, and said he could cure her
- G. sprucely as a doctor and said he could cure her
- H. sprucely as a doctor and said, he could cure her

27. A. NO CHANGE

- B. apple and, to punish her a little more,
- C. apple, and to punish her a little more,
- D. apple and to punish her a little more

28. Which of the following is an acceptable replacement for the underlined selection, “had been growing on?”

- E. grow
- F. grew
- G. had grown
- H. had continued growing

29. A. NO CHANGE

- B. as before but the poor princess was in a dreadful fright
- C. as before; therefore, the poor princess was in a dreadful fright
- D. as before; nevertheless, the poor princess was in a dreadful fright

30. E. NO CHANGE

- F. it would help and he would
- G. it would help and that he would
- H. it would help; and he would

GO ON TO THE NEXT PAGE.



Next day came, and the nose was to be sure a little

31

smaller. However, it was bigger than when the

doctor first began to meddle with it.

32

Then he thought to him, "I must frighten this

33

cunning princess a little more before my plan will

work." Therefore, he gave her another dose of the

apple and said he would call on the morrow. The

34

morrow came, and the nose was ten times bad as

34

before.

34

"My good lady," said the doctor, "Something

35

works against my medicine and is too strong for

35

36

it. However, I know by the force of my art that

36

37

it is this, you have stolen goods about you. I am

37

38

certain of it. If you do not give them back, I can do

nothing for you."

The princess denied very stoutly that she had

anything of the kind.

31. What is the best replacement for the underlined selection, "to be sure?"

- A. indeed
- B. unquestionably
- C. verifiably
- D. accurately

32. E. NO CHANGE

- F. had first began
- G. had first begun
- H. was beginning

33. A. NO CHANGE

- B. Then he thought to himself
- C. Then he thought to he
- D. Then he thought to them

34. E. NO CHANGE

- F. The morrow came and the nose was ten times bad as before.
- G. The morrow came, and the nose was ten times bad as before.
- H. The morrow came, and the nose was ten times worse than before.

35. A. NO CHANGE

- B. "My good lady," said the doctor, "Something works against
- C. "My good lady" said the doctor, "Something works against
- D. "My good lady," said the doctor, "something works against

36. E. NO CHANGE

- F. my medicine, and is too strong for it
- G. my medicine and is too strong for it
- H. my medicine, and is too strong for them

37. A. NO CHANGE

- B. art that it is this. You have stolen goods
- C. art that it is this: you have stolen goods
- D. art that it is this; you have stolen good

38. What is the meaning of the underlined word "about?"

- E. around
- F. concerning
- G. pertaining to
- H. connected to

GO ON TO THE NEXT PAGE.



“Very well,” said the doctor, “you may do as
39

you please, but I am sure I am correct. You will die

if you do not own it.” Then he went to the king,
40

and told him how the matter stood.

“Daughter,” said he, “send back the cloak, the
41

purse, and the horn, that you stole from the
41

rightful owners.”
41

Then she ordered her maid to fetch all three
42

and gave them to the doctor, and begged him to
42

give them back to the soldiers. The moment he
42

had them safe in his possession, he gave her a
43

whole pear to eat, and the nose came right. And

as for the doctor, he put on the cloak, wished the
44

king and all his court a good day and was soon
44

with his two brothers. They lived from that time
44

happily at home in their palace, except when they
45

took an airing to see the world in their coach with
45

their three dapple-grey horses.

39. A. NO CHANGE

B. “Very well,” said the doctor: “you may do as

C. “Very well”, said the doctor, “you may do as

D. “Very well.” Said the doctor. “You may do as

40. Which of the following is the best replacement for the underlined word “own?”

E. possess

F. acknowledge responsibility for

G. acknowledge possession of

H. take control of

41. A. NO CHANGE

B. send back the cloak, the purse, and the horn, all of which you stole from the rightful owners

C. send back the cloak, the purse, and the horn, which you stole from the rightful owners

D. send back the cloak, the purse, and the horn, that you had stolen from the rightful owners

42. E. NO CHANGE

F. three, and gave them to the doctor, and begging him to give them back to the soldiers

G. three and gave them to the doctor, begging him to give them back to the soldiers

H. three, gave them to the doctor, and begged him to give them back to the soldiers

43. A. NO CHANGE

B. safer

C. safely

D. more safely

44. E. NO CHANGE

F. wished the king and all his court a good day, and was soon with his two brothers

G. wished the king, and all his court, a good day and was soon with his two brothers

H. wished the king, and all his court a good day and was soon with his two brothers

45. A. NO CHANGE

B. palace, except when they took an airing in their coach to see the world

C. palace except when they took an airing to see the world in their coach

D. palace, except when they took an airing to see the world, in their coach

GO ON TO THE NEXT PAGE.



PASSAGE IV

Adapted from “Emerson’s Prose Works” in *The Works of Orestes A. Brownson: Philosophy of Religion* by Orestes Brownson (ed. 1883)

Mr. Emersons literary reputation is
46 47

established and placed beyond the reach of
47

criticism. No living writer surpasses him in his
mastery of pure and classic English; nor do any
48

equal him—neither in the exquisite delicacy and
48 49

finish of his chiseled sentences, or in the metallic
49

ring of his style. It is only as a thinker and teacher
49

that we can venture any inquiry into his merits,

and as such, we cannot suffer ourselves to be
50

imposed upon by his oracular manner, nor by

the apparent originality either of his views or his

expressions.

46. E. NO CHANGE

F. Mr. Emerson’s literary reputation

G. Mr. Emersons’ literary reputation

H. The literary of Mr. Emerson’s reputation

47. A. NO CHANGE

B. is established

C. is established, placed, and located

D. is placed and located

48. E. NO CHANGE

F. nor have any equalled him

G. nor are any able to equal him

H. nor does any equal him

49. A. NO CHANGE

B. neither in the exquisite delicacy and finish of his
chiseled sentences, nor in the metallic ring of his
style

C. neither in the exquisite delicacy and finish of his
chiseled sentences, and in the metallic ring of
his style

D. neither in the exquisite delicacy and finish of his
chiseled sentences or in the metallic ring of his
style

50. What is an adequate synonym for “suffer” as it is
used here?

E. NO CHANGE

F. allow

G. pain

H. destroy

GO ON TO THE NEXT PAGE.



Mr. Emerson has had a swarm both of
51

admirers but also of detractors. With many, he is
51 52

a philosopher and sage, almost a god; while with

others, he is regarded as an unintelligible mystic,
53

babbling nonsense fitted to captivate beardless

young men and silly maidens with pretty curls, all
54

of who constituted years ago the great body
54

of his hearers and worshipers. We rank us in

neither class, though we regard he as no ordinary
55

man. Indeed, we believe he to be one of the
55 56

deepest thinkers as well as one of the first poets of

our country. Indeed, by long acquaintance
57

have him and us been in mutual contact—if only
57

from a distance at times. We know him to be a

polished gentleman, a genial companion, and a

warmhearted friend, whose' kindness does not
58

pass over individuals and waste itself in a vague
58

philanthropy. So much, at least, we can say of the

man, and this do we base not only upon former

personal acquaintance and upon our former study
59

of his writings.
59

51. A. NO CHANGE
B. both admirers and with detractors
C. both of admirers or of detractors
D. both of admirers and of detractors

52. What is an acceptable replacement for the underlined word “many?”
E. NO CHANGE
F. many people
G. many fools
H. many reasons

53. A. NO CHANGE
B. he has been regarded
C. he had been regarded
D. he was regarded

54. E. NO CHANGE
F. all of whom
G. all of who's
H. all of whose

55. A. NO CHANGE
B. though we regard he like no ordinary man
C. though we regarded him as no ordinary man
D. though we regard him as no ordinary man

56. E. NO CHANGE
F. we believe that he to be one
G. we believe him to be one
H. we believe, him to be, one

57. A. NO CHANGE
B. by long acquaintance have we been in mutual contact with each other
C. by long acquaintance have we been in mutual contact
D. by long acquaintance, have we been in mutual contact

58. E. NO CHANGE
F. whose kindness does not pass over individuals
G. who's kindness does not pass over individuals
H. whom kindness does not pass over individuals

59. A. NO CHANGE
B. acquaintance and also upon our former study of his writings
C. acquaintance but also upon our former study of his writings
D. acquaintance but upon our former study of his writings

GO ON TO THE NEXT PAGE.



PASSAGE V

[1] The sport of lacrosse, while perhaps not as widely popular today in the United States as baseball or football, is far older. [60] [2] These games served many important cultural functions. [3] They were used to settle disputes between tribes, as festival events, and to train young men to become warriors and hunters.

[4] Hundreds of men and women from rival tribes or villages would gather to play at once. [5] The playing field was sometimes several miles long. [61] [6] The original game was very different from the organized sport played today. [62] [7] A single game would be played from dawn until sunset, and be followed by dancing and feasting.

[8] Modern, standardized versions of lacrosse started to be played in the 1850s and soon became very popular throughout Canada and the United States. [9] For over a century, it has been one of the most widely played sports in high schools in both nations. [63]

60. What would be the best sentence to insert between Sentence 1 and Sentence 2?
- E. The first modern lacrosse league was organized in Montreal in 1856.
 - F. Numerous Native American tribes, including the Iroquois, Cherokee, and Onondaga have played versions of it for at least 400 years.
 - G. The sport has quickly gained popularity in recent years.
 - H. Consequently, professional lacrosse players are not paid the same salaries as baseball players.
61. Which is the best way to combine Sentences 4 and Sentence 5?
- A. Hundreds of men and women from rival tribes would gather at once to play on fields sometimes several miles long.
 - B. On playing fields sometimes several miles long, gathering to play would be hundreds of men and women from rival tribes.
 - C. Hundreds of men and women from rival tribes would gather to play at once and they would play on fields sometimes several miles long.
 - D. Hundreds of men and women from rival tribes or villages would gather to play at once, but the playing field was sometimes several miles long.
62. In context, where is the best place to put Sentence 6?
- E. After Sentence 1
 - F. Before Sentence 3
 - G. Before Sentence 4
 - H. After Sentence 8
63. The author wishes to add a concluding sentence after Sentence 9 that relates to a statement made in the opening sentence. Which of the following sentences would achieve this most effectively?
- A. Lacrosse has played an important role in North American culture for hundreds of years.
 - B. If you haven't ever played lacrosse, you're missing out on a really fun game.
 - C. Lacrosse is a difficult sport to play because it requires much more physical endurance than baseball.
 - D. Though it may not be considered "America's pastime," lacrosse could certainly be called the original American team sport.

GO ON TO THE NEXT PAGE.



PASSAGE VI

Alfred Tarski, born on January 14, 1901, became known during his lifetime as a brilliant mathematician and teacher. He is best known for proving several advanced geometric theorems. By the time Tarski moved to the United States, much of Europe has already fallen⁶⁴ into the grips of World War II. Hundreds of mathematical⁶⁵ problems were solved by Tarski.⁶⁵

Tarski enrolled in Warsaw University in 1920. Originally wishing to study biology, mathematics⁶⁶ was the subject in which Tarski ultimately⁶⁶ excelled.⁶⁶ He graduated with honors, and⁶⁷ began his career as a math teacher. A true mathematical virtuoso, Tarski was concerned with neither the application of his research nor publishing his⁶⁸ findings.⁶⁸

64. E. NO CHANGE
F. already falls
G. had already fallen
H. will have already fallen
65. A. NO CHANGE
B. Hundreds of problems, all of them mathematical, were solved by Tarski
C. Tarski solved hundreds of mathematical problems
D. Mathematical problems—hundreds of them—were solved by Tarski
66. E. NO CHANGE
F. Tarski ultimately excelled in mathematics
G. mathematics is what Tarski excelled in
H. the subject in which Tarski ultimately excelled was mathematics
67. A. NO CHANGE
B. honors and
C. honors, and,
D. honors; and
68. E. NO CHANGE
F. making his findings public
G. having published his findings
H. the publication of his findings

GO ON TO THE NEXT PAGE.



Discoveries made by Tarski influenced the work of one of the world's greatest physicists, Albert Einstein. Einstein and Tarski had many⁶⁹ similar interests in common⁶⁹. Unlike Einstein, however, Tarski was especially fond for⁷⁰ pure mathematics. Although Tarski and Einstein were contemporaries, Einstein was the most prolific⁷¹ writer of the two.

In 1929, Tarski married fellow teacher Maria Witkowska. An affinity for mathematics ran in the family. Tarski even admitted that his wife knew more about algebra, geometry and trigonometry⁷² than he did. Tarski's two children, Jan, and⁷³ Ina⁷³, grew up to be prominent mathematicians themselves; however, neither Jan nor Ina have⁷⁴ received⁷⁴ a great deal of international attention. 75

69. A. NO CHANGE
B. shared many interests
C. shared many similar interests in common
D. had in common many similar interests, which they shared.
70. E. NO CHANGE
F. fond with
G. fond about
H. fond of
71. A. NO CHANGE
B. more prolific
C. prolific
D. best
72. E. NO CHANGE
F. algebra, geometry, and, trigonometry
G. algebra, geometry and, last but not least, trigonometry
H. algebra, geometry, and trigonometry
73. A. NO CHANGE
B. children, Jan and Ina,
C. children, Jan and Ina
D. children Jan and Ina
74. E. NO CHANGE
F. has received
G. are receiving
H. receive
75. The author is considering adding the following sentence to the passage:
"Whereas Einstein published dozens of books, Tarski published only one."
If the writer were to add this sentence to the passage, which of these choices would be the most logical place to insert it?
A. After the last sentence of paragraph 3
B. After the first sentence of paragraph 1
C. After the first sentence of paragraph 2
D. After the first sentence of paragraph 4

END OF TEST 3

STOP! DO NOT TURN THE PAGE UNTIL TOLD TO DO SO.

DO NOT RETURN TO A PREVIOUS TEST.





MATHEMATICS TEST

60 Minutes—60 Questions

DIRECTIONS: Solve each problem and choose the correct answer.

Do not linger over problems that take too much time. Solve as many as you can, then return to the others in the time you have left for this test.

You are permitted to use a calculator on this test. You may use your calculator for any problems you choose, but some of the problems may best be done without using a calculator.

Note: Unless otherwise stated, all of the following should be assumed.

1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word *line* indicates a straight line.
4. The word *average* indicates arithmetic mean.

1. Find the value of w if $w = \left(\frac{1}{3} + y^2\right)$ and $y = \frac{1}{9}$.

- A. $\frac{16}{81}$
 B. $\frac{9}{10}$
 C. $\frac{1}{12}$
 D. $\frac{4}{27}$

2. Let $F(x) = x^3 + 2x^2 - 3$ and $G(x) = x + 5$.

Find $F(G(x))$.

- E. $x^3 + 17x^2 + 95x + 172$
 F. $x^3 + 2x^2 - x - 8$
 G. $x^3 + x^2 + x + 8$
 H. $x^3 + 2x^2 + x + 2$

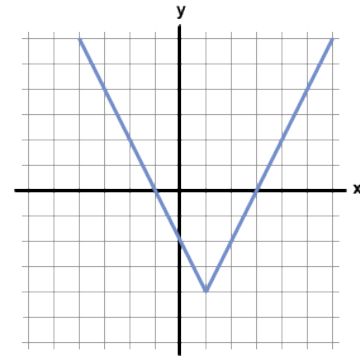
3. Circle A is given by the equation $(x - 4)^2 + (y + 3)^2 = 29$. Circle A is shifted up five units and left by six units. Then, its radius is doubled. What is the new equation for circle A?

- A. $(x + 2)^2 + (y - 2)^2 = 116$
 B. $(x - 10)^2 + (y + 8)^2 = 116$
 C. $(x - 10)^2 + (y + 8)^2 = 58$
 D. $(x + 2)^2 + (y - 2)^2 = 58$

4. The midpoint of a line segment is $(4, -1)$. If one endpoint of the line segment is $(-4, 3)$, what is the other endpoint?

- E. $(12, -5)$
 F. $(-12, 7)$
 G. $(-12, -5)$
 H. $(12, 7)$

5. Below is the graph of the function $f(x)$:



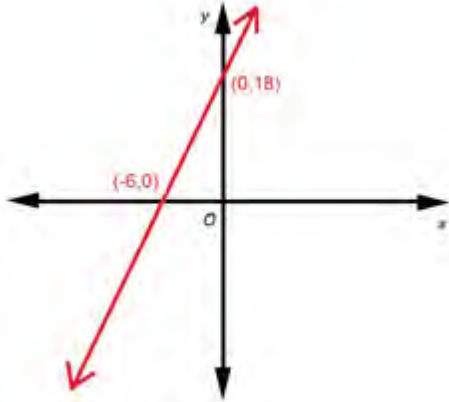
Which of the following could be the equation for $f(x)$?

- A. $f(x) = x^2 - 4x + 3$
 B. $f(x) = |2x - 2| - 4$
 C. $f(x) = |2x - 6|$
 D. $f(x) = |x - 4x| - 3$

GO ON TO THE NEXT PAGE.



6. Refer to the graphed line below. If the red line passes through the point $(N, 4)$, what is the value of N ?



- E. $N = -7\frac{1}{3}$
 F. $N = -1\frac{1}{3}$
 G. $N = -4\frac{2}{3}$
 H. $N = -3\frac{1}{3}$
7. Which line below is perpendicular to $5x + 6y = 18$?
- A. $y = \frac{5}{6}x + \frac{6}{5}$
 B. $y = \frac{6}{5}x + 3$
 C. $y = -\frac{6}{5}x + 8$
 D. $y = \frac{5}{6}x + 2$

8. Which line is parallel to the line with the equation $2x + 3y = 6$ and travels through the point $(3, 2)$?

- E. $y = -\frac{3}{2}x + 4$
 F. $y = -\frac{2}{3}x + 4$
 G. $y = -\frac{3}{2}x + 8$
 H. $y = -\frac{3}{2}x + 8$

9. Simplify: $(4r^2 + 2s^3)^2$

- A. $16r^4 + 16s^3r^2 + 4s^6$
 B. $16r^4 + 4s^6$
 C. $4r^4 + 2s^5$
 D. $16r^4 + 8s^3r^2 + 4s^6$

10. $x^2 - 1$ in factored form is equal to:

- E. $(x+1) + (x-1)$
 F. $(x-1)^2$
 G. $x(x-1)$
 H. $(x+1)(x-1)$

11. Given the inequality $|12 - 2x| > 2$ which of the following is correct?

- A. $x < -5$ or $x > 7$
 B. $5 < x < 7$
 C. $x < 5$ or $x > 7$
 D. $-5 < x < 7$

12. Simplify the following inequality: $4 - 3x \geq 22 + 2x$
- E. $x \leq -\frac{18}{5}$
 F. $x \geq -\frac{18}{5}$
 G. $x \leq \frac{2}{7}$
 H. $x \geq \frac{14}{5}$
13. Let x be a number. Increasing x by twenty percent yields the same result as decreasing the product of four and x by five. What is x ?
- A. $\frac{25}{19}$
 B. $\frac{50}{7}$
 C. $\frac{100}{19}$
 D. $\frac{25}{14}$
14. Given the equation $x^2 - 8x + 15 = 0$, what is the product of the solutions of the quadratic equation?
- E. -15
 F. 15
 G. -8
 H. 8
15. Solve for y .
 $3y + 6x = 24$
 $2y + x = 16$
- A. -8
 B. 8
 C. 4
 D. -4
16. Simplify: $\left(\frac{18a^3b^3}{3a^{-1}b^{-2}} \right)^2$
- E. $6a^4b^5$
 F. $9a^5b^4$
 G. $36a^6b^8$
 H. $36a^8b^{10}$
17. If m and n are both rational numbers and $4^m = 8^n$, what is $\frac{m}{n}$?
- A. $\frac{2}{1}$
 B. $\frac{3}{2}$
 C. $\frac{5}{3}$
 D. $\frac{4}{1}$
18. Find the value of x if $\frac{64}{27} = \left(\frac{4}{3} \right)^x$.
- E. 2
 F. 3
 G. 4
 H. 5
19. Simplify: $(xy^2 + 2x^3y^2)(xy^3 + 3x^2)$
- A. $x^2y^5 + 3x^3y^2 + 2x^4y^5 + 6x^5y^2$
 B. $xy^6 + 3x^2y^2 + 2x^3y^6 + 6x^6y^2$
 C. $6x^2y^4 + 2x^3y^2 + 3x^2y^2$
 D. $3x^4y^5 + x^3y^2 + 2xy^5 + 3x^5y^2$
20. What is the result when 5,678,732 is rounded to the nearest thousand and then put in scientific notation?
- E. 5.678×10^3
 F. 5.678×10^6
 G. 5.679×10^3
 H. 5.679×10^6
21. If $x^7y^8z^{10} < 0$, then which of the following must also be true?
- A. $x < 0$
 B. $y < 0$
 C. $zy > 0$
 D. $zx > 0$

22. Which of the following is equal to $-16i^6$?

- E. -4
- F. -16
- G. 16
- H. $16i$

23. Which of the following is equal to the expression $\sqrt{(16)(8) + (32)(20)}$?

- A. $2^7\sqrt{5}$
- B. $2^3\sqrt{6}$
- C. $2^4\sqrt{3}$
- D. $2^3\sqrt{10}$

24. Combine the following two expressions if possible.

$$\frac{x+3}{x+7} + \frac{x^2}{4-x}$$

- E. $\frac{-x^3 + 4x^2 + 8x - 4}{x^2 - 3x - 28}$
- F. $\frac{(x-3)^3}{-x^2 - 3x + 28}$
- G. $\frac{x^3 + 3x^2 - 7x - 22}{x^2 - 3x - 28}$
- H. $\frac{x^3 + 6x^2 + x + 12}{-x^2 - 3x + 28}$

25. The price of silver varies directly as the square of the mass. If 3.6g of silver is worth \$64.80, what is the value of 7.5g of silver?

- A. \$135.00
- B. \$301.75
- C. \$178.50
- D. \$281.25

26. Choose the answer that is the simplest form of the following expression of monomial quotients:

$$\frac{2x^3y^4}{10z^3} \times \frac{4z^2p}{10xy}$$

- E. $\frac{2x^2y^3p}{25z}$
- F. $\frac{8x^2y^3p}{25z^2}$
- G. $\frac{2xyp}{15z}$
- H. $\frac{2p}{25z}$

27. A function of the form $f(x) = ax^2 + b$ passes through the points (0,7) and (-2,19). What is the value of a ?

- A. -3
- B. 3
- C. 2
- D. -2

28. Add the following polynomials:

$$(5x^3 + 31x^2 - 17x - 6) + (-2x^3 + 9x^2 + 34x - 12)$$

- E. $3x^3 + 40x^2 + 17x - 18$
- F. $3x^3 + 22x^2 + 10x - 18$
- G. $7x^3 + 40x^2 + 17x + 6$
- H. $-3x^3 + 40x^2 - 17x + 6$

29. Solve for t : $t\sqrt{28} - t\sqrt{63} = \sqrt{14}$

- A. 2
- B. $-\sqrt{2}$
- C. $-\sqrt{14}$
- D. $\sqrt{14}$

30. What is the ratio of $\sqrt{105}$ to $\sqrt{\frac{12}{5}}$?

- E. $4\sqrt{7}:15$
- F. $5\sqrt{7}:2$
- G. $5\sqrt{21}:12$
- H. $\sqrt{105}:12$

GO ON TO THE NEXT PAGE.



31. If $x = 3.09$ and $y = 2.97$, what is $(x - y)^2$ equal to?

A. 0.0144
B. 0.144
C. 1.44
D. 0.04

32. Simplify:

$$0.35 + \frac{0.75}{0.2} - 0.4 \times \frac{0.5}{0.1}$$

E. $\frac{213}{10}$

F. $\frac{30}{7}$

G. $\frac{25}{4}$

H. $\frac{21}{10}$

33. A is $16\frac{2}{3}\%$ of B and $62\frac{1}{2}\%$ of C; all are positive integers. Give the smallest possible value

of A.

A. 1
B. 5
C. 8
D. 30

34. Which of the following is equivalent to $5\frac{7}{11}$?

E. $\frac{62}{11}$

F. $\frac{12}{11}$

G. $\frac{53}{11}$

H. $\frac{71}{11}$

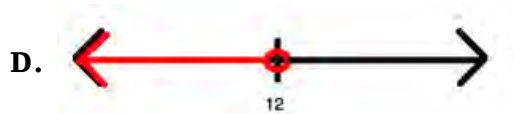
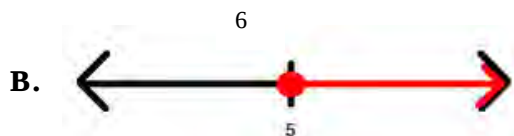
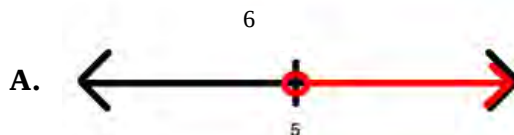
35. A large reservoir holds 200,000 gallons of water. It has an emptying pipe attached to it that allows 20 gallons to flow out of the reservoir every hour. If an additional emptying pipe of the same dimensions as the first one is attached to the reservoir, how many gallons will be left in the reservoir after three days of drainage, presuming that there is no overall change in water due to addition or evaporation?

A. 81,494 gallons
B. 157,800 gallons
C. 197,120 gallons
D. 199,880 gallons

36. What is the least common multiple of 5 and 13?

E. 65
F. 130
G. 175
H. 52

37. Which of the following is a graph for the values of x defined by the inequality $2x + 6 > 18$?





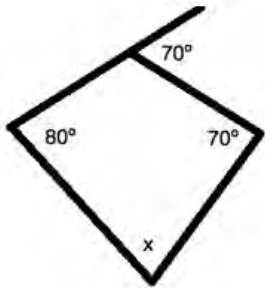
38. The game of euchre uses the 9's, 10's, jacks, queens, kings, and aces from a standard deck of 52 cards. How many 5-card euchre hands have at least 2 black cards?
- E. 8,731
F. 5,940
G. 14,000
H. 35,772
39. Find the 50th term in the following sequence.
-8, -1, 6, 13, ...
- A. 306
B. 335
C. 342
D. 350
40. Solve for x : $x = (3 - 2)^2 + 3 - 3 \cdot 2^2$
- E. $x = -4$
F. $x = 4$
G. $x = -8$
H. $x = 8$
41. A large group of professors are sitting in an auditorium. One-half of the professors are mathematicians. One-fourth of the remaining professors who are not mathematicians are philosophers. All of the other professors are sociologists. If there is a total of 464 professors seated in the auditorium, how many of them are sociologists?
- A. 58
B. 98
C. 110
D. 174
42. A university has 570 students currently enrolled in its freshman class. Last year, there were only 500 freshmen enrolled at the school. By what percentage did the number of students in the freshman class increase?
- E. 11%
F. 12%
G. 13%
H. 14%
43. The mean of five numbers is 40. The mean of the smallest two numbers is 25. What is the mean of the other three numbers?
- A. 40
B. 45
C. 50
D. 55
44. The list below shows a ninth grader's grades for the academic year. What is the difference between the median and the mode of these grades?
- 67, 73, 85, 83, 80, 73, 94, 65, 80, 73, 98, 59, 76
- E. 1
F. 2
G. 3
H. 4
45. Students at a local high school are given the option to take one gym class, one music class or one of each. Out of 100 students, 60 say that they are currently taking a gym class and 70 say that they are taking a music class. How many students are taking both?
- A. 30
B. 40
C. 50
D. 60
46. On a standard analog clock, what is the angle between the hands when the clock reads 1:20?
- E. 55°
F. 80°
G. 85°
H. 90°
47. A square has an area of 32in^2 . If a circle is inscribed within the square, what is its area?
- A. $2\sqrt{2}\text{in}^2$
B. $4\sqrt{2}\text{in}^2$
C. $8\pi\text{in}^2$
D. $16\pi\text{in}^2$



48. The base of a triangle is 8cm and the area is 48cm^2 . The height of the triangle is then decreased by 75%. What is the final area of the triangle?

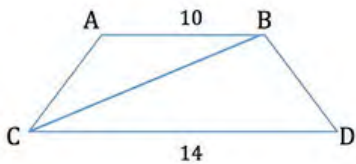
E. 9cm^2
 F. 12cm^2
 G. 14cm^2
 H. 16cm^2

49. What is the value of angle x in the figure below?



A. 50°
 B. 75°
 C. 100°
 D. 115°

50. If the height of the isosceles trapezoid below is 5 units, what is the length of the diagonal $\overline{BC}$?



E. 11
 F. 12
 G. 13
 H. 14

51. An equilateral triangle has a height of $3\sqrt{3}$. What is the perimeter of this triangle?

A. $9\sqrt{3}$
 B. $18\sqrt{3}$
 C. 9
 D. 18

52. A cylindrical grain silo is erected vertically, to rest on one of its faces. The silo is then covered with reflective paint. If the cylinder is 12m tall and has a circumference of $6\pi\text{m}$, how many square meters of paint must be used? (We'll assume the first coat is enough.)

E. $81\pi\text{m}^2$
 F. $100\pi\text{m}^2$
 G. $128\pi\text{m}^2$
 H. $140\pi\text{m}^2$

53. A 10ft ladder is leaning against a wall. If the bottom of the ladder touches the ground 5ft from the base of the wall, approximately (to the nearest whole number) how far is the top of the ladder from the base of the wall?

A. 8ft
 B. 9ft
 C. 10ft
 D. 12ft

54. A sphere with a volume of $\frac{32}{3}\pi\text{m}^3$ is inscribed in a cube. What is the surface area of the cube, in m^2 ?

E. 24m^2
 F. 48m^2
 G. 96m^2
 H. 120m^2

55. Sturgis is in charge of designing a new exhibit in the shape of a rectangular prism for a local aquarium. The exhibit will hold alligator snapping turtles and needs to have a volume of 150m^3 . Sturgis knows that the exhibit will be 15m long and have a width of 5m.

If three-quarters of the exhibit's volume will be water, what will the height (or the depth) of the water be?

A. 1.0m
 B. 1.5 m
 C. 1.75 m
 D. 2.0 m

56. A man has a rope that is 40ft long, attached to the top of a small building. He pegs the rope into the ground at an angle of 14.5° . How far away from the building did he walk horizontally to attach the rope to the ground? Round to the nearest inch.

- E. 37 feet and 5 inches
- F. 38 feet and 9 inches
- G. 39 feet and 4 inches
- H. 40 feet and 2 inches

57. What is the value of $\log_2(64)$? Round to the nearest hundredth.

- A. 4
- B. 6
- C. 8
- D. 12

58. What is the tangent of the obtuse angle formed between the x-axis and a straight line drawn from the origin to the point (4,-3)? Round to the nearest hundredth.

- E. -0.75
- F. 0.75
- G. -1.33
- H. 1.33

59. What is the area of an isosceles right triangle that has a hypotenuse of length 18cm?

- A. 40cm^2
- B. 81cm^2
- C. 121cm^2
- D. 324cm^2

60. Suppose $c > 0$.

To obtain the graph of $y = f(x) + c$, shift the graph $y = f(x)$ a distance of c units in which of the following directions?

- E. Upwards
- F. Downwards
- G. Left
- H. Right

END OF TEST 3

STOP! DO NOT TURN THE PAGE UNTIL TOLD TO DO SO.

DO NOT RETURN TO A PREVIOUS TEST.



READING TEST

35 Minutes—40 Questions

DIRECTIONS: There are several passages in this test. Each passage is accompanied by several questions. After reading a passage, choose the best answer to each question. You may refer to the passages as often as necessary.

Passage I

PROSE FICTION: This passage is adapted from *Northanger Abbey* by Jane Austen (1817)

5 The progress of the friendship between Catherine and Isabella was quick as its beginning had been warm, and they passed so rapidly through every gradation of increasing
10 tenderness that there was shortly no fresh proof of it to be given to their friends or themselves. They called each other by their first names, were always arm in arm when they walked, pinned up each other's train for the dance, and were not
15 to be divided in the set; and if a rainy morning deprived them of other enjoyments, they were still resolute in meeting in defiance of wet and dirt, and shut themselves up, to read novels together.

20 Yes, novels; for I will not adopt that ungenerous and impolitic custom so common with novel-writers, of degrading by their contemptuous censure the very performances, to the number of which they are themselves
25 adding—joining with their greatest enemies in bestowing the harshest epithets on such works, and scarcely ever permitting them to be read by their own heroine, who, if she accidentally take up a novel, is sure to turn over its insipid pages with disgust. Alas! If the heroine of one novel be
30 not patronized by the heroine of another, from whom can she expect protection and regard? I cannot approve of it. Let us leave it to the reviewers to abuse such effusions of fancy at their leisure, and over every new novel to talk in threadbare strains of the trash with which the press now groans. Let us not desert one another; we are an injured body.

35 Although our productions have afforded more extensive and unaffected pleasure than those of any other literary corporation in the world, no species of composition has been so much

45 decried. From pride, ignorance, or fashion, our foes are almost as many as our readers. And while the abilities of the nine-hundredth abridger of the History of England, or of the man who collects and publishes in a volume some dozen lines of Milton, Pope, and Prior, with a paper from the *Spectator*, and a chapter from Sterne, are eulogized by a thousand pens—
50 there seems almost a general wish of decrying the capacity and undervaluing the labor of the novelist, and of slighting the performances which have only genius, wit, and taste to recommend them. "I am no novel-reader—I seldom look into novels—Do not imagine that I often read novels—It is really very well for a novel." Such is the common cant. "And what are you reading, Miss—?" "Oh! It is only a novel!" replies the young lady, while she lays down her book with affected indifference, or momentary
55 shame. "It is only *Cecilia*, or *Camilla*, or *Belinda*"; or, in short, only some work in which the greatest powers of the mind are displayed, in which the most thorough knowledge of human nature, the happiest delineation of its varieties, the liveliest effusions of wit and humor, are conveyed to the world in the best-chosen language.

60 Now, had the same young lady been engaged with a volume of the *Spectator*, instead of such a work, how proudly would she have produced the book, and told its name; though the chances must be against her being occupied by any part of that voluminous publication, of which either
75 the matter or manner would not disgust a young person of taste: the substance of its papers so often consisting in the statement of improbable circumstances, unnatural characters, and topics of conversation which no longer concern anyone living; and their language, too, frequently so
80 coarse as to give no very favorable idea of the age that could endure it.

GO ON TO THE NEXT PAGE.



1. The first line of the second paragraph (lines 13-22) can be considered surprising because _____.
 - A. the writer does not support novelists, despite the fact that she is a novelist
 - B. the narrator starts using first-person perspective whereas the first paragraph seemed to use third-person perspective
 - C. according to the narrator, novelists and their work are typically highly regarded, but the narrator opposes this view
 - D. the narrator strongly espouses one view of novels before the dash before reconsidering the view after the semicolon
2. The first paragraph serves to _____.
 - E. introduce a new topic
 - F. set forward an argument
 - G. continue a story
 - H. describe a scene
3. At the end of the passage, the narrator presents two hypothetical scenes: one in which a hypothetical reader is _____, and another in which the same reader is _____.
 - A. making excuses for reading a novel . . . highlighting the fact that she was reading the *Spectator*
 - B. enjoying reading a well-written novel . . . unable to get through a poorly-written novel
 - C. engrossed in a novel . . . unable to get through the *Spectator*'s dense language
 - D. recommending a novel to a friend . . . recommending that a friend read the *Spectator*
4. By "literary corporation," underlined in the third paragraph (line 31), the narrator means _____.
 - E. a writer's guild
 - F. the *Spectator*
 - G. the publishing house with which the writer works
 - H. novelists
5. Based on the way in which it is used in line 47, the underlined word "cant" means _____.
 - A. question
 - B. refrain
 - C. reasoning
 - D. emphasis
6. According to the narrator, most novelists are _____.
 - E. morbid
 - F. haughty
 - G. self-deprecating
 - H. defensive when their work is criticized
7. Which of the following can we infer, based on the passage?
 - A. *Belinda*, *Cecilia*, *Camilla*, and the *Spectator* were familiar to the author's intended audience.
 - B. The *History of England* was rarely abridged.
 - C. The *Spectator* often published excerpts from new novels to drum up support for them.
 - D. Milton, Pope, Prior, and Sterne were rarely referenced in the author's era.
8. Which of the following complaints does the narrator NOT make against papers published in the *Spectator* in the passage's last paragraph?
 - E. They are often too wordy and verbose.
 - F. They often involve unlikely situations.
 - G. Their characters are often not convincingly realistic.
 - H. The subject matter of their dialogue is often notably outdated.
9. Which of the following best summarizes the passage?
 - A. The narrator abandons the story of Catherine and Isabella to argue that novelists deserve more money and fame for their work.
 - B. The narrator interrupts the story of Catherine and Isabella to defend the worth of novels.
 - C. The narrator disrupts the story of Catherine and Isabella to argue that reading the *Spectator* is a waste of time.
 - D. The narrator decries types of literature that she feels to be inferior to novels.
10. We can infer that this passage was excerpted from _____.
 - E. a novel
 - F. an article published in the *Spectator*
 - G. a book about the history of novels
 - H. an opinion piece published in a newspaper

GO ON TO THE NEXT PAGE.

Passage II

Natural Science: Adapted from *A Practical Treatise on the Hive and Honey-Bee* by Lorenzo Lorraine Langstroth (1857 ed.)

Of all the numerous enemies of the honey-bee, the Bee-Moth (*Tinea mellonella*), in climates of hot summers, is by far the most to be dreaded. So widespread and fatal have been its ravages in this country that thousands have abandoned the cultivation of bees in despair, and in districts which once produced abundant supplies of the purest honey, bee-keeping has gradually dwindled down into a very insignificant pursuit. Contrivances almost without number have been devised to defend the bees against this invidious foe, but still it continues its desolating inroads, almost unchecked, laughing as it were to scorn at all the so-called “moth-proof” hives, and turning many of the ingenious fixtures designed to entrap or exclude it into actual aids and comforts in its nefarious designs.

I should feel but little confidence in being able to reinstate bee-keeping in our country into a certain and profitable pursuit if I could not show the apiarian in what way he can safely bid defiance to the pestiferous assaults of this, his most implacable enemy. I have patiently studied its habits for years, and I am at length able to announce a system of management founded upon the peculiar construction of my hives, which will enable the careful bee-keeper to protect his colonies against the monster. The bee-moth infects our apiaries, just as weeds take possession of a fertile soil. Before explaining the means upon which I rely to circumvent the moth, I will first give a brief description of its habits.

Swammerdam, towards the close of the seventeenth century, gave a very accurate description of this insect, which was then called by the very expressive name of the “bee-wolf.” He has furnished good drawings of it, in all its changes, from the worm to the perfect moth, together with the peculiar webs or galleries that it constructs and from which the name of *Tinea galleria* or “gallery moth” has been given to it by some entomologists. He failed, however, to

discriminate between the male and female, which, because they differ so much in size and appearance, he supposed to be two different species of the wax-moth. It seems to have been a great pest in his time, and even Virgil speaks of the “*dirum tineæ genus*,” the dreadful offspring of the moth; that is the worm.

This destroyer usually makes its appearance about the hives in April or May, the time of its coming depending upon the warmth of the climate or the forwardness of the season. It is seldom seen on the wing (unless startled from its lurking place about the hive) until towards dark, and is evidently chiefly nocturnal in its habits. In dark cloudy days, however, I have noticed it on the wing long before sunset, and if several such days follow in succession, the female, oppressed with the urgent necessity of laying her eggs, may be seen endeavoring to gain admission to the hives. The female is much larger than the male, and “her color is deeper and more inclining to a darkish gray, with small spots or blackish streaks on the interior edge of her upper wings.” The color of the male inclines more to a light gray; they might easily be mistaken for different species of moths. These insects are surprisingly agile, both on foot and on the wing. The motions of a bee are very slow in comparison. “They are,” says Reaumur, “the most nimble-footed creatures that I know.” “If the approach to the apiary be observed of a moonlight evening, the moths will be found flying or running round the hives, watching an opportunity to enter, whilst the bees that have to guard the entrances against their intrusion will be seen acting as vigilant sentinels, performing continual rounds near this important post, extending their antenna to the utmost, and moving them to the right and left alternately. Woe to the unfortunate moth that comes within their reach!” “It is curious,” says Huber, “to observe how artfully the moth knows how to profit, to the disadvantage of the bees, which require much light for seeing objects; and the precautions taken by the latter in reconnoitering and expelling so dangerous an enemy.”

GO ON TO THE NEXT PAGE.



11. Based on the passage, another word for “beehive” is _____.
- A. galley
 - B. contrivance
 - C. apiary
 - D. apiarian
12. The first paragraph establishes all of the following EXCEPT _____.
- E. the author has faith in the devices used to stop the bee-moth
 - F. bee keeping has, in some areas, become a trifling hobby
 - G. the ravages of the bee-moth have dissuaded many from continuing bee keeping
 - H. in hot summers the bee-moth is the worst enemy of the honey bee
13. In the third paragraph the information about Swammerdam’s name for the moth serves to _____.
- A. suggest that the bee-moth preys on other insects besides bees
 - B. show how much of a menace it has always been to bees
 - C. show that Swammerdam was not scientific in his approach to bee keeping
 - D. mock the moth as something feared yet destroyable
14. What is the main idea of the second paragraph?
- E. The author has come up with a method for stopping some of the destruction of the bee-moth, which he will share later in the book.
 - F. There is no point trying to devise ways to save bee-keeping from the moth, and the author thinks that any advice he gives will be of little consequence.
 - G. Several people have recommended methods to prevent the attacks of the bee-moth and the author will share them.
 - H. The author is uncertain as to the future of bee-keeping until a pesticide is created to kill the bee-moth.
15. The point of view from which the passage is told can best be described as that of _____.
- A. a bug collector
 - B. a scientific illustrator
 - C. an enthusiastic bee-keeper
 - D. an expert on different types of honey
16. Which of the following statements about bees is supported by the passage?
- E. They do not keep their hives in good condition.
 - F. They will guard the entrance to their hives.
 - G. They allow the bee-moth to enter their nests.
 - H. They are not very good at caring for their young.
17. Which of these most accurately restates the meaning of “The bee-moth infects our apiaries, just as weeds take possession of a fertile soil,” a line found in the second paragraph?
- A. The bee-moth is to the bee keeper what the dandelion is to the gardener.
 - B. The description of the moth is not sufficient without considering its impact on a garden.
 - C. None other than gardeners and bee keepers can comprehend the devastations of the moth.
 - D. The impact of the bee moth on a hive is disproportionate to that of the weed on a garden.
18. It can reasonably be inferred from the passage that _____.
- E. the moths are sluggish
 - F. the bees do not guard against the moth
 - G. bee-moths have only recently begun to trouble bee-keepers
 - H. the author is a keen bee-keeper
19. As it is used in the passage, the underlined word “discriminate” in the third paragraph most nearly means _____.
- A. recognize
 - B. distinguish
 - C. compare
 - D. act in a manner that is unfairly biased

GO ON TO THE NEXT PAGE.



20. One of the main points made in the underlined section of the last paragraph is -----.
- E. a critic has stated that the bee-moth takes advantage of the bees' inability to see at night
 - F. it is not curious that bees defend against the bee-moth
 - G. Huber believes that the bee-moths would be more successful if they entered the hive during the day
 - H. the author thinks that Huber is wrong in his assertions

Passage III

Social Sciences: Adapted from "Introductory Remarks" in *The Interpretation of Dreams* by Sigmund Freud (trans. 1913)

In attempting to discuss the interpretation of dreams, I do not believe that I have overstepped the bounds of neuropathological interest. For, when investigated psychologically, the dream proves to be the first link in a chain of abnormal psychic structures whose other links—the hysterical phobia, the obsession, and the delusion—must interest the physician for practical reasons. The dream can lay no claim to a corresponding practical significance; however, its theoretical value is very great, and one who cannot explain the origin of the content of dreams will strive in vain to understand phobias, obsessive and delusional ideas, and likewise their therapeutic importance.

While this relationship makes our subject important, it is responsible also for the deficiencies in this work. The surfaces of fracture, which will be frequently discussed, correspond to many points of contact where the problem of dream formation informs more comprehensive problems of psychopathology which cannot be discussed here. These larger issues will be elaborated upon in the future.

Peculiarities in the material I have used to elucidate the interpretation of dreams have rendered this publication difficult. The work itself will demonstrate why all dreams related in scientific literature or collected by others had to

remain useless for my purpose. In choosing my examples, I had to limit myself to considering my own dreams and those of my patients who were under psychoanalytic treatment. I was restrained from utilizing material derived from my patients' dreams by the fact that during their treatment, the dream processes were subjected to an undesirable complication—the intermixture of neurotic characters. On the other hand, in discussing my own dreams, I was obliged to expose more of the intimacies of my psychic life than I should like, more so than generally falls to the task of an author who is not a poet but an investigator of nature. This was painful, but unavoidable; I had to put up with the inevitable in order to demonstrate the truth of my psychological results at all. To be sure, I disguised some of my indiscretions through omissions and substitutions, though I feel that these detract from the value of the examples in which they appear. I can only express the hope that the reader of this work, putting himself in my difficult position, will show patience, and also that anyone inclined to take offense at any of the reported dreams will concede freedom of thought at least to the dream life.

21. The author can be most accurately described as:
- A. furious and insulted
 - B. defensive and meticulous
 - C. imploring and desperate
 - D. whimsical and descriptive
22. The author discusses a topic that he plans to pursue in future work:
- E. in the first sentence of the passage
 - F. in the second paragraph of the passage
 - G. in the last sentence of the passage
 - H. in the first and last paragraphs of the passage
23. According to the author, studying phobias, obsessions, and delusions is _____, but studying dreams is not.
- A. practical
 - B. possible
 - C. useless
 - D. easy

GO ON TO THE NEXT PAGE.



24. When he uses the underlined phrase “the inevitable” in lines 44-45, the author is referring to _____.
- E. the idea that all dreams contain significant meaning
 - F. the gradual loss of detail in what one can remember about a dream
 - G. the scorn of many important psychologists upon his publication of his work on dreams
 - H. the fact that he had to publish some of his own dreams, which made him uncomfortable
25. The author of this passage is most interested in _____.
- A. the physical structure of the brain
 - B. the ability of certain dreams to foreshadow future events
 - C. the workings of the human mind
 - D. the role of criticism in science
26. Based on the way in which the underlined word “informs” is used in line 20 in the passage, the author is using it to mean:
- E. requires
 - F. tells
 - G. influences
 - H. solves
27. In the third paragraph, what does the author tell us about the omissions and substitutions he made when discussing his own dreams in the work that follow the passage?
- A. He feels that the adjusted examples would be more useful had they remained unadjusted.
 - B. He is not responsible for these; his editors are.
 - C. He was forced to make these, or journals would not accept his work.
 - D. He doesn't think that they affect the work whatsoever, and help him save face.
28. The author has written this passage in order to _____.
- E. teach the reader how to interpret his or her own dreams
 - F. respond to a specific critic who has cast doubt on his work's reliability
 - G. propose a psychological experiment
 - H. justify his work and address some of its limitations
29. The author could not rely upon the dreams related in scientific literature because _____.
- A. he couldn't be sure if material had been changed in or censored from them
 - B. he needed to interview people himself in order to discuss their emotional reactions to their dreams
 - C. not many dreams had been discussed in scientific literature, and those that had been discussed concerned a very limited number of topics
 - D. The author does not give a reason for this in the passage, but says that the rest of his work explains why this is the case
30. In the last sentence of the passage, the author attempts to _____.
- E. encourage the reader to read the work of a variety of psychologists
 - F. get the reader to empathize with him
 - G. explain why he made certain redactions to the dreams he later discusses
 - H. emphasize why his work is valuable, despite its flaws



Passage A

Adapted from “Robespierre” in *Critical Miscellanies* by John Morley (1904)

But history must be just; and the character of [Marie Antoinette] had far more concern in the disaster of the first five years of the Revolution than had the character of Robespierre. Every new document that comes to light heaps up proof that if blind and obstinate choice of personal gratification before the common weal be enough to constitute a state criminal, then the Queen of France was one of the worst state criminals that ever afflicted a nation. The popular hatred of Marie Antoinette sprang from a sound instinct. We shall never know how much or how little truth there was in those frightful charges against her, that may still be read in a thousand pamphlets. These imputed depravities far surpass anything that John Knox ever said against Mary Stuart, or that Juvenal has recorded against Messalina; and, perhaps, for the only parallel we must look to the hideous stories of the Byzantine secretary against Theodora, the too-famous empress of Justinian and the persecutor of Belisarius. We have to remember that all the revolutionary portraits are distorted by furious passion, and that Marie Antoinette may no more deserve to be compared to Mary Stuart than Robespierre deserves to be compared to Ezzelino or to Alva. It is at least certain that, from the unlucky hour when the Austrian archduchess crossed the French frontier, a childish bride of fourteen, down to the hour when the Queen of France made the attempt to recross it in resentful flight one and twenty years afterwards, Marie Antoinette was ignorant, unteachable, blind to events and deaf to good counsels, a bitter grief to her heroic mother, the evil genius of her husband, the despair of her truest advisers, and an exceedingly bad friend to the people of France. She broke out in incredible dissipations; in indiscreet visits to the masked balls at the opera, in midnight parades and mystifications on the terrace at Versailles, in insensate gambling. “The court of France is turned into a gaming-hell,” said the Emperor Joseph, the Queen’s own brother: “if they do not amend, the revolution will be cruel.”

Passage B

Adapted from “On the Death of Marie Antoinette” by Edmund Burke (1793)

It is now sixteen or seventeen years since I saw the Queen of France, then the Dauphiness, at Versailles; and surely never lighted on this orb, which she hardly seemed to touch, a more delightful vision. I saw her just above the horizon, decorating and cheering the elevated sphere she had just begun to move in, glittering like the morning star full of life and splendor and joy.

Oh, what a revolution! And what a heart must I have, to contemplate without emotion that elevation and that fall! Little did I dream, when she added titles of veneration to those of enthusiastic, distant, respectful love, that she should ever be obliged to carry the sharp antidote against disgrace concealed in that bosom; little did I dream that I should have lived to see such disasters fallen upon her, in a nation of gallant men and of cavaliers! I thought ten thousand swords must have leaped from their scabbards, to avenge even a look that threatened her with insult.

But the age of chivalry is gone; that of sophistry, economists, and calculators has succeeded, and the glory of Europe is extinguished forever. Never, never more, shall we behold that generous loyalty to rank and sex, that proud submission, that dignified obedience, that subordination of the heart, which kept alive, even in servitude itself, the spirit of an exalted freedom! The unsought grace of life, the cheap defense of nations, the nurse of manly sentiment and heroic enterprise is gone. It is gone, that sensibility of principle, that chastity of honor, which felt a stain like a wound, which inspired courage whilst it mitigated ferocity, which ennobled whatever it touched, and under which vice itself lost half its evil, by losing all its grossness.

GO ON TO THE NEXT PAGE.



31. In Passage B, the underlined statement “I thought ten thousand swords must have leaped from their scabbards, to avenge even a look that threatened her with insult” most nearly reflects the author’s _____.
- A. misery and disdain
 - B. confusion and praise
 - C. arrogance and apathy
 - D. shock and disappointment
32. The author’s tone in Passage B is primarily _____.
- E. optimistic and restrained
 - F. condescending and apathetic
 - G. celebratory and ecstatic
 - H. pessimistic and admonishing
33. Passage A compares Mary Stuart, Messalina, and Theodora to Marie Antoinette in order to emphasize that _____.
- A. Marie Antoinette is more famous
 - B. Marie Antoinette has been accused more harshly
 - C. The listed rulers, who were unpopular, all governed better than Marie Antoinette
 - D. Marie Antoinette’s current situation isn’t worse than what other historic rulers have experienced
34. The quotation that ends Passage A serves what purpose in the text?
- E. It contradicts the author’s argument.
 - F. It provides evidence that supports one of the author’s claims about Marie Antoinette.
 - G. It provides a transition to a new idea discussed in the next paragraph in the larger work from which the passage is adapted.
 - H. It suggests that Emperor Joseph was as unpopular as Marie Antoinette.
35. Based on the passage, what can the reader infer about Robespierre (underlined in line 4 in Passage A)?
- A. The author is discussing Marie Antoinette in a piece primarily about Robespierre.
 - B. Robespierre provides an example of a highly-praised figure, whereas Marie Antoinette was highly disparaged.
 - C. Robespierre was related to Marie Antoinette.
 - D. Robespierre greatly disliked Marie Antoinette.
36. Consider Passage A. Where did Marie Antoinette live before she came to France?
- E. Denmark
 - F. Belgium
 - G. Austria
 - H. The Netherlands
- The following questions ask about ~both Passage A and Passage B.**
37. The author of Passage A _____ Marie Antoinette, while the author of Passage B _____.
- A. is critical of . . . idealizes her
 - B. is biased in favor of . . . bemoans her loss
 - C. detests . . . reluctantly admits that she had a few good characteristics
 - D. praises . . . disparages her
38. Which of the authors considers Marie Antoinette’s life in the context of history?
- E. The author of Passage A
 - F. The author of Passage B
 - G. Both authors
 - H. Neither author



39. If presented with Passage B, the author of Passage A would most likely _____.
- A. point out that Passage B's argument is unsubstantiated opinion
 - B. agree with Passage B
 - C. refuse to acknowledge that Marie Antoinette was well-liked due to being unable to offer evidence to the contrary
 - D. accuse the author of Passage B of building his argument on evidence from unreliable sources
40. A historian is researching how the French population felt about Marie Antoinette in the period leading up to the French Revolution. Which aspect of which passage would this historian find most relevant?
- E. Passage A's discussion of historical pamphlets
 - F. Passage A's quotation from Emperor Joseph
 - G. The third paragraph of Passage B
 - H. Passage B's anecdote about the last time the author saw Marie Antoinette

END OF TEST 3

**STOP! DO NOT TURN THE PAGE UNTIL TOLD TO DO SO.
DO NOT RETURN TO A PREVIOUS TEST.**





SCIENCE TEST

35 Minutes—40 Questions

DIRECTIONS: There are several passages in this test. Each passage is followed by several questions. After reading a passage, choose the best answer to each question. You may refer to the passages as often as necessary.

You are NOT permitted to use a calculator on this test.

Passage I

An endocrinologist is a doctor who studies, diagnoses, and treats patients with hormone imbalances. There are many hormones involved in daily functioning. Two of the most important hormones for digestion are insulin and glucagon. These hormones are secreted by the pancreas. Insulin is released after a meal to help body cells take in sugar and convert it to energy. Glucagon is released when the body needs more energy, such as during exercise, and causes body cells to secrete sugar into the blood. A team of endocrinologists performed the following experiments:

Experiment 1

Five participants were given 15 grams of pure glucose (sugar). The participants had not eaten for four hours prior to the experiment. All five participants were healthy and had no serious medical conditions. A team of endocrinologists monitored the blood sugar and insulin levels of the participants over time. Figure 1 is a graph of the average levels in the five participants.

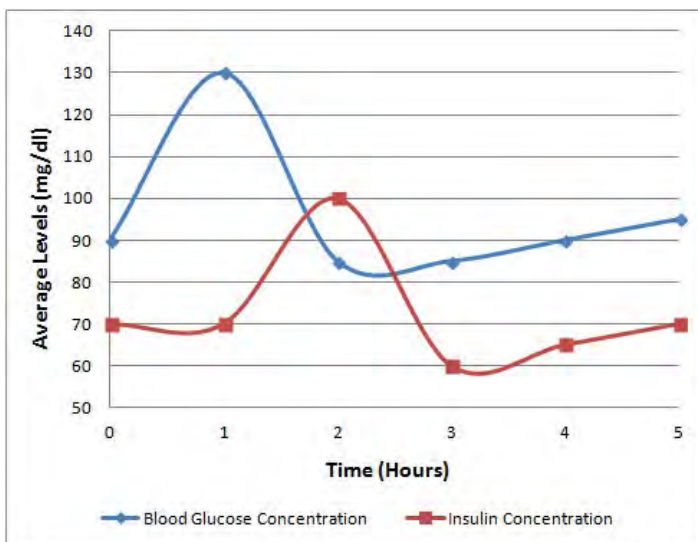


Figure 1

Experiment 2

Two participants were given 15 grams of pure glucose (sugar). The participants had not eaten for four hours prior to the experiment. One participant was healthy and the other had a hormone disease. Figure 2 is a graph of the blood sugar levels of the participants over time.

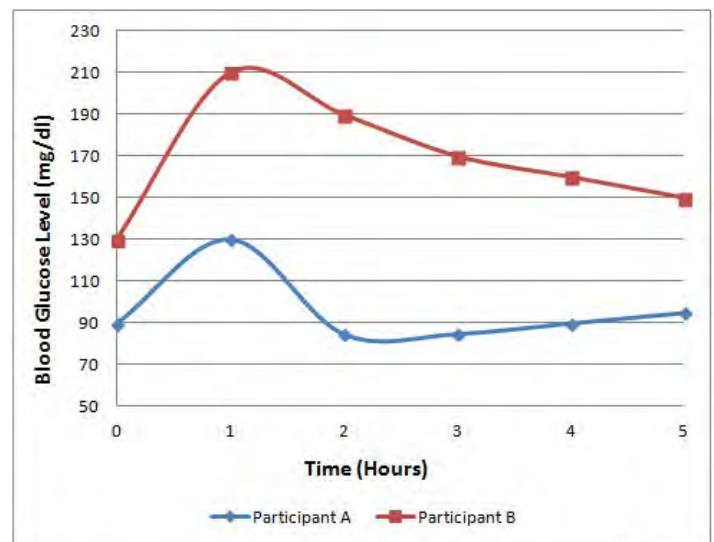


Figure 2

- Diabetes is a disease where a person may need to take insulin to help the body cells take up blood sugar. According to the passage, when should a person with diabetes take insulin?
 - 1 hour before eating
 - 5 hours before eating
 - 3 hours before eating
 - A person with diabetes should not take insulin.
- Who or what was the “control” group in Experiment 2?
 - Participant A
 - Participant B
 - 15 grams of glucose
 - Insulin levels

GO ON TO THE NEXT PAGE.



3. Experiment 1 is repeated. The team of endocrinologists also monitors glucagon levels of the participants over time. What would the graph of average glucagon levels over time look like?
- A. Linear increase
 - B. Exponential increase
 - C. Inverse of insulin graph
 - D. Horizontal line
4. At what time after eating would you expect the highest blood sugar level?
- E. 1 hour
 - F. 2 hours
 - G. 3 hours
 - H. 4 hours
5. If you were an endocrinologist, what would you recommend to Participant B?
- A. Take glucagon injections to increase blood sugar levels
 - B. Take insulin injections to increase blood sugar levels
 - C. Take insulin injections to decrease blood sugar levels
 - D. Take glucagon injections to increase blood sugar levels
6. Experiment 2 is repeated. Instead of insulin levels, Participant A's glucagon levels are recorded. What would you expect Participant A's blood glucose level to be after 1 hour?
- E. $130 \frac{mg}{dl}$
 - F. $90 \frac{mg}{dl}$
 - G. $60 \frac{mg}{dl}$
 - H. $200 \frac{mg}{dl}$

Passage II

Earth's moon rotates like a satellite around Earth. It is the fifth largest moon in the Solar System and is best seen at night. The Earth's moon is about 384,400 km from Earth and has an orbital period of twenty-seven days. Most scientists agree that the Moon formed about 4.5 billion years ago; however, there are several conflicting theories on the Moon's origin. Below two scientists discuss what they believe to be true.

Scientist 1

The Fission Theory states that the Moon and Earth were once the same formation. A part of the formation separated from Earth and became the Moon. The formation that broke off to form the Moon most likely came from the Pacific Ocean Basin. The rock densities of the Moon are similar to the rock densities of the Earth's mantle. This is because the part that broke off from the Earth to form the Moon broke off from the outer part of the Earth's mantle. The theory that the Moon and Earth formed separately is highly unlikely. For this theory to be true, Earth's gravitational field would have had to pull the moon into orbit. This is unlikely because it would have required a very particular setup. Most objects that come into the Earth's gravitational field have elliptical orbits. If the Moon were pulled into orbit with the Earth, it would have a comet-like elliptical orbit—which it does not.

Scientist 2

The Impactor Theory states that a small planet collided with the Earth just after the solar system was formed. This caused large amounts of materials from the outer shell of both planets to break off. This debris started orbiting the Earth and forming one collective body of material. That collective piece is what we now call the Moon. The lunar rocks studied are burnt, implying they were heated at one time. This would make sense because when the small planet and Earth collide, the material became heated due to impact. In addition, the Moon does not have a magnetic field like Earth, but some of the rocks on the surface of the Moon hint the Moon could have had some sort of magnetic qualities at one time. This is because the Moon was partially made up of Earth's outer rocks.

7. Which of the following best states the basis for the belief of Scientist 1?
 - A. The Moon was formed from the destruction of another planet.
 - B. The Moon was formed from a broken-off piece of the Earth's mantle.
 - C. The Moon was pulled into orbit with the Earth.
 - D. The Moon has the exact same rock composition as the Earth.
8. When it comes to the Moon, both scientists agree that:
 - E. the Moon was formed about 4.5 billion years ago
 - F. the Moon takes twenty-seven days to complete an orbit
 - G. the Moon orbits around the Earth
 - H. All of the other answers are correct.
9. What is the main conflicting viewpoint between Scientist 1 or Scientist 2?
 - A. Scientist 1 believes that the Moon formed from pieces of the Earth, while Scientist 2 believes the Moon was formed from pieces of the Earth as well as pieces of another planet.
 - B. Scientist 1 believes the Moon was formed from pieces of the Earth as well as pieces of another planet, while Scientist 2 believes that the Moon formed from pieces of the Earth.
 - C. Scientist 1 believes the Moon was formed from debris pulled into Earth's orbit, while Scientist 2 believes the Moon was formed from pieces of the Earth as well as pieces of another planet.
 - D. Scientist 1 believes that the Moon formed from pieces of the Earth, while Scientist 2 believes the Moon was formed from the collision of two planets that orbited Earth.

GO ON TO THE NEXT PAGE.

10. What do both the viewpoint of Scientist 1 and the viewpoint of Scientist 2 have in common?
- E. Both agree that pieces of the Earth were used in the formation of the Moon.
 - F. A small planet collided with the Earth prior to the existence of the Moon.
 - G. It is unlikely, but possible, that the Moon and Earth formed separately.
 - H. They both agree that the Moon should have a comet-like elliptical orbit.
11. If research concluded that the Moon's composition was the same as the Earth's composition, which viewpoint would this support?
- A. Scientist 1's
 - B. Scientist 2's
 - C. Both Scientist 1's and Scientist 2's
 - D. Neither Scientist 1's nor Scientist 2's

Passage III

Sound waves travel through a medium by mechanically disturbing the particles of that medium. As particles in the medium are displaced by the sound wave, they in turn act upon neighboring particles. In this fashion, the wave travels through the medium through a parallel series of disturbed particles. Like in other forms of motion, the rate at which the sound wave travels can be measured by dividing the distance over which the wave travels by the time required for it to do so.

Study 1

A group of students hypothesizes that the velocity of sound is dependent upon the density of the medium through which it passes. They propose that with more matter in a given space, each particle needs to travel a shorter distance to disturb the adjacent particles. Using two microphones and a high speed recording device, the students measured the delay from the first microphone to the second. They chose a variety of media, shown in Table 1, and measured the velocity of sound through each using their two-microphone setup. The results are found in Table 1.

Table 1		
Velocity of Sound in Various Media		
Media	Density (kg per cubic meter)	Velocity (m/s)
Air	1.0	343.0
Pure Water	1,000.0	1,493.0
Sea Water	1,020.0	1,533.0
Glass	2,600.0	4,540.0
Iron	7,870.0	5,130.0
Lead	11,350.0	1,158.0

Study 2

The students wanted to test their hypothesis by using the same medium at different densities. To do this, they heated pure water to various temperatures and repeated the procedure described in Study 1. Their results can be found in Table 2.

Table 2		
Velocity of Sound in Water		
Temperature (°C)	Density (kg per cubic meter)	Velocity (m/s)
0	999.8395	1,402.39
10	999.7026	1,447.28
20	998.2071	1,482.36
30	995.6502	1,509.14
40	992.2	1,528.88
50	988.04	1,542.57
60	983.2	1,552.00
80	971.8	1,555.00
100	958.37	1,543.05

12. According to the data in Study 1, as density increases, what happens to the velocity of sound?
 - E. It increases
 - F. It decreases
 - G. It either increases or decreases
 - H. It does not change
13. According to Study 2, over what temperature interval does velocity begin to decrease as water temperature rises?
 - A. 0°C to 10°C
 - B. 50°C to 60°C
 - C. 60°C to 80°C
 - D. 80°C to 100°C
14. Assume that density of a substance is the only contributing factor to velocity of sound through that substance. If the students' hypothesis in Study 1 is correct, what might they have predicted for the velocity of sound through lead? (Assume all other values in Table 1 remained the same.)
 - E. $1300 \frac{m}{s}$
 - F. $4200 \frac{m}{s}$
 - G. $5100 \frac{m}{s}$
 - H. $6500 \frac{m}{s}$

GO ON TO THE NEXT PAGE.





15. Which study provides stronger evidence against the students' prediction and why?
- A. Study 1, because using different media shows that velocity increases as density increases
 - B. Study 1, because using different media shows that velocity increases as density decreases
 - C. Study 2, because using the same medium shows that velocity increases as density decreases
 - D. Study 2, because using the same medium shows that velocity decreases as density decreases
16. According to Study 2, water at which of the following temperatures yields the greatest velocity of sound?
- E. 0°
 - F. 60°
 - G. 80°C
 - H. 100°C
17. In Study 1, if the students were to double the length of the samples of media, what would happen to the velocity of sound through those media?
- A. It would remain constant.
 - B. It would increase.
 - C. It would decrease.
 - D. More information is necessary.



Passage IV

Understanding the biological features of different bacteria that allow them to grow in unwelcoming environments is necessary to treat and prevent human disease. Modern scientific laboratories, such as those in major hospitals, take blood, urine, and mucus samples from patients and culture them for bacterial growth. During the culturing process, laboratory technicians stain the growing bacteria for a component of their cell wall, the structure that provides shape and rigidity to the bacterium, through a process called Gram staining. Bacteria are typically classified as Gram positive or Gram negative, a distinction that is important in selecting the most effective antibiotic for treatment. Gram positive bacteria appear purple under a microscope, while Gram negative bacteria appear pink. However, some bacteria do not Gram stain and cannot be seen under a microscope when prepared this way.

Technicians also grow the bacteria on various types of plates containing special growth nutrients to determine which bacteria are causing a specific illness. If a bacterium is able to grow on a selective plate, meaning a plate that contains additional nutrients required for a specific bacterium to grow if it is present in the culture, doctors are able to determine the exact cause of a patient's illness and prescribe targeted antibiotics to eliminate the infection. Bacteria that commonly cause human illness, their growth requirements, and their appearance on specific growth media are presented below in Table 1.

Scientists can take the bacteria cultured on the plate and further analyze their enzymes. Three enzymes—urease, catalase, and beta-lactamase—are important for bacterial survival against the

human immune system. Urease is responsible for producing urea, a basic molecule that can counteract the bactericidal (bacteria-killing) activity of stomach acid. Catalase, on the other hand, helps bacteria neutralize toxic substances released from human immune cells, allowing them to survive oxidative stress in high-oxygen areas. Finally, beta-lactamase allows Gram Positive bacteria to break down antibiotics called penicillins. While this ability to break down penicillin and its related antibiotic ampicillin was not initially present, bacteria, especially *E. coli*, have adapted by developing the new enzyme beta-lactamase that opens the ring responsible for penicillin's bactericidal activity, rendering the antibiotic ineffective. This and other examples of antibiotic resistance are becoming more common and are making treatment of serious human diseases very challenging.

18. A bacterium that stains pink in a Gram stain and requires lactose to grow is most likely to cause what disease?
- E. Sinus infection
 - F. Urinary tract infection
 - G. Pneumonia
 - H. Diarrhea
19. A patient with a sinus infection goes to the doctor and a culture of mucus is taken to determine the species of bacteria causing the disease. The technician appropriately selects the Chocolate Agar base but forgets to add Factor X. What effect will this have on the bacterial culture?
- A. No growth
 - B. Normal growth
 - C. Excess growth
 - D. Cannot be predicted

GO ON TO THE NEXT PAGE.

Table 1

Disease	Organism	Color Under Microscope	Selective Medium Required
Sinus infection	<i>H. influenzae</i>	Pink	Chocolate Agar (Factors V and X)
Pneumonia (lung infection)	<i>S. pneumoniae</i>	Purple	None
Stomach infection	<i>H. pylori</i>	None Visible	Sheep's blood
Urinary tract infection	<i>E. coli</i>	Pink	Lactose
Diarrhea*	<i>B. cerrius</i> *	Purple	Lactose

*When answering questions about this passage, assume that only *B. cerrius* causes diarrhea.





20. A patient with watery diarrhea comes to the doctor after eating spoiled food at a family event. The bacterium most likely responsible for causing the diarrhea would require what growth medium or media?
- E. Chocolate agar
 - F. Lactose
 - G. Lactose and chocolate agar
 - H. Sheep's blood
21. A technician stains a slide using the Gram stain procedure and sees nothing upon looking under the microscope. Which growth medium could be required to determine if a particular bacterium is causing disease?
- A. Chocolate agar
 - B. Lactose
 - C. Sheep's blood
 - D. None of the other answers
22. Which of the following bacteria is most likely to produce urease?
- E. *B. cereus*
 - F. *E. coli*
 - G. *H. pylori*
 - H. *S. pneumoniae*
23. Assume a new growth medium was created that contained a mixture of sheep's blood, lactose, and Factors X and V (chocolate agar) but was also supplemented with penicillin. What type of bacteria could likely be cultured on this new medium?
- A. *B. cereus*
 - B. *E. coli*
 - C. *H. pylori*
 - D. *S. pneumoniae*



Passage V

Two scientists wanted to test the solubility of different substances. Solubility is a measure of how many moles of a given substance (known as the “solute”) can dissolve in a given volume of another substance (known as the “solvent”). The solvent can also be thought of as the substance present in greater amount, while the solute can be seen as the substance present in lesser amount. The scientists performed the following experiments to investigate this property.

Experiment 1

The scientists tested the number of moles of several substances that could be completely dissolved in 50mL of water at various temperatures. They made their solutions by slowly adding amounts of each substance to beakers sitting on a hot plate containing water and a stirring rod until no more of the substance dissolved in the solution. The beakers were weighed before and after the additions and the difference in mass was calculated to be the added mass of the substance. The researchers then calculated the number of moles that dissolved for each trial using the molecular mass and the recorded mass for each trial. Results are recorded in Table 1.

Table 1			
Substance Added	Temperature (°C)	Grams Added	Moles Added
NaCl	10°	1.27g	0.02173
NaCl	30°	1.76g	0.03011
NaCl	50°	2.34g	0.04004
KNO ₃	10°	0.93g	0.00920
KNO ₃	30°	1.66g	0.01642
KNO ₃	50°	3.45g	0.03412
CuSO ₄	10°	1.24g	0.00777
CuSO ₄	30°	2.56g	0.01604
CuSO ₄	50°	4.75g	0.01603
AgCl	10°	0.0075g	5.233×10^{-5}
AgCl	30°	0.0045g	3.1398×10^{-5}
AgCl	50°	0.0023g	1.6047×10^{-5}

Experiment 2

In this experiment, the scientists wanted to test the solubility of in a variety of liquids at several temperatures. Their procedure was similar to that of Experiment 1, but with a range of liquids and only one solid. The results are compiled in Table 2.

Table 2			
Liquid	Temperature (°C)	Grams NaCl Added	Moles NaCl Added
Water	10°	1.25g	0.02139
Water	30°	1.81g	0.03097
Water	50°	2.41g	0.04124
Ethanol	10°	0.75g	0.01283
Ethanol	30°	0.98g	0.01677
Ethanol	50°	1.02g	0.01745
HCl	10°	5.05g	0.08641
HCl	30°	6.32g	0.10814
HCl	50°	7.03g	0.12029
Methanol	10°	2.47g	0.04226
Methanol	30°	3.23g	0.05527
Methanol	50°	4.56g	0.07802

24. How did the procedures of the two experiments differ?
- E. In Experiment 1, temperature was varied, while in Experiment 2, it was held constant.
 - F. In Experiment 1, temperature was held constant, while in Experiment 2, temperature was varied.
 - G. In Experiment 1, several solvents were tested, while Experiment 2 tested several solutes.
 - H. In Experiment 1, several solutes were tested, while Experiment 2 tested several solvents.

GO ON TO THE NEXT PAGE.



25. In Experiment 2, which of the following combinations of temperature and solvent dissolved the greatest number of moles of NaCl?
- A. Water at 10°C
 - B. Ethanol at 30°C
 - C. HCl at 50°C
 - D. Methanol at 50°C
26. Which of the following correctly ranks the solutes from Experiment 1 in decreasing order of solubility in water at 50°C?
- E. KNO_3 , CuSO_4 , AgCl , NaCl
 - F. NaCl , KNO_3 , CuSO_4 , AgCl
 - G. AgCl , NaCl , KNO_3 , CuSO_4
 - H. CuSO_4 , NaCl , KNO_3 , AgCl
27. Which of the following best explains the relationship between temperature and solubility of AgCl?
- A. As temperature decreases, solubility of AgCl increases.
 - B. As temperature increases, solubility of AgCl increases.
 - C. As temperature decreases, solubility of AgCl decreases.
 - D. As temperature increases, solubility of AgCl increases, then decreases.
28. Suppose the scientists conducted a third experiment in which they dissolved NaCl in 100mL of methanol at 100°C. Which of the following might have been the number of grams dissolved?
- E. 3.42g
 - F. 4.24g
 - G. 4.56g
 - H. 6.46g

**Passage VI**

The period of a simple pendulum is defined as the amount of time that it takes for a pendulum to swing from one end to the other and back. In studying the period of a simple pendulum, two students express their opinions.

Student 1

The period of a pendulum depends on two factors: the mass of the pendulum's bob (the object swinging at the end of the pendulum) and the length of the pendulum. The height at which the pendulum is originally dropped does not affect the period.

Student 2

The period of a pendulum only depends on the length of the pendulum. Varying the mass and the height at which the pendulum is originally dropped does not affect how long the pendulum takes to swing across.

The two students ran a series of trials to measure the period of a simple pendulum using varying weights and lengths. The students did not measure height as a factor. The results of the trials can be seen in Table 1 below:

Table 1		
Mass (kg)	Length (m)	Period (s)
6 kg	0.25 m	1 s
6 kg	1 m	2 s
10 kg	4 m	4 s
10 kg	9 m	6 s
14 kg	9 m	6 s

29. On which of the following points would the students most likely disagree?
- A. Two children of different masses swinging on identical swings would show the exact same swinging period.
 - B. Two children of identical masses swinging on swings of different length would show different swinging periods.
 - C. A child swinging at a height of one meter would show the same period of swinging as a child swinging at a height of two meters.
 - D. Length of a pendulum is not important to consider when measuring period.
30. During an earthquake, several chandeliers in a mansion begin to swing. Some of the chandeliers are quite small while others, such as one found in the dining room, are very large. However, all of the chandeliers hang the same exact distance from the ceiling. What would the two students predict would happen?
- E. Student 1: Every chandelier would display a different period of swinging
Student 2: Every chandelier would have the same exact period of swinging
 - F. Student 1: Every chandelier would have the same exact period of swinging
Student 2: Every chandelier would display a different period of swinging
 - G. Both Student 1 and Student 2: Every chandelier would have the same exact period of swinging.
 - H. Both Student 1 and Student 2: Every chandelier would display a different period of swinging



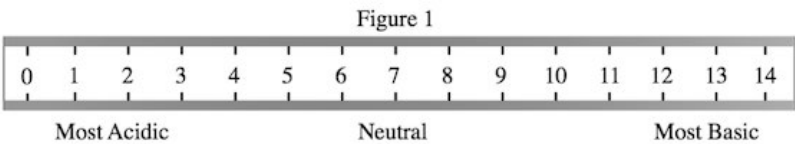


31. Before analyzing the data collected, the two students go out into a local playground and use the swing set to test their hypotheses in an approximate manner. Student 1 and Student 2 are almost exactly the same mass, so Student 2 swings wearing his backpack full of books. Both students begin swinging from the same height and swing exactly three times each in exactly twelve seconds. Whose hypothesis has been supported in this brief trial?
- A. Student 1's
 - B. Student 2's
 - C. Both Student 1's and Student 2's
 - D. Neither student's
32. According to the data provided, what would we predict would happen if an experiment compared the periods of a pendulum made of a lead weight on a meter-long cord and a pendulum made of a tennis ball on a three-meter-long cord?
- E. The period of the tennis ball pendulum would be longer than the period of the lead ball pendulum.
 - F. The period of the tennis ball pendulum would be shorter than the period of the lead ball pendulum.
 - G. The period of the tennis ball pendulum would be the same as the period of the lead ball pendulum.
 - H. The period of the tennis ball pendulum would be different than the period of the lead ball pendulum. We do not know whether it would be shorter or longer.
33. According to the data presented, what is the apparent relationship between mass m and period T ?
- A. The two variables have a positive linear correlation
 - B. The two variables have a positive non-linear correlation
 - C. The two variables have a negative linear correlation
 - D. The two variables are not related.
34. According to the data, what is the apparent relationship between length l and period T ?
- E. The two variables have a positive linear correlation
 - F. The two variables have a positive non-linear correlation
 - G. The two variables have a negative linear correlation
 - H. The two variables are not related.



Passage VII

A student wished to study the acidity and basicity of various household ingredients and chemicals using her own, homemade pH indicator. A pH indicator is a substance that changes colors to indicate the acidity or basicity of a chemical solution. Acids can be defined as substances that donate hydrogen ions, or H^+ , while bases are substances that accept H^+ ions. The strength of these acids and bases can be measured using the pH scale as shown in Figure 1.



Experiment 1

The student placed a leaf of red cabbage in a blender with one liter of water and blended until the cabbage had been liquefied. She then strained the purple mixture and bottled it. The student then added one drop of her homemade cabbage pH indicator to a variety of household chemicals listed in Table 1. She recorded the known pH of these chemicals as well as the color the indicator turned when added to these chemicals.

Table 1		
Household Chemical	Known pH	Color of Indicator
Toilet Bowl Cleaner	1.0	Red
Soda Pop	2.5	Light Pink
Lemon Juice	3.0	Medium Pink
Vinegar	4.5	Dark Pink
Water	7.0	Purple-Blue
Baking Powder	8.0	Dark Blue
Baking Soda	10.0	Light Blue
Laundry Detergent	12.0	Green
Drain Cleaner	14.0	Yellow

Experiment 2

The student wanted to see how baking soda would react in the presence of other household chemicals. She combined baking soda in water separately with each of the other chemicals used in Experiment 1. Some combinations would create bubbling while some other combinations wouldn't. She recorded the results in Table 2 below.

Table 2	
Household Chemical	Reaction?
Toilet Bowl Cleaner	Yes
Soda Pop	Yes
Lemon Juice	Yes
Vinegar	Yes
Water	No
Baking Powder	No
Laundry Detergent	No
Drain Cleaner	No

35. If four solutions were made from household chemicals and the red cabbage indicator and resulted in the four following colors, which solution contained the most acidic chemical?
 - A. Light Pink
 - B. Dark Pink
 - C. Yellow
 - D. Green
36. Which of the following answer choices lists the four acids lemon juice, vinegar, toilet bowl cleaner, and soda pop in order from weakest to strongest?
 - E. Toilet Bowl Cleaner, Soda Pop, Lemon Juice, Vinegar
 - F. Vinegar, Soda Pop, Lemon Juice, Toilet Bowl Cleaner
 - G. Vinegar, Lemon Juice, Soda Pop, Toilet Bowl Cleaner
 - H. Toilet Bowl Cleaner, Vinegar, Soda Pop, Lemon Juice

GO ON TO THE NEXT PAGE.



37. The student will attempt to color in Figure 1 with the appropriate color the indicator will turn at various pHs. Which answer choice lists the colors in the correct order, from left, or low pHs, to right, or high pHs?
- A. Purple, Blue, Green, Yellow, Red, Pink
 - B. Pink, Red, Yellow, Green, Blue, Purple
 - C. Yellow, Green, Blue, Purple, Pink, Red
 - D. Red, Pink, Purple, Blue, Green, Yellow
38. What can be inferred from the results of Experiment 2?
- E. Baking soda only produces bubbles in the presence of an acid.
 - F. Baking soda only produces bubbles in the presence of a base.
 - G. Baking soda only produces bubbles in the presence of a chemical with a neutral pH.
 - H. Baking soda does not bubble in the presence of any solutions.
39. A new indicator, called Methyl Red, is also used to test the household chemicals from Experiments 1 and 2. It is found that the indicator turns red in the presence of toilet bowl cleaner, soda pop, or lemon juice; it turns orange in the presence of vinegar; it turns yellow in the presence of the rest of the chemicals. Which of the following pH ranges most likely contain the pH value at which Methyl Red has its color transition, or pH at which the indicator will stop being red and change to yellow?
- A. 0 – 2
 - B. 2 – 4
 - C. 4 – 6
 - D. 6 – 8
40. A universal indicator is a pH indicator that is a mix of several different indicators that have distinct color changes in various ranges of the pH scale in order to precisely tell the pH of any solution. While the red cabbage indicator is a good indicator for most pHs, it has one range that does not have drastic enough color changes to precisely tell the pH within this region. What pH range should a supplemental indicator have drastic colors changes in to improve the red cabbage indicator?
- E. 2 – 4
 - F. 6 – 8
 - G. 8 – 10
 - H. 12 – 14

END OF TEST 4

STOP! DO NOT RETURN TO ANY OTHER TEST.



ENGLISH TEST: ANSWER KEY

1. A. NO CHANGE

- B. of Asia. The
C. of Asia, the quite
D. for Asia. The

2. E. NO CHANGE

- F. that incited
G. sparked, initiated, and
incited
→ H. incited

3.

- A. NO CHANGE
B. cautious
C. caution
D. and cautiously

4. E. NO CHANGE

- F. have posed
→ G. poses
H. are posing

The first sentence in this passage contains two independent clauses: 1) “During the final months of 2007, the prices of basic grains nearly doubled in Northern Africa, Latin America, and much of Asia,” and 2) “The high prices caused a global food crisis.”

The insertion of a comma between two independent clauses without a coordinating conjunction such as “and” or “but” is called a comma splice. The original text and the answer choice “Asia, the quite” contain comma splices and are therefore incorrect. Although the answer choice “for Asia. The” avoids a comma splice, it is incorrect because it uses an incorrect preposition before the word “Asia.” Only the answer choice “of Asia. The” avoids a comma splice and other errors.

In the context, “sparked” and “incited” are synonyms. To use both words is redundant. The answer choice “sparked, initiated, and incited” is incorrect because it adds even more redundancy to the phrase, and the answer choice “that incited” leads to a sentence fragment. The correct answer, “incited,” avoids the redundancy problem and does not create a sentence fragment.

The original text is correct. The adverb phrase “extremely cautiously” correctly modifies the verb “proceed.” Answer choice “cautious” is incorrect because “cautious” is an adjective whereas the modifier of the verb “proceed” must be an adverb. Similarly, answer choice “caution” is incorrect because it does not contain an adverb to modify the verb “proceed.” Although answer choice “and cautiously” contains the adverb “and cautiously,” the insertion of the word “and” leads to the sentence becoming ungrammatical.

The subject of the sentence “The construction of private farms pose a serious financial threat to farmers in The United States” is “construction,” and the predicate is “pose.” Since “construction” is singular and “pose” is plural, the original text contains a subject-verb agreement error. The answer choices “has posed” and “is posing” contain subject-verb agreement errors as well—both are plural predicates when we have a singular subject. The only answer choice in the singular form is “poses.” The correct way to express the sentence is “The construction of private farms poses a serious financial threat to farmers in The United States.”



ENGLISH TEST: ANSWER KEY

5. A. NO CHANGE
 → B. the most crops
 C. a greater number of
 D. that great number

6. E. NO CHANGE
 F. others;
 → G. others?
 H. others,

7.
 → A. NO CHANGE
 B. depend
 C. are depending
 D. have been depended

8. E. NO CHANGE
 F. soil, and last but not least, elevation
 → G. soil, and elevation
 H. soil, and, of course, elevation

When comparing two items, use the comparative form of an adjective such as “better,” “softer,” or “more transparent.” When comparing three or more items, use the superlative form of an adjective, such as “best,” “softest,” or “most transparent.” We know from context that there are five countries being compared, so we must use the superlative form. In this case, the superlative is “most.” The correct way to express the phrase is therefore “Of the five most industrialized nations, the United States exports the most crops.”

The phrase “How do some countries cope with food crises better than others” is an interrogative—that is, a question. We know it is a question because 1) it starts with an interrogative word, “how,” and 2) because the question support word “do” is inserted. The appropriate punctuation to follow an interrogative is a question mark, so the answer choice “others?” is correct.

Consider the given sentence: “The rate at which countries adopt innovations depends significantly on environmental factors.” We see that the subject is “rate” and the predicate is “depends.” Since rate is singular and depends is singular, they match in number (i.e. they “agree”). Thus, no change is necessary, and the correct answer is “NO CHANGE.”

Lists with three or more items must be separated by commas. The original text contains an error because the list is missing a comma between items. The sentence should read: “climate, soil, and elevation.” The answer choice “soil, and elevation” is correct.



ENGLISH TEST: ANSWER KEY

9. A. NO CHANGE

→ B. new technologies that are suited for one particular region over another from gaining worldwide popularity

C. new technologies that is suited for one particular region over another from gaining worldwide popularity

D. new technologies from gaining worldwide popularity that have been suited for one particular region over another

10.

→ E. NO CHANGE

F. Therefore

G. However

H. As a result

11. A. NO CHANGE

B. that used in North America

C. those used in North America's

→ D. those used in North America

12. E. NO CHANGE

F. to be growing

→ G. from growing

H. of growth

The original text contains a misplaced modifier. We know that the phrase “that are suited for one particular region over another” must modify “new technologies” because the modifier phrase uses the plural form “are,” and “new technologies” is the only plural noun in the sentence. The correct way to express the sentence is “The variability in environment inhibits new technologies that are suited for one particular region over another from gaining worldwide popularity.”

We are asked about a sentence that describes a specific instance of differences between North America and Europe—namely, each continent’s use of pesticides. In the preceding sentence, the author discusses the differences in agricultural practices across regions. The pesticide acidity is an example of the interregional differences, so the answer choice “For example” is the most logical way to connect the two sentences.

Consider the original sentence, “The pesticides used in Europe are much more acidic than North America.” It contains a flawed comparison because it compares the pesticides used in Europe to the entire continent of North America. The proper way to express the sentence is to say “The pesticides used in Europe are much more acidic than those used in North America” because in that case, we are comparing pesticides to pesticides. Note: we must say “more acidic than those” rather than “more acidic than that” because “pesticides” is a plural noun.

The correct idiomatic expression is “prevent from,” so “prevent from growing” is the correct answer choice. The answer choices with “prevent to” and “prevent of” contain incorrect uses of this idiom.



ENGLISH TEST: ANSWER KEY

13. A. NO CHANGE
B. Three committees were appointed
C. Three committees were appointed;
→ D. Three committees were appointed:

14. E. NO CHANGE
→ F. Mr. Samuel Adams was appointed to the committee of confederation.
G. Mr. Samuel Adams was appointed upon the committee of confederation.
H. On the committee of confederation was Mr. Samuel Adams appointed.

15. A. NO CHANGE
B. The committee of independence had been
→ C. The committee of independence was comprised of
D. The committee of independence included

The list following this introduction functions to enumerate the committees that were appointed. For this purpose, a colon is the best option, as it signals such an enumeration.

The sense of the sentence is that Samuel Adams was appointed to the committee of confederation. While the sentence is decipherable in the form in the paragraph, it would be best to reorder the sentence and replace “on” with “to.”

The style in this sentence is a bit antiquated as it stands. If we were to reorder the sentence, it would be like saying, “They (Jefferson, Adams, et al.) were the committee.” Among the options provided, the best two are those that use “comprised of” and “included.” The better option of these two is “comprised of” because it means consisted of. The word “included” could imply that others were members of the committee as well. This is not warranted in the original English.



ENGLISH TEST: ANSWER KEY

16. E. NO CHANGE

F. Mr. Jefferson had been now about a year a member of Congress, but had attended his duty in the house a very small part of the time. When there, had never spoken in public.

→ G. Mr. Jefferson had been now about a year a member of Congress; however, he had attended his duty in the house a very small part of the time and, when there, had never spoken in public.

H. Mr. Jefferson had been now about a year a member of Congress but had attended his duty in the house a very small part of the time, and, when there, had never spoken in public.

17. A. NO CHANGE

→ B. I never heard him utter three sentences together.

C. I never would hear him utter three sentences together.

D. I never could hear him utter three sentences together.

This sentence presents a great occasion to use a semicolon. These ideas are all related, so it is best to keep them together without a period until the very end of the thought. The point is that Jefferson had been a member of Congress for a year but hadn't been very public in his activity. Since there is a second conjunction ("and,"), it is best to split the sentence at the "but," using a semicolon and the word "however" to express the sense of the "but."

Clearly, the present tense (as written) is incorrect. After this, it is necessary to pay attention to the moods of the verb. The two modal verbs "could" and "should" make no sense, given the author's intention. He is writing factual, indicative sentences. Thus, the best option is the simple, indicative "heard."



ENGLISH TEST: ANSWER KEY

18. E. NO CHANGE

- F. Mr. Jefferson had the reputation of a masterly pen for he had been chosen a delegate in Virginia in consequence of a very handsome public paper that he had written for the House of Burgesses that had given him the character of a fine writer.
- G. Mr. Jefferson had the reputation of a masterly pen, for he had been chosen a delegate in Virginia in consequence of a very handsome public paper which he had written for the House of Burgesses that had given him the character of a fine writer.
- H. Mr. Jefferson had the reputation of a masterly pen; he had been chosen a delegate in Virginia in consequence of a very handsome public paper which he had written for the House of Burgesses and which had given him the character of a fine writer.

The most significant issue in this sentence is the paralleling of the two uses of “which” in the second independent clause. They both modify the word “paper”:

The paper...

(1) “which he had written for the House of Burgesses”

AND

(2) “which had given him the character of a fine writer.”

To help clear up this sentence, the best option (among those provided) is the one that makes this parallel explicit by using the conjunction “and.” This helps to prevent the reader from being confused about the antecedent for the second “which.”

19. A. NO CHANGE

- B. Another reason was that Mr. Richard Henry Lee was not beloved by most of his colleagues from Virginia
- C. Additionally, Mr. Richard Henry Lee was not beloved by the most of his colleagues from Virginia
- D. Therefore Mr. Richard Henry Lee was not beloved by most of his colleagues from Virginia

The main issue in the sentence is the use of the superlative “most.” As it is used here, it should not have the definite article “the.” The sentence simply is stating that Mr. Lee was not beloved by most (the majority) of his colleagues. The additional “the” only makes the sentence more opaque and difficult to understand.



ENGLISH TEST: ANSWER KEY

20. To what does “this” refer?

- E. The potential supplanting of Lee by Jefferson
- F. The rivalry between Lee and his colleagues
- G. The overt hatred of Lee by his colleagues
- H. Jefferson’s excellent writing abilities

21. If you wanted to split this paragraph into two separate paragraphs, which sentence would best begin the second paragraph?

- A. “Mr. Jefferson had been now about a year . . .”
- B. “The committee of independence . . .”
- C. “Mr. Jefferson had the reputation of . . .”
- D. “During the whole time that I . . .”

22. Which of the following words would best replace the word “stand” in the last sentence?

- E. NO CHANGE
- F. position
- G. endure
- H. argue

23. A. NO CHANGE

- B. known to all his kingdom that
- C. known to all his kingdom: that
- D. known to all his kingdom; that

This is an example of potential (though not complete) ambiguity in using a demonstrative pronoun. The “this” is referring back to the setting up of Jefferson as a rival to supplant Lee. (This is stated directly at the end of the previous sentence.) The best option among those provided is, “The potential supplanting of Lee by Jefferson,” since it does at least imply the rivalry (by means of the act of supplanting).

Although the choice is a bit subjective, the best option among those provided is the sentence that marks the shift to focus on Jefferson himself. The first paragraph would thus describe the set of committees that were established. The second would discuss Jefferson’s character and his selection. The most tempting wrong answer is likely, “Mr. Jefferson had the reputation of a masterly pen.” That is not acceptable because it is part of a set of reasons being provided. It would be better to keep these together with their explanatory introduction.

The word “stand” is a bit too informal for a potential reader. The author clearly means to say that Mr. Jefferson was unable to “withstand” or “undertake” verbal dispute. The best option for this would be “endure,” which is more literal and direct than the somewhat informal “stand.”

The word “that” is here being used as a conjunction that introduces an indirect quotation. You could write this sentence in a form like: Then the king spoke to all his kingdom, “Etc . . .” However, as used here, “that” introduces the clause describing indirectly what he said to the kingdom. This requires no comma, as it is a necessary part of the main clause, helping to specify exactly what he made known to the kingdom.



ENGLISH TEST: ANSWER KEY

24. E. NO CHANGE

F. he whom healed
her of this dreadful
diseaseG. whomever could heal
her of this dreadful
disease→ H. whoever would heal
her of this dreadful
disease

25. A. NO CHANGE

→ B. disease would be richly
rewardedC. disease will be richly
rewardedD. disease could be richly
rewarded

26. A. NO CHANGE

B. sprucely, as a doctor,
and said he could cure
her→ C. sprucely as a doctor
and said he could cure
herD. sprucely as a doctor
and said, he could cure
her

27. A. NO CHANGE

→ B. apple and, to punish
her a little more,C. apple, and to punish
her a little more,D. apple and to punish
her a little more

The error with the sentence as written is its improper use of the objective form of “whoever.” When “who” is the object of a verb or a preposition, it takes the form “whom.” (For example: “The man to whom the shirt was given . . .”) The same holds true for “whoever” and “whomever.” Here, the word “whomever” is being incorrectly used as the subject of the relative clause, when it the sentence says that whoever heals her will be rewarded.

Although you could stylistically argue that “should” could be used as the modal for the main verb of this sentence, we can make it a bit clearer by changing this word. Notice that this selection hides a conditional statement: If someone heals her, then they will be rewarded. The modal “would” expresses the conditional nature of the king’s offer.

As written, the problem with this selection is its unnecessary use of the comma before the conjunction “and.” The conjunction is only in the predicate, compounding the old soldier’s two actions: (1) dressing up and (2) saying that he can cure her. In such a case, you do not need to use a comma.

The easiest way to see the proper form is to simplify the sentence. In its most basic form, it can be written: “He chopped . . . and . . . gave . . .”

The conjunction (“and”) is being used to make a compound predicate. This does not require a comma. Therefore, you can eliminate the comma before “and.” However, the expression, “to punish her . . . more,” is an explanatory clause that should be set off from the second half of the predicate. Therefore, the two commas surrounding it should be retained.



ENGLISH TEST: ANSWER KEY

28. Which of the following is an acceptable replacement for the underlined selection, “had been growing on?”
- E. grow
 - F. grew
 - G. had grown
 - H. had continued growing
29. A. NO CHANGE
- B. as before, but the poor princess was in a dreadful fright
 - C. as before; therefore, the poor princess was in a dreadful fright
 - D. as before; nevertheless, the poor princess was in a dreadful fright
30. E. NO CHANGE
- F. it would help and he would
 - G. it would help and that he would
 - H. it would help; and he would
31. What is the best replacement for the underlined selection, “to be sure?”
- A. indeed
 - B. unquestionably
 - C. verifiably
 - D. accurately

From the context, you can guess that the expression “growing on” means continued to grow. The author wishes to express this action as something that happened earlier in the past but also as a process as well. (That is, the growing occurred over a period of time.) The best way to do this is to keep the past perfect by using the form “had continued” but use the participle “growing” to explain what “had continued” happening over a period of time.

Clearly, the author does relate the princess’s fright to the growth of the nose. Therefore, the word “therefore” would function as the best connective between the two sentences that are connected by the semicolon. “But” and “nevertheless” implies contradiction that isn’t present; only “therefore” correctly associates the growth of the nose with the princess’s being “in a dreadful fright.”

There are two indirect statements being made by the “doctor” in this sentence:

1. That he was sure that it would help
2. That he would call on the next day

To make it clear that this is a compound set of indirect statements, use an additional “that” as the appropriate conjunction to indicate the fact. Otherwise, it is possible to misunderstand exactly what is being connected by the conjunction “and.”

Given the tone of the passage, the expression “to be sure” expresses the idea that the nose was in fact smaller, though this might seem implausible to the reader. The word that best expresses this emphatic “in fact” is the option “indeed.”



ENGLISH TEST: ANSWER KEY

32. **E. NO CHANGE**
F. had first began
→ **G.** had first begun
H. was beginning

33. **A. NO CHANGE**
→ **B.** Then he thought to himself
C. Then he thought to he
D. Then he thought to them

34. **E. NO CHANGE**
F. The morrow came and the nose was ten times bad as before.
G. The morrow came, and the nose was ten times bad as before.
→ **H.** The morrow came, and the nose was ten times worse than before.

35. **A. NO CHANGE**
B. "My good lady." said the doctor, "Something works against
C. "My good lady" said the doctor, "Something works against
→ **D.** "My good lady," said the doctor, "something works against

Notice the sequence of events in the sentence: (1) First, the soldier dressed as a doctor began to meddle with the nose. Then, (2) it was bigger, even after that meddling. To express the fact that (1) was earlier in the past, you need to use the past perfect tense. This is formed by using "had" with the perfect passive participle "begun." (Note, that this participle is "begun" and not "began.")

From the context, we can tell that the author means to say that the doctor is thinking internally (that is, thinking "to himself"). This calls for the use of a reflexive pronoun, namely "himself." Such a pronoun indicates that the action of the verb "returns" or "reflects" back on to the subject.

As written, the mistake in the sentence is its lack of the appropriate correlative conjunction "as" to accompany the "as" that precedes "before." The comma is appropriate, for the sentence is a compounding of two independent clauses. The comparison should be made by using "worse." This requires "than" instead of "as," but it is presented this way in answer choice D.

As written, the problem with the selection is the fact that the word "something" is capitalized. When a quotation is being continued in the middle of a sentence, you do not capitalize the beginning of the new portion of the quote.



ENGLISH TEST: ANSWER KEY

36. E. NO CHANGE

F. my medicine, and is too strong for it

→ G. my medicine and is too strong for it

H. my medicine, and is too strong for them

37. A. NO CHANGE

B. art that it is this. You have stolen goods

→ C. art that it is this: you have stolen goods

D. art that it is this; you have stolen good

38. What is the meaning of the underlined word “about?”

→ E. around

F. concerning

G. pertaining to

H. connected to

39.

→ A. NO CHANGE

B. “Very well,” said the doctor: “you may do as

C. “Very well”, said the doctor, “you may do as

D. “Very well.” Said the doctor. “You may do as

As written, the only error in the sentence is its inappropriate use of “to.” The author wishes to say that something is overly strong, counteracting the doctor’s medicine. This means that the thing in question is too strong. The word “to” may be used as a preposition or in conjunction with an infinitive. However, it does not perform the “to a high degree” modification expressed by “too.”

Because of the use of the demonstrative pronoun “this,” you need to keep the two clauses linked together. “This” refers to the remark that the doctor makes soon thereafter. It is like he is saying, “The problem is this . . .” Such an expression uses the colon in order to indicate that the continuation, “You have . . .” explains what is meant by “this.”

Although it is not one of the primary meanings of the preposition “about,” the word *can* mean “around.” It is likely that you have seen the adverbial form of the word used in this sense, as in: “He left his books scattered all about.” Context provides the necessary clues for this meaning, for the doctor soon says, “If you do not give them back . . .” The “them” refers to the goods that are somewhere in the presence of the princess.

The selection is correct as written. All of the wrong answers misuse punctuation in various ways. The first comma should lie within the quotation marks. You could place a period after “doctor”; however, you would still need to retain the connection (via comma) with the initial portion of the quotation.



ENGLISH TEST: ANSWER KEY

40. Which of the following is the best replacement for the underlined word “own?”

E. possess

→ F. acknowledge responsibility for

G. acknowledge possession of

H. take control of

41. A. NO CHANGE

→ B. send back the cloak, the purse, and the horn, all of which you stole from the right owners

C. send back the cloak, the purse, and the horn, which you stole from the right owners

D. send back the cloak, the purse, and the horn, that you had stolen from the right owners

42. E. NO CHANGE

F. three, and gave them to the doctor, and begging him to give them back to the soldiers

G. three and gave them to the doctor, begging him to give them back to the soldiers

→ H. three, gave them to the doctor, and begged him to give them back to the soldiers

This passage is slightly old-fashioned in its tone; therefore, we should not be surprised that a word like “own” might be used in an older, less standard manner (as it is here). The word own can mean “acknowledge ownership of / possession of / [or even] responsibility for.” Think of the contemporary expression “own up to your crime.” It means “accept responsibility for your crime.” Since the author uses the word “it,” we know that he is not talking about the possessions. (He spoke of them in the plural in the preceding paragraphs.) Instead, he wants the princess to take responsibility for the general state of affairs. That is, he wants her to “own up” to the fact that she has stolen goods (not to “own” the goods themselves).

As written, the sentence is slightly confusing in its use of the relative pronoun “that.” Although common sense might indicate that it refers to all of the items that are to be sent back, it is still possible to read this sentence as only applying the relative clause “that you stole from the right owners” to the horn. Replacing “that” with “all of which” helps to draw attention to the fact that the clause refers to all of the items.

Since the author wishes to express a sequence of events, we can do this by treating the predicate like we would a list of (e.g.) nouns. Just as you would separate three nouns with commas (e.g. apples, oranges, and goblins), you can separate each of the verbs (with their objects, etc) in the predicate: (1) ordered . . .three, (2) gave . . . doctor, (3) begged. This option is slightly better than the option that uses the present active participle (“begging”), because the correct answer still expresses the sequential order in clear terms: First (1), then (2), and then (finally) (3).



ENGLISH TEST: ANSWER KEY

43. A. NO CHANGE

B. safer

→ C. safely

D. more safely

44. E. NO CHANGE

→ F. wished the king and
all his court a good
day, and was soon
with his two brothersG. wished the king, and
all his court, a good
day and was soon
with his two brothersH. wished the king, and
all his court a good
day and was soon
with his two brothers

45. A. NO CHANGE

→ B. palace, except when
they took an airing in
their coach to see the
worldC. palace except when
they took an airing to
see the world in their
coachD. palace, except when
they took an airing to
see the world, in their
coach

46. E. NO CHANGE

→ F. Mr. Emerson's
literary reputationG. Mr. Emersons'
literary reputationH. The literary of Mr.
Emerson's reputation

The word “safe” is an adjective, but there is no justification for an adjective to be used here. It clearly is meant to describe how the doctor “had” the items. That is, it is meant to say, “He had them safely (in his possession).” Although the author’s expression is somewhat awkward here, we can at least help make the passage more readable by changing the incorrect form of “safe” to the correct “safely.”

As written, the sentence does not delineate the multiple verbs in the predicate very well. Looking at the sentence in a simplified fashion, we can see that it states that the doctor: (1) put . . . (2) wished . . . (3) was with . . . Therefore, it is best to separate the verbs appropriately by placing a comma before the “and” that directly precedes the final verb (“was”).

The problem with the sentence is the confusing use of the prepositional phrase “in their coach.” This is being used adverbially to describe where they would “take an airing.” It is not an adjectival prepositional phrase demarcating a particular “world”—i.e. the one that is in their coach (whatever nonsense that might even mean)!

Since “literary” is an adjective modifying the noun “reputation,” we need to keep the general structure of the sentence as it stands. The problem with the selection as written is its lack of use of an apostrophe for the possessive form of “Emerson.” Since “Emerson” ends as it does, it requires an “s” to form the possessive. Thus, the appropriate form is “Mr. Emerson’s.”



ENGLISH TEST: ANSWER KEY

47.

- A. NO CHANGE
B. is established
C. is established, placed, and located
D. is placed and located

There are two matters to be considered regarding the verb in question. First, notice that it is comprised of the helping verb “is” and the perfect passive participle “established.” In English, when we use “to be” and this participle, we form the passive voice. When “to be” is in the present tense, the whole verb is in the present tense—as in this case. The verb is called “passive” because the subject “receives” the action. The reputation is placed; it is not that the reputation is doing the placing (so to speak).

48. E. NO CHANGE

- F. nor have any equaled him
G. nor are any able to equal him
→ H. nor does any equal him

The problem here is the implied subject. The adjective “any” is being used “substantively.” That is, it is functioning as the noun of the sentence. The question you should ask yourself is, “Any what?” Based on the information provided, the “any” refers to “living writer”—it is singular in the earlier independent clause. Therefore, let’s place it into the sentence as it stands: “nor do any living writer equal him.” Notice the blatant error! The subject is singular—“writer”—but the verb is plural—“do.” The verb should be changed to be “does” so that the sentence reads, “nor does any living writer equal him.”

49. A. NO CHANGE

- B. neither in the exquisite delicacy and finish of his chiseled sentences, nor in the metallic ring of his style
C. neither in the exquisite delicacy and finish of his chiseled sentences, and in the metallic ring of his style
D. neither in the exquisite delicacy and finish of his chiseled sentences or in the metallic ring of his style

The issue here is the use of the correlative conjunctions, “Neither . . . nor.” The sentence does not use “nor” properly. The “or” should be replaced with “nor.”



ENGLISH TEST: ANSWER KEY

50. What is an adequate synonym for “suffer” as it is used here?

E. NO CHANGE

→ F. allow

G. pain

H. destroy

51. A. NO CHANGE

B. both admirers and with detractors

C. both of admirers or of detractors

→ D. both of admirers and of detractors

52. What is an acceptable replacement for the underlined word “many?”

E. NO CHANGE

→ F. many people

G. many fools

H. many reasons

53.

→ A. NO CHANGE

B. he has been regarded

C. he had been regarded

D. he was regarded

54. E. NO CHANGE

→ F. all of whom

G. all of who's

H. all of whose

The author's style is a bit old-fashioned, so the word “suffer” is being used here in an older sense. In this older sense, “suffer” means *to tolerate* or *allow*. In old translations of the Bible, there is a passage in which Jesus says something of the form, “Suffer the little children to come unto me.” This does not mean anything related to pain or suffering in the sense that we usually use the word. Instead, it means “allow.”

The problem in this selection is the use of the correlative conjunction set “both . . . and . . .” As written, the sentence incorrectly uses “but also.” Note, however, that the form, “Both of admirers and not of detractors” also is incorrect because of the immediate negation (“not”) after “and.”

The adjective “many” is being used substantively in this sentence. That is, it is being treated as though it were a noun. It implies a noun, though to ascertain what noun, we need to consider the broader context. Although the author is somewhat dismissive regarding Mr. Emerson's admirers, he is by no means completely negative. Therefore, the option “many fools” is inappropriate. However, the other two options are incorrect as well, for clearly he is referring to groups of human beings.

As written, the verb is in the present passive. Mr. Emerson is being regarded by others; thus, he is receiving the action of the others. The first independent clause speaks of him in the present tense. In that sentence, the verb is merely the copula “is,” using two adjectives as predicate nominatives. Given the information that we have, it is best to infer that the author intends to keep these two tenses the same. Therefore, “is regarded” is the best option.

The subordinate clause is describing the young men and maidens who had been Emerson's hearers at an earlier time. Those persons are the implied subject of the subordinate clause. (It is they who had constituted the group.) When “who” is the object of a preposition (here, “of”), the form “whom” must be used. This is similar to the case of “he.” That is, we do not say, “I took it from he.” Instead, we say, “I took it from him.”



ENGLISH TEST: ANSWER KEY

55. A. NO CHANGE

B. though we regard he like no ordinary man

C. though we regarded him as no ordinary man

→ D. though we regard him as no ordinary man

56. E. NO CHANGE

F. we believe that he to be one

→ G. we believe him to be one

H. we believe, him to be, one

57. A. NO CHANGE

→ B. by long acquaintance have we been in mutual contact with each other

C. by long acquaintance have we been in mutual contact

D. by long acquaintance, have we been in mutual contact

58. E. NO CHANGE

→ F. whose kindness does not pass over individuals

G. who's kindness does not pass over individuals

H. whom kindness does not pass over individuals

The main issue with the sentence as written is the fact that the direct object of “regard” is not in the proper objective form. The personal pronoun “he” should be rendered “him” when it functions as an object. The incorrect answer that has “him” but likewise changes the verb “regard” to “regarded” is less acceptable than the correct answer because there is nothing in the sentence (or the context) to justify shifting the tense into the past.

The problem here is the fact that “he” is functioning as the object of the verb “believe”—even though you are likely to read the sentence, “We believe that he is.” That is the sense of the formation “him to be,” but since we have a belief *about him*, it is necessary to render “he” in the proper object form, namely, *him*. The incorrect option with commas adds extra punctuation where it is not necessary.

Clearly, the problem with the selection is the fact that “him and us” are *not* appropriate forms of the personal pronouns that are to be used for “have been.” Although the sentence is inverted, these two are the subjects of the verb. Therefore they should be in the form “he and we.” This is awkward, however. It is perhaps most tempting merely to state, “we.” However, notice that the author has used “we” to refer to himself. (This creates a slightly less personal tone than using “I.”) In order to retain this tone, it is best to indicate that this new use of “we” indicates more than just the author himself. This is best done in the correct answer, which adds “with each other” to the end of the clause.

The relative pronoun is being used here in a possessive sense, describing the “kindness” mentioned in the clause and linking it to “friend” by noting that he has a kindness of a certain quality. The appropriate form for “who” in this case is “whose”—not “who’s” or “whom.”



ENGLISH TEST: ANSWER KEY

59. A. NO CHANGE

B. acquaintance and
also upon our former
study of his writings

→ C. acquaintance but also
upon our former study
of his writings

D. acquaintance but
upon our former
study of his writings

60. What would be the
best sentence to insert
between Sentence 1 and
Sentence 2?

E. The first modern
lacrosse league was
organized in Montreal
in 1856.

→ F. Numerous Native
American tribes,
including the Iroquois,
Cherokee, and
Onondaga have played
versions of it for at
least 400 years.

G. The sport has quickly
gained popularity in
recent years.

H. Consequently,
professional lacrosse
players are not paid
the same salaries as
baseball players.

As written, the sentence does not use the correlative conjunctions “not only . . . but also” correctly. It has the form, “not only upon . . . and upon.” The correct option is the one that corrects the second member of the correlative pair, replacing it with “but also.”

The sentence “Numerous Native American tribes, including the Iroquois, Cherokee, and Onondaga have played versions of it for at least 400 years” creates the best transition between the two other sentences and adds key information which clarifies the passage, introducing where and when the game originated.



ENGLISH TEST: ANSWER KEY

61. Which is the best way to combine Sentences 4 and Sentence 5?

- A. Hundreds of men and women from rival tribes would gather at once to play on fields sometimes several miles long.
- B. On playing fields sometimes several miles long, gathering to play would be hundreds of men and women from rival tribes.
- C. Hundreds of men and women from rival tribes would gather to play at once and they would play on fields sometimes several miles long.
- D. Hundreds of men and women from rival tribes or villages would gather to play at once, but the playing field was sometimes several miles long.

62. In context, where is the best place to put Sentence 6?

- E. After Sentence 1
- F. Before Sentence 3
- G. Before Sentence 4
- H. After Sentence 8

Simplicity is the best option here. Keep the sentence direct, eliminate redundancies, and complicate the sentence structure as little as possible. The correct answer is “Hundreds of men and women from rival tribes would gather at once to play on fields sometimes several miles long.”

This sentence would be best used to begin the second paragraph, as it introduces the topic of differences between the modern game and the first versions of it, and this topic is elaborated on in Sentences 4, 5, and 7.



ENGLISH TEST: ANSWER KEY

63. The author wishes to add a concluding sentence after Sentence 9 that relates to a statement made in the opening sentence. Which of the following sentences would achieve this most effectively?

- A. Lacrosse has played an important role in North American culture for hundreds of years.
- B. If you haven't ever played lacrosse, you're missing out on a really fun game.
- C. Lacrosse is a difficult sport to play because it requires much more physical endurance than baseball.

- D. Though it may not be considered "America's pastime," lacrosse could certainly be called the original American team sport.

64. E. NO CHANGE

F. already falls

- G. had already fallen

H. will have already fallen

The sentence "Though it may not be considered 'America's pastime,' lacrosse could certainly be called the original American team sport" is best because it echoes both the reference to baseball and the assertion of lacrosse's longevity made in Sentence 1.

When describing two past events, one of which took place before the other, the appropriate tense for the chronologically earlier event is the past perfect (e.g., "had eaten" or "had heard").

The sentence mentions two past events: 1) Tarski's move to the US and 2) much of Europe falling into the grips of WWII. The "already" tells us that Europe fell into the grips of WWII before Tarski moved. The sentence should therefore read "By the time Tarski moved the United States, much of Europe had already fallen into the grips of World War II."

The answer choice "has fallen" is incorrect because it uses the present perfect, a tense reserved for continuous past events that persist up until and including the present.

The answer choice "will have fallen" is incorrect because it uses the future perfect, a tense reserved for continuous events that are going to happen in the future.

The answer choice "to fall" is incorrect because it is an infinitive and does not convey either a specific or relative time at which the falling occurs.



ENGLISH TEST: ANSWER KEY

65. A. NO CHANGE

B. Hundreds of problems, all of them mathematical, were solved by Tarski

→ C. Tarski solved hundreds of mathematical problems

D. Mathematical problems—hundreds of them—were solved by Tarski

66. E. NO CHANGE

→ F. Tarski ultimately excelled in mathematics

G. mathematics is what Tarski excelled in

H. the subject in which Tarski ultimately excelled was mathematics

67. A. NO CHANGE

→ B. honors and

C. honors, and,

D. honors; and

The ACT prefers the active voice over the passive voice. We know that the sentence “Hundreds of mathematical problems were solved by Tarski.” contains the passive because it has a form of the verb “to be” (i.e. “were”) followed by a past participle (i.e. “solved”).

The correct way to express the sentence in active voice is “Tarski solved hundreds of mathematical problems.”

Each of the other answer choices contains a passive construction and is therefore incorrect.

The sentence begins with the modifier phrase “Originally wishing to study biology,” which must modify the element immediately following it.

It would not make very much sense for mathematics to study biology, but that is the meaning of the original sentence. Since it was Tarski—not mathematics—who originally wished to study biology, we must put “Tarski” rather than “mathematics” immediately after the modifier phrase.

The correct way to express the sentence is therefore “Originally wishing to study biology, Tarski excelled in mathematics,” so the correct answer is “Tarski ultimately excelled in mathematics.”

Each of the other answer choices contains a misplaced modifier error.

We see that the subject of the sentence is “He.” The sentence goes on to list two predicates: (1) “graduated with honors” and (2) “began his career as a math teacher.” When we have two predicates (i.e. a “complex predicate”) and one subject, the predicates must be separated by a conjunction (such as “and” or “but”), and there should not be a comma between the two predicates.

The correct way to express the sentence is “He graduated with honors and began his career as a math teacher.” The correct answer is therefore “honors and.” Each of the other answer choices introduces unnecessary punctuation between the two predicates.



ENGLISH TEST: ANSWER KEY

68. E. NO CHANGE

F. making his findings public

G. having published his findings

→ H. the publication of his findings

When we have a “neither . . . nor” list, it is important to make the items in the list parallel—that is, to make the items in the list share a linguistic structure.

In this case, the “neither . . . nor” list is “neither the application of his research nor publishing his findings.” We see that the first item—“application of his research”—is a noun whereas the second item—“publishing his findings”—is an -ING verb phrase (a “gerund” phrase). To make the “neither . . . nor” list parallel, we must change the bolded portion to a noun phrase to match the first item in the list.

The correct way to express the sentence is therefore “A true mathematical virtuoso, Tarski was concerned with neither the application of his research nor the publication of his findings.” The correct answer is “the publication of his findings.” None of the other answer choices leads to parallelism of the “neither . . . nor” list.

69. A. NO CHANGE

→ B. shared many interests

C. shared many similar interests in common

D. had in common many similar interests, which they shared.

When we consider the answer choices, we see that “in common,” “similar,” and “share” each conveys the same information by itself—that there was overlap between the interests.

The correct answer—“shared many interests”—contains only one of these terms, so it is not redundant. The other answer choices, however, contain two or more of these terms, making them redundant.

70. E. NO CHANGE

F. fond with

G. fond about

→ D. fond of

English features many fixed expressions, such as “figure out” or “look for,” in which certain verbs are paired with specific prepositions. In this case, the correct fixed expression is “fond of,” so it is the correct answer choice. The other answer choices are distortions of this expression.

71. A. NO CHANGE

→ B. more prolific

C. prolific

D. best

Adjectives have comparative and superlative forms: the comparative form (e.g. “bigger” or “more persuasive”) and the superlative form (e.g. “biggest” or “most persuasive”).

When comparing two items, the appropriate form is the comparative form.

When comparing three or more items, the appropriate form is the superlative form.

Since the sentence is comparing two items (as indicated by the phrase “of the two”), the comparative form is appropriate.

The correct way to express the sentence is therefore “Although Tarski and Einstein were contemporaries, Einstein was the more prolific writer of the two.” Each of the other answer choices contains the superlative rather than comparative form, and is therefore incorrect.



ENGLISH TEST: ANSWER KEY

72. E. NO CHANGE

F. algebra, geometry,
and, trigonometry

G. algebra, geometry
and, last but not
least, trigonometry

→ H. algebra, geometry,
and trigonometry

73. A. NO CHANGE

→ B. children, Jan and Ina,

C. children, Jan and Ina

D. children Jan and Ina

74. E. NO CHANGE

→ F. has received

G. are receiving

H. receive

In a list with three items, every single item should be separated by a comma, and the final element should include an “and” after the comma. In this case, “algebra, geometry, and trigonometry” is the appropriate format of the list and the correct answer.

Answer choices “geometry and” and “geometry and, last but not least, trigonometry” are incorrect because they do not contain a comma before the word “and.”

Although answer choice “geometry, and, trigonometry” contains a comma before the word “and,” it is nevertheless incorrect because it contains an extra comma after the word “and.”

The appropriate way to write a list with two elements is simply to insert an “and” between them without any punctuation. The list containing “Jan” and “Ina” should therefore read “Jan and Ina.”

The phrase “Jan and Ina” serves to define the phrase right before it—“Tarski’s two children”—so “Jan and Ina” should be surrounded by commas.

The correct form is therefore “Tarski’s two children, Jan, and Ina, grew up to be prominent mathematicians themselves . . .” The correct answer is “children, Jan and Ina,”

Answer choice “Jan, and Ina” is incorrect because it contains an unnecessary comma between the two items in the list. Answer choices “children, Jan and Ina” and “children Jan and Ina” are incorrect because the phrase “Jan and Ina” is missing one or more surrounding commas.

The sentence contains two singular subjects—1) Jan and 2) Ina—separated by the word “or.” The verb should also be in singular because each of the subjects is singular.

Answer choice “has received” is correct because it is in singular form.

The other answer choices—“have received,” “are receiving,” and “receive”—are plural forms and are therefore incorrect.



ENGLISH TEST: ANSWER KEY

75. The author is considering adding the following sentence to the passage:

“Whereas Einstein published dozens of books, Tarski published only one.”

If the writer were to add this sentence to the passage, which of these choices would be the most logical place to insert it?

- **A.** After the last sentence of paragraph 3
- B.** After the first sentence of paragraph 1
- C.** After the first sentence of paragraph 2
- D.** After the first sentence of paragraph 4

The correct answer is “The last sentence of Paragraph 3.”

Paragraph 3 contains the discussion of Tarski’s relationship with Albert Einstein. The very end of paragraph 3 draws the contrast of Einstein being a more prolific writer than Tarski, which means that Einstein produced more written works than Tarski. The sentence under consideration—“Whereas Einstein published dozens of books, Tarski published only one”—provides a specific example of this contrast, so it is most logical to insert the sentence as the final sentence of paragraph 3.





MATHEMATICS TEST: ANSWER KEY

1. Find the value of w if

$$w = \frac{1}{3} + y^2 \text{ and } y = \frac{1}{9}.$$

→ A. $\frac{16}{81}$

B. $\frac{9}{10}$

C. $\frac{1}{12}$

D. $\frac{4}{27}$

2. Let $F(x) = x^3 + 2x^2 - 3$ and $G(x) = x + 5$.

Find $F(G(x))$.

E. $x^3 + 17x^2 + 95x + 172$

F. $x^3 + 2x^2 - x - 8$

G. $x^3 + x^2 + x + 8$

→ H. $x^3 + 2x^2 + x + 2$

3. Circle A is given by the equation $(x-4)^2 + (y+3)^2 = 29$. Circle A is shifted up five units and left by six units. Then, its radius is doubled. What is the new equation for circle A?

→ A. $(x+2)^2 + (y-2)^2 = 116$

B. $(x-10)^2 + (y+8)^2 = 116$

C. $(x-10)^2 + (y+8)^2 = 58$

D. $(x+2)^2 + (y-2)^2 = 58$

In order to solve for w , first substitute into the equation for $y = \frac{1}{9}$:

$$w = \left(\frac{1}{3} + y\right)^2$$

$$w = \left(\frac{1}{3} + \frac{1}{9}\right)^2$$

Then, find the Least Common Multiple (LCM) of the two fractions and generate equivalent fractions with the same denominator:

$$w = \left(\frac{3}{9} + \frac{1}{9}\right)^2$$

Finally, simplify the equation:

$$w = \left(\frac{4}{9}\right)^2$$

$$w = \frac{16}{81}$$

$F(G(x))$ is a composite function where the expression $G(x)$ is substituted in for x in $F(x)$.

$$F(G(x)) = (x+5)^3 + 2(x+5)^2 - 3 = x^3 + 17x^2 + 95x + 172$$

$$G(F(x)) = x^3 + x^2 + 2$$

$$F(x) - G(x) = x^3 + 2x^2 - x - 8$$

$$F(x) + G(x) = x^3 + 2x^2 + x + 2$$

The general equation of a circle is $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) represents the location of the circle's center, and r represents the length of its radius.

Circle A first has the equation of $(x-4)^2 + (y+3)^2 = 29$. This means that its center must be located at $(4, -3)$, and its radius is $\sqrt{29}$.

We are then told that circle A is shifted up five units and then left by six units. This means that the y -coordinate of the center would increase by five, and the x -coordinate of the center would decrease by 6. Thus, the new center would be located at $(4-6, -3+5)$, or $(-2, 2)$.

We are then told that the radius of circle A is doubled, which means its new radius is $2\sqrt{29}$.

Now, that we have circle A's new center and radius, we can write its general equation using $(x-h)^2 + (y-k)^2 = r^2$.

$$(x-(-2))^2 + (y-2)^2 = (2\sqrt{29})^2 = 2^2(\sqrt{29})^2 = 4(29) = 116$$

$$(x+2)^2 + (y-2)^2 = 116$$

The answer is $(x+2)^2 + (y-2)^2 = 116$.



MATHEMATICS TEST: ANSWER KEY

4. The midpoint of a line segment is $(4, -1)$. If one endpoint of the line segment is $(-4, 3)$, what is the other endpoint?

→ **E.** $(12, -5)$
F. $(-12, 7)$
G. $(-12, -5)$
H. $(12, 7)$

The midpoint formula can be used to solve this problem, where the midpoint is the average of the two coordinates.

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

We are given the midpoint and one endpoint. Plug these values into the formula.

$$(4, -1) = \left(\frac{-4 + x}{2}, \frac{3 + y}{2} \right)$$

$$4 = \frac{-4 + x}{2} \text{ and } -1 = \frac{3 + y}{2}$$

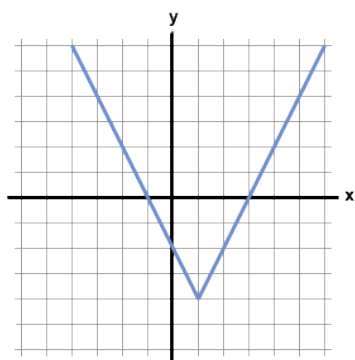
Solve for the variables to find the coordinates of the second endpoint.

$$8 = -4 + x \text{ and } -2 = 3 + y$$

$$12 = x \text{ and } -5 = y$$

The final coordinates of the other endpoint are $(12, -5)$.

5. Below is the graph of the function $f(x)$:



Which of the following could be the equation for $f(x)$?

A. $f(x) = x^2 - 4x + 3$
→ **B.** $f(x) = |2x - 2| - 4$
C. $f(x) = |2x - 6|$
D. $f(x) = |x - 4x| - 3$

First, because the graph consists of pieces that are straight lines, the function must include an absolute value, whose functions usually have a distinctive “V” shape. Thus, we can eliminate $f(x) = x^2 - 4x + 3$ from our choices. Furthermore, functions with x^2 terms are curved parabolas, and do not have straight line segments. This means that $f(x) = |x - 4x| - 3$ is not the correct choice.

Next, let's examine $f(x) = |2x - 6|$. Because this function consists of an absolute value by itself, its graph will not have any negative values. An absolute value by itself will only yield non-negative numbers. Therefore, because the graph dips below the x-axis (which means $f(x)$ has negative values), $f(x) = |2x - 6|$ cannot be the correct answer.

By process of elimination, the answer must be $f(x) = |2x - 2| - 4$. We can verify this by plugging in several values of x into this equation. For example, $f(1) = |2 - 2| - 4 = -4$, which corresponds to the point $(1, -4)$ on the graph above. Likewise, if we plug 3 or -1 into the equation $f(x) = |2x - 2| - 4$, we obtain zero, meaning that the graph should cross the x-axis at 3 and -1 . According to the graph above, this is exactly what happens.

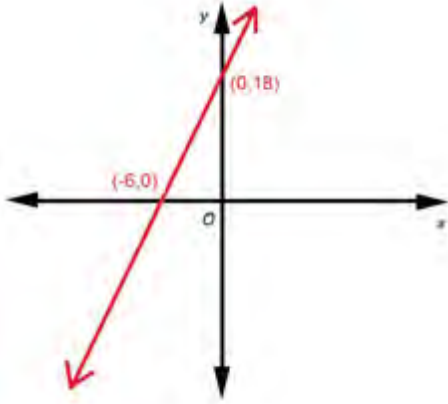
The answer is $f(x) = |2x - 2| - 4$.





MATHEMATICS TEST: ANSWER KEY

6. Refer to the graphed line below. If the red line passes through the point $(N, 4)$, what



- E. $N = -7\frac{1}{3}$
 F. $N = -1\frac{1}{3}$
→ G. $N = -4\frac{2}{3}$
 H. $N = -3\frac{1}{3}$

7. Which line below is perpendicular to $5x + 6y = 18$?

- A. $y = \frac{5}{6}x + \frac{6}{5}$
→ B. $y = \frac{6}{5}x + 3$
 C. $y = -\frac{6}{5}x + 8$
 D. $y = \frac{5}{6}x + 2$

One way to answer this is to first find the equation of the line.

The slope of a line, given two points (x_1, y_1, x_2, y_2) can be calculated using the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Set $x_1 = -6$, $y_1 = 0$, and $y_2 = 18$:

$$m = \frac{18 - 0}{0 - (-6)} = \frac{18}{6} = 3$$

The line has slope 3 and y-intercept $(0, 18)$, so we can substitute $m = 3$, $b = 18$ in the slope-intercept form:

$$y = mx + b$$

$$y = 3x + 18$$

Now substitute 4 for y and N for x and solve for N:

$$4 = 3N + 18$$

$$-14 = 3N$$

$$N = -\frac{14}{3} = -4\frac{2}{3}$$

The definition of a perpendicular line is one that has a negative, reciprocal slope to another.

For this particular problem, we must first manipulate our initial equation into a more easily recognizable and useful form: slope-intercept form or $y = mx + b$.

$$5x + 6y = 18$$

$$6y = -5x + 18$$

$$y = -\frac{5}{6}x + 3$$

According to our $y = mx + b$ formula, our slope for the original line is $-\frac{5}{6}$. We are looking for an answer that has a perpendicular slope, or an





MATHEMATICS TEST: ANSWER KEY

opposite reciprocal. The opposite reciprocal of $-\frac{5}{6}$ is $\frac{6}{5}$. Flip the original and multiply it by -1 .

Our answer will have a slope of $\frac{6}{5}$. Search the answer choices for $\frac{6}{5}$ in the m position of the $y = mx + b$ equation.

$$y = \frac{6}{5}x + 3 \text{ is our answer.}$$

(As an aside, the negative reciprocal of 4 is $-\frac{1}{4}$. Place the whole number over one and then flip/negate. This does not apply to the above problem, but should be understood to tackle certain permutations of this problem type where the original slope is an integer.)

8. Which line is parallel to the line with the equation $2x + 3y = 6$ and travels through the point $(3, 2)$?

E. $y = -\frac{3}{2}x + 4$

→ F. $y = -\frac{2}{3}x + 4$

G. $y = -\frac{3}{2}x + 8$

H. $y = -\frac{3}{2}x + 8$

9. Simplify: $(4r^2 + 2s^3)^2$

→ A. $16r^4 + 16s^3r^2 + 4s^6$

B. $16r^4 + 4s^6$

C. $4r^4 + 2s^5$

D. $16r^4 + 8s^3r^2 + 4s^6$

Find the slope of the given line: $y = mx + b$ (slope intercept form)

$$y = -\frac{2}{3}x + 2, \text{ therefore, the slope is } -\frac{2}{3}$$

Parallel lines have the same slope, so now we need to find the equation of a line with slope $-\frac{2}{3}$ and going through point $(3, 2)$ by substituting values into the point-slope formula.

$$2 = -\frac{2}{3}(3) + b$$

$$\text{So, } b = 4$$

$$\text{Thus, the new equation is } y = -\frac{2}{3}x + 4.$$

Expand the equation and use the FOIL method:
 $(4r^2 + 2s^3)(4r^2 + 2s^3)$

$$\text{First: } 4r^2 \cdot 4r^2 = 16r^4$$

$$\text{Outside: } 4r^2 \cdot 2s^3 = 8s^3r^2$$

$$\text{Inside: } 2s^3 \cdot 4r^2 = 8s^3r^2$$

$$\text{Last: } 2s^3 \cdot 2s^3 = 4s^6$$

Sum the terms:

$$16r^4 + 8s^3r^2 + 8s^3r^2 + 4s^6$$

$$16r^4 + 16s^3r^2 + 4s^6$$





MATHEMATICS TEST: ANSWER KEY

10. $x^2 - 1$ in factored form is equal to:

E. $(x + 1) + (x - 1)$

F. $(x - 1)^2$

G. $x(x - 1)$

→ H. $(x + 1)(x - 1)$

$x^2 - 1$ is a special type of binomial. Notice that both terms are perfect squares and they are separated by a subtraction symbol; this is known as the difference of squares.

The purpose of this question is to understand the rules of algebra and recognize different forms of expressions. The correct answer is $(x + 1)(x - 1)$. This is because when evaluated, it equals $x^2 - 1$.

When both terms of the factored form have the same coefficients with different signs, there is no number of x 's in the simplified version of the expression.

11. Given the inequality $|12 - 2x| > 2$ which of the following is correct?

A. $x < -5$ or $x > 7$

B. $5 < x < 7$

→ C. $x < 5$ or $x > 7$

D. $-5 < x < 7$

First separate the inequality $|12 - 2x| > 2$ into two equations.

$$12 - 2x > 2$$

$$12 - 2x < -2$$

Solve the first inequality.

$$12 - 2x > 2$$

$$-2x > -10$$

$$x < 5$$

Solve the second inequality.

$$12 - 2x < -2$$

$$-2x < -14$$

$$x > 7$$

Thus, $x < 5$ or $x > 7$.

12. Simplify the following inequality: $4 - 3x \geq 22 + 2x$

→ E. $x \leq -\frac{18}{5}$

F. $x \geq -\frac{18}{5}$

G. $x \leq \frac{2}{7}$

H. $x \geq \frac{14}{5}$

For the most part, you can treat inequalities just like equations. (It is not exact, as you will see below.) Thus, start by isolating your variables. Subtract $2x$ from both sides:

$$4 - 5x \geq 22$$

Next, subtract 4 from both sides:

$$-5x \geq 18$$

Finally—here you need to be careful—divide by -5 . When you divide or multiply by a negative value in inequalities, you need to flip the inequality sign.

Thus, you get:

$$x \leq -\frac{18}{5}$$





MATHEMATICS TEST: ANSWER KEY

13. Let x be a number. Increasing x by twenty percent yields the same result as decreasing the product of four and x by five. What is x ?

A. $\frac{25}{19}$

B. $\frac{50}{7}$

C. $\frac{100}{19}$

→ D. $\frac{25}{14}$

The problem tells us that increasing x by twenty percent gives us the same thing that we would get if we decreased the product of four and x by five. We need to find expressions for these two situations, and then we can set them equal and solve for x .

Let's find an expression for increasing x by twenty percent. We could represent this as $x + 20\%$ of $x = x + 0.2x = 1.2x = \frac{6x}{5}$.

Let's find an expression for decreasing the product of four and x by five. First, we must find the product of four and x , which can be written as $4x$. Then we must decrease this by five, so we must subtract five from $4x$, which could be written as $4x - 5$.

Now we must set the two expressions equal to one another.

$$\frac{6x}{5} = 4x - 5$$

Subtract $\frac{6x}{5}$ from both sides. We can rewrite $4x$ as $\frac{20x}{5}$ so that it has a common denominator with $\frac{6x}{5}$.

$$0 = \frac{20x}{5} - \frac{6x}{5} - 5 = \frac{14x}{5} - 5$$

$$0 = \frac{14x}{5} - 5$$

Now we can add five to both sides.

$$5 = \frac{14x}{5}$$

Now we can multiply both sides by $\frac{5}{14}$, which is the reciprocal of $\frac{14}{5}$.

$$5\left(\frac{5}{14}\right) = \left(\frac{14x}{5}\right)\left(\frac{5}{14}\right) = x$$

$$\frac{25}{14} = x$$

The answer is $\frac{25}{14}$.





MATHEMATICS TEST: ANSWER KEY

14. Given the equation $x^2 - 8x + 15 = 0$, what is the product of the solutions of the quadratic equation?

E. -15

→ F. 15

G. -8

H. 8

We are initially presented with a quadratic equation, $x^2 - 8x + 15 = 0$. To begin we must factor this equation.

The multiples of 15 are (15 and 1) and (3 and 5). The only multiples that add to or subtract from -8 are 3 and 5. Hence we use these as our binomial numbers: $(x \pm 5)$ and $(x \pm 3)$. We must now decide on the signs. Because we need to add or subtract 5 and 3 to get to , both signs must be negative: $(x - 5)(x - 3) = 0$.

From this point we need to switch gears to find solutions to the equation. What numbers would make this equation equal 0?

At this point split the equation into two parts.

$(x - 3) = 0$ and $(x - 5) = 0$ and solve.

$x = 3$ and $x = 5$. Both of these numbers inserted into the original equation will produce a result of 0.

Now the question itself is asking for the product of the solutions to the equation, or $3 \cdot 5$, which equals 15, therefore 15 is our answer.

15. Solve for y.

$$\begin{aligned} 3y + 6x &= 24 \\ 2y + x &= 16 \end{aligned}$$

A. -8

→ B. 8

C. 4

D. -4

Solve the system of equations using substitution.

First, isolate one of the variables. Since we are solving for y, we are going to isolate x in the second equation.

$$\begin{aligned} 2y + x &= 16 \\ x &= -2y + 16 \end{aligned}$$

Replace x with $-2y + 16$ in our first equation.

$$\begin{aligned} 3y + 6x &= 24 \\ 3y + 6(-2y + 16) &= 24 \end{aligned}$$

Now we can solve to isolate y.

$$\begin{aligned} 3y + (-12y + 96) &= 24 \\ -9y + 96 &= 24 \\ -9y &= -72 \\ y &= 8 \end{aligned}$$

16. Simplify: $\frac{18a^3b^3}{3a^{-1}b^{-2}}^2$

E. $6a^4b^5$

F. $9a^5b^4$

G. $36a^6b^8$

→ H. $36a^8b^{10}$

Step 1: Simplify the fraction. When dividing exponents subtract the exponents on the bottom from the exponents on the top.

$$\frac{18a^3b^3}{3a^{-1}b^{-2}}^2 = (6a^4b^5)^2$$

Step 2: Distribute the exponent. When raising an exponent to a power, multiply them together.

$$(6a^4b^5)^2 = 36a^8b^{10}$$





MATHEMATICS TEST: ANSWER KEY

17. If m and n are both rational numbers and $4^m = 8^n$, what is $\frac{m}{n}$?

- A. $\frac{2}{1}$
 → B. $\frac{3}{2}$
 C. $\frac{5}{3}$
 D. $\frac{4}{1}$

18. Find the value of x if $\frac{64}{27} = \left(\frac{4}{3}\right)^x$.

- E. 2
 → F. 3
 G. 4
 H. 5

19. Simplify:
 $(xy^2 + 2x^3y^2)(xy^3 + 3x^2)$

- A. $x^2y^5 + 3x^3y^2 + 2x^4y^5 + 6x^5y^2$
 B. $xy^6 + 3x^2y^2 + 2x^3y^6 + 6x^6y^2$
 C. $6x^2y^4 + 2x^3y^2 + 3x^2y^2$
 D. $3x^4y^5 + x^3y^2 + 2xy^5 + 3x^5y^2$

This question is asking you for the ratio of m to n . To figure it out, the easiest way is to figure out when 4 to an exponent equals 8 to an exponent. The easiest way to do that is to list the first few results of 4 to an exponent and 8 to an exponent and check to see if any match up, before resorting to more drastic means of finding a formula.

$$\begin{array}{ll}
 4^1 = 4 & 8^1 = 8 \\
 4^2 = 16 & 8^2 = 64 \\
 4^3 = 64 & 8^3 = 512 \\
 4^4 = 256 & 8^4 = 4096 \\
 4^5 = 1024 & 8^5 = 32768
 \end{array}$$

And, would you look at that: $4^3 = 8^2$. Therefore, $\frac{m}{n} = \frac{3}{2}$.

When a fraction is raised to an exponent, both the numerator and denominator are raised to that exponent. Therefore, the equation can be

rewritten as $\frac{64}{27} = \frac{4^x}{3^x}$. From here we can proceed one of two ways. We can either solve for x in $64 = 4^x$ or in $27 = 3^x$. Let's solve the first equation.

We simply multiply 4 by itself until we reach a value of 64. $4^1 = 4$, $4^2 = 4 \cdot 4 = 16$, $4^3 = 4 \cdot 4 \cdot 4 = 64$, and so on. Since $4^3 = 64$, we know that $x = 3$.

We can repeat this process for the second equation to get, confirming our previous answer. However, since the ACT is a timed test, it is best to only solve one of the equations and move on. Then, if you have time left once all of the questions have been answered, you can come back and double check your answer by solving the other equation.

First, merely FOIL out your values. Thus:

$$\begin{aligned}
 &(xy^2 + 2x^3y^2)(xy^3 + 3x^2) \text{ becomes} \\
 &(xy^2 \cdot xy^3 + xy^2 \cdot 3x^2 + 2x^3y^2 \cdot xy^3 + 2x^3y^2 \cdot 3x^2)
 \end{aligned}$$

Now, just remember that when you multiply similar bases, you *add* the exponents. Thus, simplify to:

$$x^2y^5 + 3x^3y^2 + 2x^4y^5 + 6x^5y^2$$





MATHEMATICS TEST: ANSWER KEY

20. What is the result when 5,678,732 is rounded to the nearest thousand and then put in scientific notation?

E. 5.678×10^3
 F. 5.678×10^6
 G. 5.679×10^3

→ H. 5.679×10^6

First, when we round to the nearest thousand we get 5,679,000 since we round up when the next digit is greater than 5.

Then, to put it in scientific notation, we arrange the digits so that a decimal point creates a number between 1 and 10. We get 5.679.

Then, we want the exponent of the 10 to be the number of times the decimal needs to move to the right. This is 6 times.

Thus, we get our answer. 5.679×10^6

21. If $x^7 y^8 z^{10} < 0$, then which of the following must also be true?

→ A. $x < 0$
 B. $y < 0$
 C. $zy > 0$
 D. $zx > 0$

We know that the expression must be negative. Therefore one or all of the terms x^7 , y^8 and z^{10} must be negative; however, even powers always produce positive numbers, so y^8 and z^{10} will both be positive. Odd powers can produce both negative and positive numbers, depending on whether the base term is negative or positive. In this case, x^7 must be negative, so x must be negative. Thus, the answer is $x < 0$.

22. Which of the following is equal to $-16i^6$?

E. -4
 F. -16

→ G. 16
 H. $16i$

Remember that since $i = \sqrt{-1}$, you know that i^2 is -1. Therefore, i^4 is $-1 \cdot -1$ or 1. This makes our question very easy.

i^6 is the same as $i^4 \cdot i^2$ or $1 \cdot -1$

Thus, we know that $-16i^6$ is the same as $-16 \cdot -1$ or 16.

23. Which of the following is equal to the expression

$$\sqrt{(16)(8) + (32)(20)}?$$

A. $2^7 \sqrt{5}$
 B. $2^3 \sqrt{6}$

→ C. $2^4 \sqrt{3}$
 D. $2^3 \sqrt{10}$

First, break down the components of the square root:

$$\sqrt{(2^4)(2^3) + (2^5)(2^2) \cdot 5}$$

Combine like terms. Remember, when multiplying exponents, add them together:

$$\sqrt{(2^7) + (2^7) \cdot 5}$$

Factor out the common factor of 2^7 :

$$\sqrt{(2^7)(1+5)}$$

$$\sqrt{(2^7) \cdot 6}$$

Factor the 6:

$$\sqrt{(2^7) \cdot 2 \cdot 3}$$

Combine the factored 2 with the 2^7 :

$$\sqrt{(2^8) \cdot 3}$$

Now, you can pull $\sqrt{2^8}$ out from underneath the square root sign as $2^4: 2^4 \sqrt{3}$





MATHEMATICS TEST: ANSWER KEY

24. Combine the following two expressions if possible.

$$\frac{x+3}{x+7} + \frac{x^2}{4-x}$$

E. $\frac{-x^3 + 4x^2 + 8x - 4}{x^2 - 3x - 28}$

F. $\frac{(x-3)^3}{-x^2 - 3x + 28}$

G. $\frac{x^3 + 3x^2 - 7x - 22}{x^2 - 3x - 28}$

→ H. $\frac{x^3 + 6x^2 + x + 12}{-x^2 - 3x + 28}$

25. The price of silver varies directly as the square of the mass. If 3.6g of silver is worth \$64.80, what is the value of 7.5g of silver?

A. \$135.00

B. \$301.75

C. \$178.50

→ D. \$281.25

26. Choose the answer that is the simplest form of the following expression of monomial quotients:

$$\frac{2x^3y^4}{10z^3} \times \frac{4z^2p}{10xy}$$

→ E. $\frac{2x^2y^3p}{25z}$

F. $\frac{8x^2y^3p}{25z^2}$

G. $\frac{2xyp}{15z}$

H. $\frac{2p}{25z}$

find a common trinomial than it is to look for individual least common denominators. Let's do that here:

$$\frac{x+3}{x+7} + \frac{x^2}{4-x} = \frac{x+3}{x+7} \cdot \frac{(4-x)}{(4-x)} + \frac{x^2}{4-x} \cdot \frac{(x+7)}{(x+7)}$$

FOIL and simplify.

$$\frac{-x^2 + x + 12}{-x^2 - 3x + 28} + \frac{x^3 + 7x^2}{-x^2 - 3x + 28}$$

$$\frac{-x^2 + x + 12}{-x^2 - 3x + 28} + \frac{x^3 + 7x^2}{-x^2 - 3x + 28} = \frac{x^3 + 6x^2 + x + 12}{-x^2 - 3x + 28}$$

Thus, our answer is $\frac{x^3 + 6x^2 + x + 12}{-x^2 - 3x + 28}$.

This is a direct variation problem of the form $y = kx^2$. The first set of data, 3.6g and \$64.80, is used to calculate the proportionality constant, k . So $64.80 = k(3.6)^2$, and solving the equation gives $k = 5$.

Now we move to the new data, 7.5 g, and we get $y = 5(7.5)^2$ to yield an answer of \$281.25.

\$135.00 is the answer obtained if using proportions. This is an error because it does not take into consideration the squared elements of the

problem. $\frac{2x^3y^4}{10z^3} \cdot \frac{4z^2p}{10xy}$

To simplify, first multiply across:

$$\frac{8x^3y^4z^2p}{100z^3xy}$$

Then, reduce:

$$\frac{2x^2y^3p}{25z}$$





MATHEMATICS TEST: ANSWER KEY

27. A function of the form $f(x) = ax^2 + b$ passes through the points $(0, 7)$ and $(-2, 19)$. What is the value of a ?

A. -3
 B. 3
 C. 2
 D. -2



The easiest way to solve for a is to begin by plugging each pair of coordinates into the function.

Using our first point, we will plug in 0 for x and 7 for $f(x)$. This gives us the equation

$$7 = a(0)^2 + b.$$

Squaring 0 gives us 0, and multiplying this by a still gives 0, leaving only b on the right side, such that

$$7 = b.$$

We now know the value of b , and we can use this to help us find a . Substituting our second set of coordinates into the function, we get

$$19 = a(-2)^2 + b$$

which simplifies to

$$19 = 4a + b.$$

However, since we know $b = 7$, we can substitute to get

$$19 = 4a + 7$$

subtracting 7 from both sides gives

$$12 = 4a$$

and dividing by 4 gives our answer

$$3 = a.$$

28. Add the following polynomials:

$$(5x^3 + 31x^2 - 17x - 6) \\ + (-2x^3 + 9x^2 + 34x - 12)$$



E. $3x^3 + 40x^2 + 17x - 18$
 F. $3x^3 + 22x^2 + 10x - 18$
 G. $7x^3 + 40x^2 + 17x + 6$
 H. $-3x^3 + 40x^2 - 17x + 6$

This is a problem where elimination can be help you save a little time. You can eliminate options quickly by simplifying one power at a time and comparing your work with the answer choices.

To begin, reorder the problem so that all like terms are next to each other. When doing so, keep an eye on your signs so that you don't accidentally make a mistake.

$$(5x^3 - 2x^3) + (31x^2 + 9x^2) + (-17x + 34x) + (-6 - 12)$$

From here, combine each pair of terms. As you do so, compare your work with the answer choices.

$(5x^3 - 2x^3) = 3x^3$ Eliminate any answer choices that have a different x^3 term.

$(31x^2 + 9x^2) = 40x^2$ Eliminate any answer choices that have a different x^2 term.

$(-17x + 34x) = 17x$ Eliminate any answer choices that have a different x term.

$(-6 - 12) = -6 - 12 = -18$ Eliminate any answer choices that have a different constant term.





MATHEMATICS TEST: ANSWER KEY

29. Solve for t :
 $t\sqrt{28} - t\sqrt{63} = \sqrt{14}$

- A. 2
 B. $-\sqrt{2}$
 C. $-\sqrt{14}$
 D. $\sqrt{14}$

30. What is the ratio of $\sqrt{105}$ to $\sqrt{\frac{12}{5}}$?

- E. $4\sqrt{7}:15$
 F. $5\sqrt{7}:2$
 G. $5\sqrt{21}:12$
 H. $\sqrt{105}:12$

Once you put all of your solutions together, the correct answer looks like this:

$$3x^3 + 40x^2 + 17x - 18$$

Begin by breaking apart the square roots on the left side of the equation:

$$t\sqrt{4 \cdot 7} - t\sqrt{9 \cdot 7} = \sqrt{14}$$

$$2t\sqrt{7} - 3t\sqrt{7} = \sqrt{14}$$

You can combine like terms on the left side:

$$-t\sqrt{7} = \sqrt{14}$$

Solve by dividing both sides by $-\sqrt{7}$:

$$t = \frac{-\sqrt{14}}{\sqrt{7}}$$

This simplifies to:

$$t = -\sqrt{2}$$

The ratio of two numbers is merely the division of the two values. Therefore, for the information given, we know that the ratio of

$$\sqrt{105} \text{ to } \sqrt{\frac{12}{5}}$$

can be rewritten:

$$\frac{\sqrt{105}}{\sqrt{\frac{12}{5}}}$$

Now, we know that the square root in the denominator can be “distributed” to the numerator and denominator of that fraction:

$$\sqrt{\frac{12}{5}} = \frac{\sqrt{12}}{\sqrt{5}}$$

Thus, we have:

$$\frac{\sqrt{105}}{\frac{\sqrt{12}}{\sqrt{5}}}$$

To divide fractions, you multiply by the reciprocal:

$$\frac{\sqrt{105}}{\frac{\sqrt{12}}{\sqrt{5}}} = \frac{\sqrt{105}}{1} \cdot \frac{\sqrt{5}}{\sqrt{12}}$$

Now, since there is one 5 in 105, you can rewrite the numerator:





MATHEMATICS TEST: ANSWER KEY

$$5\sqrt{21}$$

This gives you:

$$\frac{5\sqrt{21}}{\sqrt{12}}$$

Rationalize the denominator by multiplying both numerator and denominator by $\sqrt{12}$:

$$\frac{5\sqrt{21}}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}}$$

Let's be careful how we write the numerator so as to make explicit the shared factors:

$$\frac{5\sqrt{3 \cdot 7}}{\sqrt{12}} \cdot \frac{\sqrt{3 \cdot 4}}{\sqrt{12}} = \frac{5 \cdot 3 \cdot 2\sqrt{7}}{12}$$

Now, reduce:

$$\frac{5 \cdot 3 \cdot 2\sqrt{7}}{12} = \frac{5\sqrt{7}}{2}$$

$$5\sqrt{7} : 2$$

31. If $x = 3.09$ and $y = 2.97$, what is $(x - y)^2$ equal to?

- A. 0.0144
B. 0.144
C. 1.44
D. 0.04

First, we must figure out what $(x - y)$ is equal to.

We do $3.09 - 2.97 = 0.12$.

Now to find out what 0.12^2 is equal to, we look at $0.12 \cdot 0.12$.

We move our decimal points over so that we are dealing with only whole numbers. This gives us $12 \cdot 12 = 144$.

Finally, we count how many spaces we moved our decimals in total, and we move the decimal in our answer back that many spaces. To get from 0.12 to 12, we moved our decimal two spaces to the right. Because we did this for each of the 0.12 values, our total spaces we moved the decimal was 4 to the right.

Therefore, we must take 144 and move the decimal 4 places to the left. This gives us 0.0144.





MATHEMATICS TEST: ANSWER KEY

32. Simplify:

$$0.35 + \frac{0.75}{0.2} - 0.4 \times \frac{0.5}{0.1}$$

E. $\frac{213}{10}$

F. $\frac{30}{7}$

G. $\frac{25}{4}$

→ H. $\frac{21}{10}$

Begin by multiplying all of your decimal fractions by $\frac{100}{100}$:

$$0.35 + \frac{0.75}{0.2} \cdot \frac{100}{100} - 0.4 \cdot \frac{0.5}{0.1} \cdot \frac{100}{100}$$

Simplify:

$$0.35 + \frac{75}{20} - 0.4 \cdot \frac{50}{10} = 0.35 + \frac{15}{4} - 0.4 \cdot 5$$

Now perform the multiplication:

$$0.35 + \frac{15}{4} - 2$$

The easiest thing to do next is to subtract 2 from 0.35:

$$0.35 + \frac{15}{4} - 2 = \frac{15}{4} - 1.65$$

Next, convert 1.65 into the fraction $\frac{165}{100}$:

$$\frac{15}{4} - \frac{165}{100}$$

Now, the common denominator can be 100:

$$\frac{15}{4} \cdot \frac{25}{25} - \frac{165}{100} = \frac{375}{100} - \frac{165}{100}$$

Simplify:

$$\frac{375}{100} - \frac{165}{100} = \frac{210}{100} = \frac{21}{10}$$

33. A is $16\frac{2}{3}\%$ of B and $62\frac{1}{2}\%$ of C; all are positive integers. Give the smallest possible value of A.

A. 1

→ B. 5

C. 8

D. 30

A is $16\frac{2}{3}\%$ of B and $62\frac{1}{2}\%$ of C, so

$$A = \frac{16\frac{2}{3}}{100} B = \frac{16\frac{2}{3} \cdot 3}{100 \cdot 3} B = \frac{50}{300} B = \frac{1}{6} B$$

so $B = 6A$.

$$A = \frac{62\frac{1}{2}}{100} C = \frac{62\frac{1}{2} \cdot 2}{100 \cdot 2} C = \frac{125}{200} C = \frac{5}{8} C$$

so $C = \frac{8}{5} A$.

A must be divisible by 5, so the least possible value is 5 itself.





MATHEMATICS TEST: ANSWER KEY

34. Which of the following is equivalent to $5\frac{7}{11}$?

→ E. $\frac{62}{11}$

F. $\frac{12}{11}$

G. $\frac{53}{11}$

H. $\frac{71}{11}$

35. A large reservoir holds 200,000 gallons of water. It has an emptying pipe attached to it that allows 20 gallons to flow out of the reservoir every hour. If an additional emptying pipe of the same dimensions as the first one is attached to the reservoir, how many gallons will be left in the reservoir after three days of drainage, presuming that there is no overall change in water due to addition or evaporation?

- A. 81,494 gallons
B. 157,800 gallons
→ C. 197,120 gallons
D. 199,880 gallons

36. What is the least common multiple of 5 and 13?

- E. 65
F. 130
G. 175
H. 52

Remember that to convert mixed fractions, you can treat it like an addition. Thus

$$5\frac{7}{11} = 5 + \frac{7}{11}$$

You then find the common denominator of the two, which is 11:

$$5 + \frac{7}{11} = \frac{55 + 7}{11} = \frac{62}{11}$$

The rate of draining is 40 gallons per hour once the new pipe is added. Recall that:

$W = RT$, where W is the total work output. For our data, this means the total amount of water. Now, we are measuring our rate in hours, so we should translate the three days' time into hours. This is easily done:

$$3 \cdot 24 = 72$$

Now, based on this, we can set up the equation:

$$W = 40 \cdot 72 = 2880$$

Now, this means that there will be 200000 – 2880 or 197120 gallons in the reservoir after three days.

To find the least common multiple of two numbers, if they are both prime, simply multiply the numbers.

Thus, we get that:

$$5 \cdot 13 = 65$$

We could also write out the multiples of each number and see what numbers they have in common. The least common multiple will be the common number that appears first in both sets.

$$5 : 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, \mathbf{65}$$

$$13 : 13, 26, 39, 52, \mathbf{65}$$

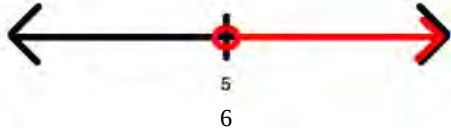




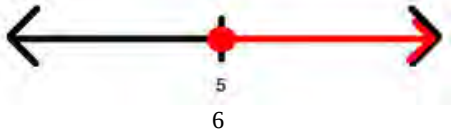
MATHEMATICS TEST: ANSWER KEY

37. Which of the following is a graph for the values of x defined by the inequality $2x + 6 > 18$?

→ A.



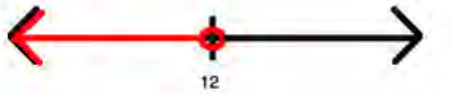
B.



C.



D.



38. The game of euchre uses the 9's, 10's, jacks, queens, kings, and aces from a standard deck of 52 cards. How many 5-card euchre hands have at least 2 black cards?

E. 8,731

F. 5,940

G. 14,000

→ H. 35,772

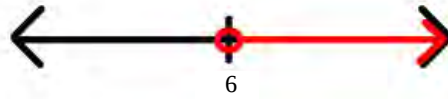
Begin by solving for x :

$$2x > 18 - 6$$

$$2x > 12$$

$$x > 6$$

Now, this is represented by drawing an open circle at 6 and graphing upward:



The hand could have 2, 3, 4, or 5 black cards. There are 12 black cards and 12 red cards, so the numbers of combinations for the four cases are as follows.

$$2 \text{ black cards: } C(12, 2) \times C(12, 3) = 14520$$

$$3 \text{ black cards: } C(12, 3) \times C(12, 2) = 14520$$

$$4 \text{ black cards: } C(12, 4) \times C(12, 1) = 5940$$

$$5 \text{ black cards: } C(12, 5) \times C(12, 0) = 792$$

The total number of euchre hands that have at least two black cards is the total of these four cases, 35772.





MATHEMATICS TEST: ANSWER KEY

39. Find the 50th term in the following sequence.

-8, -1, 6, 13, ...

- A. 306
 → B. 335
 C. 342
 D. 350

A sequence is simply a list of numbers that follow some sort of consistent rule in getting from one number in the list to the next one. Sequences generally fall into three categories: arithmetic, geometric, or neither.

In arithmetic sequences, I add the same number each time to get from one number to the next. In other words, the difference between any two consecutive numbers in my list is the same.

In geometric sequences, I multiply by the same number each time to get from one number to the next. In other words, the ratio between any two consecutive numbers in my list is the same.

Finally, sequences that are neither, still follow some rule, but it just happens not to be one of these two.

Looking at our sequence, we might quickly notice that each number is simply 7 more than the number before. In other words, I can find the next number by adding 7 each time. Hence, our sequence is arithmetic.

Unfortunately, we need to find the 50th term in this sequence, and the problem only got us through the first four. A simple (yet way too time-consuming approach) would be to keep adding 7 until we get to term number 50. Not only is that the long way, we also risk losing count and ending up on the wrong term. So what's the easier way?

The easier way hinges on the fact that I am simply adding 7 over and over again. If I want to find the 2nd term, I start with the 1st term and add 7 once.

$$-8 + 7 = -1$$

To find the 3rd term, I add 7 twice.

$$-8 + 7 + 7 = 6$$

You might already see the pattern. For the 4th term I would add 7 three times, for the 5th four times, 6th five times, etc.

Notice that to find any term, I simply add 7 one less time than the number of the term. Therefore, to find the 50th term, I would add 7 forty-nine times.

But adding 7 forty-nine times is the same as adding forty-nine 7s. But forty-nine 7s are the same as 49 times 7.

$$49 \cdot 7 = 343$$

Therefore, to find the 50th term, I simply need to add 343 to our starting value.

$$-8 + 343 = 335$$





MATHEMATICS TEST: ANSWER KEY

40. Solve for x ;
 $x = (3 - 2)^2 + 3 - 3 \cdot 2^2$

E. $x = -4$

F. $x = 4$

→ G. $x = -8$

H. $x = 8$

We begin with

$$x = (3 - 2)^2 + 3 - 3 \cdot 2^2$$

We follow the order of P.E.M.D.A.S.

Parenthesis:

$$x = 1^2 + 3 - 3 \cdot 2^2$$

Exponents:

$$x = 1 + 3 - 3 \cdot 4$$

Multiplication:

$$x = 1 + 3 - 12$$

No division needed, so addition:

$$x = 4 - 12$$

Finally, subtraction:

$$x = -8$$

41. One half of a group of professors is made up of mathematicians, and one fourth of the remaining portion is made up of philosophers. The remaining portion is made up of sociologists. If there are 464 professors in the room, how many are sociologists?

A. 58

B. 98

C. 110

→ D. 174

To begin, remember with percentages that “of” means multiplication and “is” means “equals.” Now, we know that one fourth of the remaining half are philosophers. This means that the philosophers really are:

$$\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

So, there are $1 - \frac{1}{2} - \frac{1}{8}$ sociologists as a percentage. This is:

$$1 - \frac{1}{2} - \frac{1}{8} = \frac{8}{8} - \frac{4}{8} - \frac{1}{8} = \frac{3}{8}$$

Therefore, $\frac{3}{8}$ of the 464 are sociologists, or:

$$\frac{3}{8} \cdot 464 = 174 \text{ are sociologists.}$$

42. A university has 570 students currently enrolled in its freshman class. Last year, there were only 500 freshman enrolled at the school. By what percentage did the number of students in the freshman class increase?

E. 11%

F. 12%

G. 13%

→ H. 14%

To find the percentage increase, use the following formula:

Increased # of students this year / # of students last year

If there were 500 students last year, and this year there are 570, the number of students increased by 70. Then, knowing this, you can find what percentage of the original number (500) 70 represents.

Therefore:

$$\frac{570 - 500}{500} = \frac{70}{500} = \frac{14}{100} = 0.14$$

The number of students increased by 14%.





MATHEMATICS TEST: ANSWER KEY

43. The mean of five numbers is 40. The mean of the smallest two numbers is 25. What is the mean of the other three numbers?

A. 40
B. 45
→ C. 50
D. 55

44. The list below shows a ninth grader's grades for the academic year. What is the difference between the median and the mode of these grades?

67, 73, 85, 83, 80, 73, 94, 65, 80, 73, 98,
59, 76

E. 1
F. 2
→ G. 3
H. 4

45. Students at a local high school are given the option to take one gym class, one music class or one of each. Out of 100 students, 60 say that they are currently taking a gym class and 70 say that they are taking a music class. How many students are taking both?

→ A. 30
B. 40
C. 50
D. 60

The equation for the mean of a group of numbers is to find the sum of all of the numbers and then divide by how many numbers are in the group. This means that if we know the mean and how many numbers are in the group, we can find the sum of those numbers.

$$\frac{\text{Sum of all five numbers}}{5} = 40 \rightarrow \text{Sum of all five numbers} = 200$$

$$\frac{\text{Sum of two smallest numbers}}{2} = 25 \rightarrow \text{Sum of two smallest numbers} = 50$$

Subtracting the sum of the two smallest numbers from the sum of all five gives us the sum of the remaining three. We can then divide by three to find the mean of those three remaining numbers.

$$200 - 50 = 150$$

$$\frac{150}{3} = 50$$

To find the median, sort the numbers from smallest to largest:

59, 65, 67, 73, 73, 73, 76, 80, 80, 83, 85, 94, 98

The median is the middle value in a list of numbers, it is the number separating the higher half of a data sample or a list of numbers from the lower half.

The median of the grades is 76.

The mode is the value occurring most often. The most occurring value in the list of numbers given is 73. So, the mode is 73.

$$\text{Median} - \text{Mode} = 76 - 73 = 3$$

This problem can be solved two ways, with a formula or with reasoning.

Using the formula, the intersection of the Venn diagram for which classes students take is:

$$\text{Total Students} = (\text{Gym} + \text{Music} - \text{Both})$$

$$100 = (60 + 70 - x)$$

$$x = 30$$

By using reasoning, it is clear that 60 + 70 is greater than 100 by 30. It is assumed that this extra 30 students come from students who were counted twice because they took both classes.





MATHEMATICS TEST: ANSWER KEY

46. On a standard analog clock, what is the angle between the hands when the clock reads 1:20?

E. 55°
→ F. 80°
 G. 85°
 H. 90°

47. A square has an area of 32in^2 . If a circle is inscribed within the square, what is its area?

A. $2\sqrt{2}\text{in}^2$
 B. $4\sqrt{2}\text{in}^2$
→ C. $8\pi\text{in}^2$
 D. $16\pi\text{in}^2$

48. The base of a triangle is 8cm and the area is 48cm^2 . The height of the triangle is then decreased by 75%. What is the final area of the triangle?

E. 9cm^2
→ F. 12cm^2
 G. 14cm^2
 H. 16cm^2

To find the degrees of a clock hand, first find the angle between each hour-long sections. Since there are 12 evenly spaced sections, we find that each section has an angle of: $\frac{360^\circ}{12} = 30^\circ$.

At 1:20, the hour hand has gone one-third of the way between the 1 and the 2. Thus, there are two-thirds of 30° between the hour hand and the 12. $\frac{2}{3} \cdot 30^\circ = 20^\circ$ There are 60° between 12 and 2, where the minute hands is. Thus there's a total of $20^\circ + 60^\circ = 80^\circ$ between the hands.

The diameter of the circle is the length of a side of the square. Therefore, first solve for the length of the square's sides. The area of the square is $A = s^2$ or $32 = s^2$. Taking the square root of both sides, we get $s = \sqrt{32} = \sqrt{25} = 4\sqrt{2}$. Now, based on this, we know that $2r = 4\sqrt{2}$ or $r = 2\sqrt{2}$. The area of the circle is πr^2 or $\pi(2\sqrt{2})^2 = 4 \cdot 2\pi = 8\pi$.

The formula for the area of a triangle is

$$\frac{1}{2}b \cdot h$$

If the area is equal to 48cm^2 and the base is 8cm, then the initial height is:

$$48 = \frac{1}{2} \cdot 8 \cdot h$$

$$h = 12$$

If 12 is decreased by 75% then $12 \cdot 0.75 = 9$ and $12 - 9 = 3$. The final height is 3cm.

Therefore the final area is

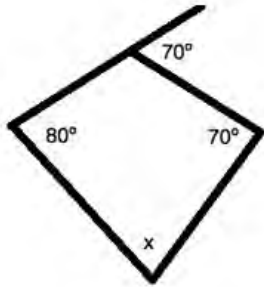
$$\frac{1}{2} \cdot 8 \cdot 3 = 12\text{cm}^2$$





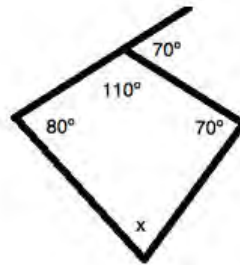
MATHEMATICS TEST: ANSWER KEY

49. What is the value of angle x in the figure below?



- A. 50°
 B. 75°
 → C. 100°
 D. 115°

Begin by noticing that the upper-right angle of this figure is supplementary to 70° . This means that it is $180^\circ - 70^\circ = 110^\circ$:



Now, a quadrilateral has a total of 360° . This is computed by the formula $\text{degrees} = 180 \cdot (s - 2)$, where s represents the number of sides. Thus, we know:

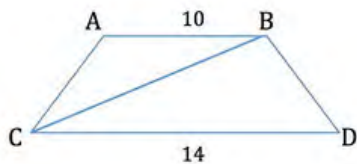
$$110^\circ + 70^\circ + 80^\circ + x = 360^\circ$$

$$\text{This is the same as } 260^\circ + x = 360^\circ.$$

Solving for x , we get:

$$x = 100^\circ$$

50. If the height of the isosceles trapezoid below is 5 units, what is the length of the diagonal BC ?



- E. 11
 F. 12
 → G. 13
 H. 14

To find the diagonal, we must subtract the top base from the bottom base:

$$14 - 10 = 4$$

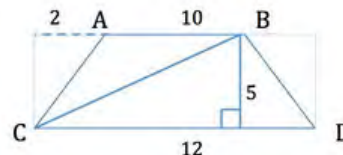
This leaves us with 4, which is the sum of the distance to the left and right of the top base. Taking half of that ($4 \div 2 = 2$) gives us the length of the distance to only the left side.

This means that the base of a triangle that includes that diagonal is equal to

$$2 + 10 = 12$$

Since the height is 5, we can solve this problem either using the Pythagorean Theorem or by remembering that this is a special right triangle (5-12-13 triangle). Therefore, the hypotenuse is 13.

See the figure below for clarification:





MATHEMATICS TEST: ANSWER KEY

51. An equilateral triangle has a height of $3\sqrt{3}$. What is the perimeter of this triangle?

A. $9\sqrt{3}$
 B. $18\sqrt{3}$
 C. 9

→ D. 18

Since this is an equilateral triangle, all of the sides and all of the angles are equal. Each angle is 60° since the three angles must add up to 180° .

To find the perimeter of the triangle, we need to find out the length of one side of the triangle. The height splits the triangle's base in half and creates two right triangles. Create expressions for the length of the two unknown sides in this new triangle. The value we are looking for is the side length so we'll make that x . The base of the new triangle is half the length of a side so we'll make that $\frac{x}{2}$.

Use the Pythagorean Theorem to find the value of or the side length:

$$a^2 + b^2 = c^2$$

In this case:

$$\left(\frac{x}{2}\right)^2 + (3\sqrt{3})^2 = x^2$$

$$\left(\frac{x}{2}\right)^2 + (3)^2(\sqrt{3})^2 = x^2$$

$$\frac{x^2}{4} + 27 = x^2$$

$$(4)\frac{x^2}{4} + (4)(27) = 4x^2$$

$$x^2 + 108 = 4x^2$$

$$3x^2 = 108$$

$$x^2 = 36$$

$$x = 6$$

Multiply the side length you found by 3 to get the perimeter:

$$6 \cdot 3 = 18$$

52. A cylindrical grain silo is erected vertically, to rest on one of its faces. The silo is then covered with reflective paint. If the cylinder is 12m tall and has a circumference of 6π m, how many square meters of paint must be used? (We'll assume the first coat is enough.)

→ E. $81\pi\text{m}^2$
 F. $100\pi\text{m}^2$
 G. $128\pi\text{m}^2$
 H. $140\pi\text{m}^2$

The formula for the surface area of a cylinder is $SA = Ch + 2\pi r^2$, where C is the circumference of the cylinder and h is the height.

In this case, we must also remember not to include one of the two πr^2 measurements, since the bottom face will not be painted. So our modified formula will be:

$$SA = Ch + \pi r^2$$

Since the circumference is 6π , we know the radius is 3, as $C = 2\pi r$.

Plug in all these values to the equation to solve for the paintable surface area.

$$SA = Ch + \pi r^2$$

$$SA = 6\pi(12) + 9\pi = 81\pi\text{m}^2$$





MATHEMATICS TEST: ANSWER KEY

53. A 10ft ladder is leaning against a wall. If the bottom of the ladder touches the ground 5ft from the base of the wall, approximately (to the nearest whole number) how far is the top of the ladder from the base of the wall?

A. 8ft
 → B. 9ft
 C. 10ft
 D. 12ft

To answer this, we first need to understand that we are trying to find the leg of a right triangle. The ladder, when leaned against the wall, forms a right triangle where the ladder is the hypotenuse, the wall is one leg, and the ground between the ladder and the wall is the other leg.

To find how high up the ladder touches the wall, we use the Pythagorean Theorem, which is $c^2 = a^2 + b^2$.

c is the length of the hypotenuse (the ladder itself), b is the distance of the ladder base from the wall, and a is how high the ladder touches on the wall.

We then rearrange to solve for one of the legs by subtracting b^2 from both sides.

$$c^2 = a^2 + b^2 \rightarrow c^2 - b^2 = a^2 + b^2 - b^2 \rightarrow c^2 - b^2 = a^2$$

We can now plug in our values of 10 for the hypotenuse and one of our legs (in this case, b)

$$c^2 - b^2 = a^2 \rightarrow 10^2 - 5^2 = b^2 \rightarrow 100 - 25 = 75 = b^2$$

We can then take the square root of this equation to get an answer for b :

$$\sqrt{75} = \sqrt{b^2} \rightarrow 8.66 = b$$

The question asked us to approximate, so we must round to the nearest whole number. To do this, we round a number up one place if the last digit is a 5, 6, 7, 8, or 9, and we round it down if the last digit is a 1, 2, 3, or 4. Therefore:

$$8.66 \rightarrow 9$$

Therefore, our answer is 9ft.

54. A sphere with a volume of $\frac{32}{3}\pi\text{m}^3$ is inscribed in a cube. What is the surface area of the cube, in m^2 ?

E. 24m^2
 F. 48m^2
 → G. 96m^2
 H. 120m^2

We must first find the radius of the sphere in order to solve this problem. Since we already know the volume, we will use the volume formula to do this.

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$\frac{32}{3}\pi = \frac{4}{3}\pi r^3$$

$$\frac{32}{3} = \frac{4}{3}r^3$$

$$\frac{3}{4} \cdot \frac{32}{3} = r^3$$

$$8 = r^3$$

$$r = 2$$

With the radius of the sphere in hand, we can now apply it to the cube.





MATHEMATICS TEST: ANSWER KEY

55. Sturgis is in charge of designing a new exhibit in the shape of a rectangular prism for a local aquarium. The exhibit will hold alligator snapping turtles and needs to have a volume of 150m^3 . Sturgis knows that the exhibit will be 15m long and have a width of 5m.

If three-quarters of the exhibit's volume will be water, what will the height (or the depth) of the water be?

- A. 1.0m
 B. 1.5 m
 C. 1.75 m
 D. 2.0 m

The radius of the sphere is half the distance from the top to the bottom of the cube (or half the distance from one side to another). Therefore, the radius represents half of a side length of a square. So, in this case,

$$\text{side} = 2 \cdot 2 = 4$$

The formula for the surface area of a cube is:

$$SA_{\text{cube}} = 6s^2$$

$$6s^2 = 6 \cdot (4)^2 = 6 \cdot 16 = 96$$

The surface area of the cube is 96m^2 .

The trickiest part of this question is the wording. This problem is asking for the height of the water in the exhibit if the exhibit is three-quarters full. We can find this at least two different ways.

1.) The longer way requires that we begin by finding three quarters of the total volume:

$$150\text{m}^3 \cdot \frac{3}{4} = 112.5\text{m}^3$$

Now we go back to our volume equation, and since we are again looking for height, we want it solved for h :

$$V = l \cdot w \cdot h$$

becomes

$$\frac{V}{l \cdot w} = h$$

$$h = \frac{112.5\text{m}^3}{15\text{m} \cdot 5\text{m}} = 1.5\text{m}$$

2) The easier way requires that we recognize a key detail. If we take three-quarters of the volume without changing our length or width, our new height will just be three-quarters of the total height. We can solve for the total height of the exhibit by using the volume equation and rearranging it to solve for h :

$$\frac{V}{l \cdot w} = h$$

At this point, we can substitute in our given values and solve for h :

$$\frac{150\text{m}^3}{15\text{m} \cdot 5\text{m}} = 2\text{m}$$

So, the total height of the exhibit is 2m. We can now easily solve for three-quarters of the total height:





MATHEMATICS TEST: ANSWER KEY

$$h' = h \cdot \frac{3}{4}$$

$$h' = 2 \text{ m} \cdot \frac{3}{4} = 1.5 \text{ m}$$

56. A man has a rope that is 40ft long, attached to the top of a small building. He pegs the rope into the ground at an angle of 14.5° . How far away from the building did he walk horizontally to attach the rope to the ground? Round to the nearest inch.

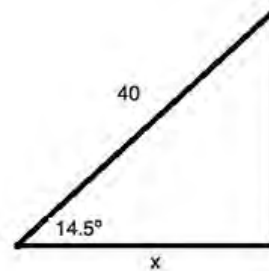
E. 37 feet and 5 inches

→ F. 38 feet and 9 inches

G. 39 feet and 4 inches

H. 40 feet and 2 inches

Begin by drawing out this scenario using a little right triangle:



We know that the cosine of an angle is equal to the ratio of the side *adjacent* to that angle to the *hypotenuse* of the triangle. Thus, for our triangle, we know:

$$\cos(14.5^\circ) = \frac{x}{40}$$

Using your calculator, solve for x :

$$x = 40 \cos(14.5^\circ)$$

This is 38.72590561512431. Now, take the decimal portion in order to find the number of inches involved.

$$0.72590561512431 \cdot 12 = 8.71086738149172$$

Thus, rounded, your answer is 38 feet and 9 inches.

57. What is the value of $\log_2(64)$? Round to the nearest hundredth.

A. 4

→ B. 6

C. 8

D. 12

You could solve this by using your calculator. Remember that you will have to translate this into:

$$\frac{\log(64)}{\log(2)}$$

Another way you can solve it is by noticing that $64 = 2^6$.

This means you can rewrite your logarithm:

$$\log_2(2^6)$$

Applying logarithm rules, you can factor out the power:

$$\log_2(2^6) = 6 \log_2(2)$$

For any value n , $\log_n(n) = 1$. Therefore, $\log_2(2) = 1$. So, your answer is 6.





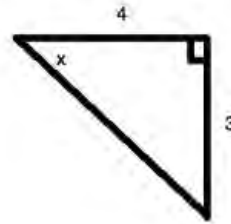
MATHEMATICS TEST: ANSWER KEY

58. What is the tangent of the obtuse angle formed between the x-axis and a straight line drawn from the origin to the point (4, -3)? Round to the nearest hundredth.

→ E. -0.75
F. 0.75
G. -1.33
H. 1.33

Recall that when you calculate a trigonometric function for an obtuse angle like this, you always use the x-axis as your reference point for your angle. (Hence, it is called the “reference angle.”)

Now, it is easiest to think of this like you are drawing a little triangle in the fourth quadrant of the Cartesian plane. It would look like:



So, the tangent of an angle is:

$\frac{\text{opposite}}{\text{adjacent}}$

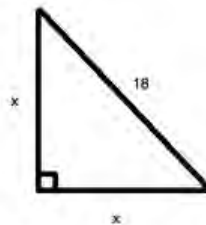
or, for your data, $\frac{3}{4}$ or 0.75. However, since (4, -3) is in the

fourth quadrant, your value must be negative. (The tangent function is negative in that quadrant.) This makes the correct answer -0.75.

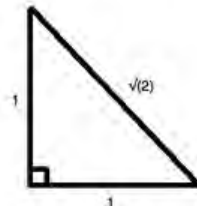
59. What is the area of an isosceles right triangle that has a hypotenuse of length 18cm?

→ A. 40cm^2
B. 81cm^2
C. 121cm^2
D. 324cm^2

Based on the information given, you know that your triangle looks as follows:



This is a 45-45-90 triangle. Recall your standard 45-45-90 triangle:



You can set up the following ratio between these two figures: $\frac{x}{1} = \frac{18}{\sqrt{2}}$

Now, the area of the triangle will merely be $\frac{1}{2}x^2$ (since both the base and the height are x). For your data, this is:

$$A = \frac{1}{2} \cdot \frac{18}{\sqrt{2}} \cdot \frac{18}{\sqrt{2}} = \frac{18 \cdot 18}{4} = 9 \cdot 9 = 81 \text{ cm}^2$$





MATHEMATICS TEST: ANSWER KEY

60. Suppose $c > 0$.

To obtain the graph of $y = f(x) + c$, shift the graph $y = f(x)$ a distance of c units in which of the following directions?

- **E.** Upwards
F. Downwards
G. Left
H. Right

There are four shifts of the graph $y = f(x)$:

$y = f(x) + c$ shifts the graph c units upwards.

$y = f(x) - c$ shifts the graph c units downwards.

$y = f(x + c)$ shifts the graph c units to the left.

$y = f(x - c)$ shifts the graph c units to the right.



READING TEST: ANSWER KEY

1. The first line of the second paragraph (lines 13–22) can be considered surprising because _____.
- A. the writer does not support novelists, despite the fact that she is a novelist
 - B. the narrator starts using first-person perspective whereas the first paragraph seemed to use third-person perspective
 - C. according to the narrator, novelists and their work are typically highly regarded, but the narrator opposes this view
 - D. the narrator strongly espouses one view of novels before the dash before reconsidering the view after the semicolon
2. The first paragraph serves to _____.
- E. introduce a new topic
 - F. set forward an argument
 - G. continue a story
 - H. describe a scene

The first line of the second paragraph is as follows:

“Yes, novels; for I will not adopt that ungenerous and impolitic custom so common with novel-writers, of degrading by their contemptuous censure the very performances, to the number of which they are themselves adding—joining with their greatest enemies in bestowing the harshest epithets on such works, and scarcely ever permitting them to be read by their own heroine, who, if she accidentally take up a novel, is sure to turn over its insipid pages with disgust.”

This is a surprising line because of the shift in perspective that takes place between the first and second paragraphs. The first paragraph seems to use third-person perspective, describing “the progress of the friendship between Catherine and Isabella” without using “I”; however, at the start of the second paragraph, the narrator interjects into the story to convey her strong opinions about how novelists should be treated with more respect, especially in comparison to other types of writers, who (according to the narrator) get more respect than they deserve. None of the other answer choices are supported by the passage.

The first paragraph tells the reader how Catherine and Isabella got to be better friends and lists some of the activities they participated in together. Thus, we can say that the first paragraph serves to “continue a story,” as the passage appears to have been taken from a story in which the reader is expected to be familiar with Catherine and Isabella already; they are not being introduced for the first time here in the context of the entire work. The passage does not support any of the other answer choices. The second paragraph introduces a new topic and sets forward an argument, arguably a “counter-argument” to the common complaints about novelists, but the first paragraph does not do these things, and neither does it “describe a scene,” as it describes multiple activities that Catherine and Isabella do together, not just a single scene in detail.



READING TEST: ANSWER KEY

3. At the end of the passage, the narrator presents two hypothetical scenes: one in which a hypothetical reader is _____, and another in which the same reader is _____.

- A. making excuses for reading a novel . . . highlighting the fact that she was reading the *Spectator*
- B. enjoying reading a well-written novel . . . unable to get through a poorly-written novel
- C. engrossed in a novel . . . unable to get through the *Spectator's* dense language
- D. recommending a novel to a friend . . . recommending that a friend read the *Spectator*

4. By “literary corporation,” underlined in the third paragraph (line 31), the narrator means _____.

- E. a writer’s guild
- F. the *Spectator*
- G. the publishing house with which the writer works

- H. novelists

5. Based on the way in which it is used in line 47, the underlined word “cant” means _____.

- A. question
- B. refrain
- C. reasoning
- D. emphasis

The passage concludes with the narrator comparing two hypothetical scenes: the first begins without warning when the narrator says, “‘And what are you reading, Miss—?’ ‘Oh! It is only a novel!’ replies the young lady, while she lays down her book with affected indifference, or momentary shame.” The second begins in the last paragraph, when the narrator says, “Now, had the same young lady been engaged with a volume of the *Spectator*, instead of such a work, how proudly would she have produced the book, and told its name.” From these excerpts, we can tell that the narrator presents two hypothetical scenes: “making excuses for reading a novel . . . highlighting the fact that she was reading the *Spectator*.”

Let’s examine the line in which the narrator uses the phrase “literary corporation”: “Although our productions have afforded more extensive and unaffected pleasure than those of any other literary corporation in the world, no species of composition has been so much decried.” From the context of the previous paragraph, you can see that by “literary corporation,” the narrator is referring to novelists.

Let’s consider the sentence in which the narrator uses the word “cant”: “Such is the common cant.” Well, that doesn’t tell us very much; let’s consider some sentences around this one: “‘I am no novel-reader—I seldom look into novels—Do not imagine that I often read novels—It is really very well for a novel.’ Such is the common cant.” This tells us a bit more. The narrator is here providing us example instances of the common things people say about novels before declaring that those are “the common cant.” Most of the provided statements are declarative, not questions, so “question” can’t be the correct answer. The statements also don’t tell us why the speakers don’t read novels, so “reasoning” can’t be correct either. None of the statements make promises, so “promise” can also be discarded as a potential answer choice. This leaves us with “emphasis” and “refrain.” “Emphasis” doesn’t make that much sense, and given that the examples provided are quotations, “refrain” is the best answer, as it specifically means *repeated statements, often in the context of a poem or song*.



READING TEST: ANSWER KEY

6. According to the narrator, most novelists are _____.

E. morbid

F. haughty

→ G. self-deprecating

H. defensive when their work is criticized

7. Which of the following can we infer, based on the passage?

→ A. *Belinda*, *Cecilia*, *Camilla*, and the *Spectator* were familiar to the author's intended audience.

B. The *History of England* was rarely abridged.

C. The *Spectator* often published excerpts from new novels to drum up support for them.

D. Milton, Pope, Prior, and Sterne were rarely referenced in the author's era.

According to the narrator, most novelists are self-deprecating. One can see that this is her opinion in the following statement from paragraph two: "Yes, novels; for I will not adopt that ungenerous and impolitic custom so common with novel-writers, of degrading by their contemptuous censure the very performances, to the number of which they are themselves adding—joining with their greatest enemies in bestowing the harshest epithets on such works." To paraphrase the narrator here, she is saying that she won't join with other novelists in criticizing novels along with other people. If most novelists criticize novels even while writing them, they are best called "self-deprecating," which means *highlighting one's own faults or telling jokes that make the person telling them look bad*.

Let's consider each of these answer choices to figure out which one is correct.

B - This is not true; the narrator only mentions the *History of England* in the third paragraph, where she references "the nine-hundredth abridger of the *History of England*." This tells us that the *History of England* was often abridged.

C - This is not true; the narrator explicitly contrasts the *Spectator* with novels and never once suggests that the *Spectator* published novel excerpts.

D - The passage does not support this inference, as when the narrator references Milton, Pope, Prior, and Sterne in the third paragraph, she does so in a way that assumes that the reader recognizes those writers; she doesn't explain who they are whatsoever.

This leaves us with the correct answer, A: "*Belinda*, *Cecilia*, *Camilla*, and the *Spectator* were familiar to the author's intended audience." The narrator uses *Belinda*, *Cecilia*, and *Camilla* as examples of a novel in the hypothetical situation she constructs at the end of the third paragraph, and the *Spectator* as an example of a widely-praised but (in her opinion) undeserving work in the contrasted scene in the last paragraph. She does not take the time to introduce these works and publication to her readers, but instead assumes that they will be familiar with them. Thus, it's reasonable to assume that they were familiar to her audience.



READING TEST: ANSWER KEY

8. Which of the following complaints does the narrator NOT make against papers published in the *Spectator* in the passage's last paragraph?

- E. They are often too wordy and verbose.
- F. They often involve unlikely situations.
- G. Their characters are often not convincingly realistic.
- H. The subject matter of their dialogue is often notably outdated.

The narrator complains about the *Spectator*'s features in the passage's last paragraph, stating that "the substance of its papers so often [consists] in the statement of improbable circumstances, unnatural characters, and topics of conversation which no longer concern anyone living; and their language, too, [is] frequently so coarse as to give no very favorable idea of the age that could endure it."

Which specific complaints can we draw out of this invective? Well, the narrator complains about its papers involving "the statement of improbable circumstances," so we can ignore the answer choice "They often involve unlikely situations." The next thing the narrator mentions is "unnatural characters," so we can ignore the answer choice "Their characters are often not convincingly realistic." After this, the narrator mentions "topics of conversation which no longer concern anyone living," so we can ignore the answer choice "The subject matter of their dialogue is often notably outdated." Finally, the narrator complains that "their language . . . [is] frequently so coarse as to give no very favorable idea of the age that could endure it," so she definitely complains that "Their language is too rough." This means that the only feature the narrator does not complain about is that the *Spectator*'s papers "are often too wordy and verbose," making this the correct answer.

9. Which of the following best summarizes the passage?

- A. The narrator abandons the story of Catherine and Isabella to argue that novelists deserve more money and fame for their work.
- B. The narrator interrupts the story of Catherine and Isabella to defend the worth of novels.
- C. The narrator disrupts the story of Catherine and Isabella to argue that reading the *Spectator* is a waste of time.
- D. The narrator decries types of literature that she feels to be inferior to novels.

In order to "best describe the passage," an answer choice needs to relate to each of its parts. This means that it needs to adequately capture the sudden shift from the first paragraph to the latter three. This means that the answer choices "Catherine and Isabella strengthen their friendship" and "The narrator decries types of literature that she feels to be inferior to novels" cannot be correct, as the first only refers to the first paragraph and the second to an aspect of the rest of the passage. The rest of the answer choices are all similar in beginning with something indicating that the narrator disrupts the story she is initially telling in order to do something. Is that something "to defend the worth of novels," "to argue that reading the *Spectator* is a waste of time," or "to argue that novelists deserve more money and fame for their work." The latter two of these listed answers only capture part of the argument that the narrator makes over the course of the passage's last three paragraphs, but the answer choice "The narrator interrupts the story of Catherine and Isabella to defend the worth of novels" relates to the shift between the first and second paragraphs as well as adequately captures the narrator's aim in the rest of the passage, so it is the correct answer.



READING TEST: ANSWER KEY

10. We can infer that this passage was excerpted from _____.
- E. a novel
 F. an article published in the *Spectator*
 G. a book about the history of novels
 H. an opinion piece published in a newspaper

11. Based on the passage, another word for “beehive” is _____.
- A. galley
 B. contrivance
 → C. apiary
 D. apiarian

12. The first paragraph establishes all of the following EXCEPT _____.
- E. the author has faith in the devices used to stop the bee-moth
 F. bee keeping has, in some areas, become a trifling hobby
 G. the ravages of the bee-moth have dissuaded many from continuing bee keeping
 H. in hot summers the bee-moth is the worst enemy of the honey bee

13. In the third paragraph the information about Swammerdam’s name for the moth serves to _____.
- A. suggest that the bee-moth preys on other insects besides bees
 → B. show how much of a menace it has always been to bees
 C. show that Swammerdam was not scientific in his approach to bee keeping
 D. mock the moth as something feared yet destroyable

We can tell that this passage was excerpted from a novel because of the way in which the first paragraph so jarringly transitions into the rest of the passage, as well as from the fact that the narrator is a novelist.

In the second paragraph, the author says, “The bee-moth infects our Apiaries,” and in the last paragraph, “If the approach to the Apiary be observed of a moonlight evening . . .” From this, we can infer that “Apiary” is another word for “beehive,” as the bee-moth infects the hives. We can also learn from this that an “Apiarian” is *a beehive keeper or someone who keeps bees*.

The author does not believe the devices created to stop the bee-moth work, as he states in the first paragraph, “Contrivances almost without number, have been devised, to defend the bees against this invidious foe, but still it continues its desolating inroads, almost unchecked, laughing as it were to scorn, at all the so-called ‘moth-proof’ hives, and turning many of the ingenious fixtures designed to entrap or exclude it, into actual aids and comforts in its nefarious designs.” So, instead of being kept out of the beehives or killed by the traps or preventative measures, the moth instead uses them to get to the hive.

The author refers to Swammerdam’s name for the moths as “the very expressive name” and as we can see the name “bee-wolf” tells us as a reader that even in Swammerdam’s time the bee was a menace. The name, and its inclusion in the passage, serves to prove that the bee-moth has been long thought of as a menace to bees.



READING TEST: ANSWER KEY

14. What is the main idea of the second paragraph?

- E. The author has come up with a method for stopping some of the destruction of the bee-moth, which he will share later in the book.
- F. There is no point trying to devise ways to save bee keeping from the moth and the author thinks that any advice he gives will be of little consequence.
- G. Several people have recommended methods to prevent the attacks of the bee-moth and the author will share them.
- H. The author is uncertain as to the future of bee keeping until a pesticide is created to kill the bee-moth.

15. The point of view from which the passage is told can best be described as that of _____.

- A. a bug collector
- B. a scientific illustrator
- C. an enthusiastic bee-keeper
- D. an expert on different types of honey

16. Which of the following statements about bees is supported by the passage?

- E. They do not keep their hives in good condition.
- F. They will guard the entrance to their hives.
- G. They allow the bee-moth to enter their nests.
- H. They are not very good at caring for their young.

In the second paragraph, the author says that he would not feel confident in trying to reinstate the art of bee keeping in his country without giving some good advice as to how to stop the moth in some way, and that he has come up with some methods, which he will describe later in the book. These methods do not appear in the passage apart from the reference to them in the second paragraph.

We can tell that the author is an enthusiastic bee keeper by the subject matter of the text and the intricate details he goes into. It happens to be true that the author is a reverend, but if we are only inferring his identity from the passage itself, we could not infer this, because the passage only discusses bee-keeping and doesn't mention anything that would make us think he is a reverend.

The third paragraph supports this answer where it says, "the bees that have to guard the entrances against their intrusion, will be seen acting as vigilant sentinels, performing continual rounds near this important post, extending their antenna to the utmost, and moving them to the right and left alternately. Woe to the unfortunate moth that comes within their reach!" This tells us that bees guard the entrance to their hives.



READING TEST: ANSWER KEY

17. Which of these most accurately restates the meaning of “The bee-moth infects our apiaries, just as weeds take possession of a fertile soil,” a line found in the second paragraph?
- A. The bee moth is to the bee keeper what the dandelion is to the gardener.
- B. The description of the moth is not sufficient without considering its impact on a garden.
- C. None other than gardeners and bee keepers can comprehend the devastations of the moth.
- D. The impact of the bee moth on a hive is disproportionate to that of the weed on a garden.

18. It can reasonably be inferred from the passage that _____.
- E. the moths are sluggish
- F. the bees do not guard against the moth
- G. bee-moths have only recently begun to trouble bee-keepers
- H. the author is a keen bee-keeper

19. As it is used in the passage, the underlined word “discriminate” in the third paragraph most nearly means _____.
- A. recognize
- B. distinguish
- C. compare
- D. act in a manner that is unfairly biased

The author is making a comparison by using the simile of the moth being like a weed. So, the moth has a similar effect on a hive as a weed—like a dandelion—has on a garden.

We know that someone wrote about the bee-moth in the seventeenth century and called it “the ‘bee-wolf,’” so it doesn’t seem likely that the problems these moths cause bee keepers only developed recently. We also know that the moths are quite fast from a description of their speed in the last paragraph. The author also states that the bees post “sentinels” to guard against the moths. The only thing we can really infer is that the author is a keen bee keeper, as he says in the passage that: “I have patiently studied [the bee-moth’s] habits for years, and I am at length able to announce a system of management founded upon the peculiar construction of my hives.” This tells us he has kept and studied bees for many years.

The full sentence in which the word “discriminate” appears is, “[Swammerdam] failed, however, to discriminate between the male and female [bee-moth], which, because they differ so much in size and appearance, he supposed to be two different species of the wax-moth.” We can tell that the correct synonym for “discriminate” in this context is “distinguish,” as Swammerdam did not distinguish between the male and the female bee-moth.



READING TEST: ANSWER KEY

20. One of the main points made in the underlined section of the last paragraph is _____.
- E. a critic has stated that the bee-moth takes advantage of the bees' inability to see at night
- F. it is not curious that bees defend against the bee-moth
- G. Huber believes that the bee-moths would be more successful if they entered the hive during the day
- H. the author thinks that Huber is wrong in his assertions
21. The author can be most accurately described as _____.
- A. furious and insulted.
- B. defensive and meticulous.
- C. imploring and desperate.
- D. whimsical and descriptive.
22. The author discusses a topic that he plans to pursue in future work _____.
- E. in the first sentence of the passage.
- F. in the second paragraph of the passage.
- G. in the last sentence of the passage.
- H. in the first and last paragraphs of the passage.
23. According to the author, studying phobias, obsessions, and delusions is _____, but studying dreams is not.
- A. practical
- B. possible
- C. useless
- D. easy

The end of the last paragraph tells us that Huber has said that the bee-moth and the bees are curious in their behavior, as the moths seem to know that the bee cannot see well at night and the bees are quite determined to expel the moths from their nests.

The author can certainly be called "defensive," as from the first sentence of the passage, he is attempting to justify his study as worthwhile and legitimate. We can also call him meticulous due to the careful way in which he addresses potential criticisms of his work and details how he selected with care the dreams he discusses in his work. Thus, "defensive and meticulous" is the best answer. While he is attempting to urge the reader to accept his work, "imploring and desperate" is far too strong a description to be accurate. Nothing in the passage suggests that the author is "furious and insulted," "whimsical and descriptive," or "unreliable and suspicious."

In the second paragraph of the passage, the author says, "The surfaces of fracture, which will be frequently discussed, correspond to many points of contact where the problem of dream formation informs more comprehensive problems of psychopathology which cannot be discussed here. These larger issues will be elaborated upon in the future." This is the only point in the passage where the author refers to the topic of future work, so "in the second paragraph of the passage" is the correct answer.

The author makes this comparison in the first paragraph when he says, "For, when investigated psychologically, the dream proves to be the first link in a chain of abnormal psychic structures whose other links—the hysterical phobia, the obsession, and the delusion—must interest the physician for practical reasons. The dream can lay no claim to a corresponding practical significance; however, its theoretical value is very great." Here, the author claims that studying phobias, obsessions, and delusions is practical, whereas studying dreams is "of theoretical value."



READING TEST: ANSWER KEY

24. When he uses the underlined phrase “the inevitable,” the author is referring to _____.

E. the idea that all dreams contain significant meaning
F. the gradual loss of detail in what one can remember about a dream
G. the scorn of many important psychologists upon his publication of his work on dreams

→ H. the fact that he had to publish some of his own dreams, which made him uncomfortable

25. The author of this passage is most interested in _____.

A. the physical structure of the brain
B. the ability of certain dreams to foreshadow future events

→ C. the workings of the human mind
D. the role of criticism in science

26. Based on the way in which the underlined word “informs” is used in the passage, the author is using it to mean _____.

E. requires
F. tells

→ G. influences
H. solves

The author uses the phrase “the inevitable” in the third paragraph when he states, “This was painful, but unavoidable; I had to put up with the inevitable in order to demonstrate the truth of my psychological results at all.” But what is he actually discussing at this point? To figure this out, we need to consider the preceding sentence: “On the other hand, in discussing my own dreams, I was obliged to expose more of the intimacies of my psychic life than I should like, more so than generally falls to the task of an author who is not a poet but an investigator of nature.” From this, we can correctly say that in this context, “the inevitable” refers to “the fact that [the author] had to relate some of his own dreams in his work, which made him uncomfortable.”

The author of this passage is clearly interested in studying dreams from a psychological standpoint. He does not mention the physical structure of the brain at all, nor does he make any comparison between human brains and the brains of other mammals. While he is interested in dreams, he does not discuss any ability of dreams to foreshadow future events. While he attempts to address potential flaws in his work, his primary interest is not in the role of criticism in science; it is in “the workings of the human mind.” While this is the most general answer choice, the specificity of each of the other options makes each incorrect.

The author uses the word “informs” in the following sentence, found in the second paragraph: “The surfaces of fracture, which will be frequently discussed, correspond to many points of contact where the problem of dream formation informs more comprehensive problems of psychopathology which cannot be discussed here.” Paraphrasing, the author is stating that the problem of dream formation does something to bigger problems that the author can’t talk about here. What might one problem do to bigger problems? “Requires,” “ignores,” and “tells” don’t make sense and so cannot be correct, despite the fact that “informs” can mean “tells” in other contexts. The author is not suggesting that the problem of dream formation “solves” the bigger problems he refers. This leaves us with “influences,” the correct answer.



READING TEST: ANSWER KEY

27. In the third paragraph, what does the author tell us about the omissions and substitutions he made when discussing his own dreams in the work that follow the passage?

- A. He feels that the adjusted examples would be more useful had they remained unadjusted.
- B. He is not responsible for these; his editors are.
- C. He was forced to make these, or journals would not accept his work.
- D. He doesn't think that they affect the work whatsoever, and help him save face.

28. The author has written this passage in order to _____.

- E. teach the reader how to interpret his or her own dreams
- F. respond to a specific critic who has cast doubt on his work's reliability
- G. propose a psychological experiment
- H. justify his work and address some of its limitations

29. The author could not rely upon the dreams related in scientific literature because _____.

- A. he couldn't be sure if material had been changed in or censored from them
- B. he needed to interview people himself in order to discuss their emotional reactions to their dreams
- C. not many dreams had been discussed in scientific literature, and those that had been discussed concerned a very limited number of topics
- D. The author does not give a reason for this in the passage, but says that the rest of his work explains why this is the case

Of the omissions and substitutions the author made when discussing his own dreams in the work he is describing, the author writes in the third paragraph, "To be sure, I disguised some of my indiscretions through omissions and substitutions, though I feel that these detract from the value of the examples in which they appear." Thus, while many answer choices sound reasonable, the correct answer is "He feels that the adjusted examples would be more useful had they remained unadjusted."

The author begins his work from a defensive standpoint, arguing in the first sentence that he has not "overstepped the bounds of neuropathological interest" in investigating and writing about the interpretation of dreams. He then goes on to address some of his study's limitations in the second and third paragraphs. Thus, we can say that the author's purpose in writing this passage is to "justify his work and address some of its limitations." None of the other answer choices are supported by the passage.

The author discusses how he selected dreams to analyze in the third paragraph. About dreams in scientific literature, he says, "The work itself will demonstrate why all dreams related in scientific literature or collected by others had to remain useless for my purpose." Thus, while some of the answer choices may sound plausible, the passage does not tell us precisely why the author could not use the dreams in scientific literature and does not give us enough information to allow us to claim that one of the listed reasons is in fact the actual reason why the author avoided using these reported dreams. The correct answer is thus "The author does not give a reason for this in the passage, but says that the rest of his work explains why this is the case."



READING TEST: ANSWER KEY

30. In the last sentence of the passage, the author attempts to _____.

- E. encourage the reader to read the work of a variety of psychologists
- F. get the reader to empathize with him
- G. explain why he made certain redactions to the dreams he later discusses
- H. emphasize why his work is valuable, despite its flaws

31. In Passage B, the underlined statement “I thought ten thousand swords must have leaped from their scabbards, to avenge even a look that threatened her with insult” most nearly reflects the author’s _____.

- A. misery and disdain
- B. confusion and praise
- C. arrogance and apathy
- D. shock and disappointment

32. The author’s tone in Passage B is primarily _____.

- E. optimistic and restrained
- F. condescending and apathetic
- G. celebratory and ecstatic
- H. pessimistic and admonishing

The last sentence of the passage states, “I can only express the hope that the reader of this work, putting himself in my difficult position, will show patience, and also that anyone inclined to take offense at any of the reported dreams will concede freedom of thought at least to the dream life.” Here, the author relates how he hopes the reader will receive his work, suggesting that the reader is mentioned in the correct answer. We can ignore “explain why he made certain redactions to the dreams he later discusses,” as the sentence doesn’t mention this at all—it’s a point made earlier in the last paragraph. The author is not attempting to get the reader to read the work of a variety of psychologists or to conduct his or her own scientific experiments, as neither of these points are mentioned or suggested at all. In choosing between the remaining two answer choices, “emphasize why his work is valuable, despite its flaws” and “get the reader to empathize with him,” the latter is the best answer. The author is not so much arguing for his work’s value in spite of flaws as he is attempting to get the reader to consider his situation, “putting [him- or herself] in [the author’s] difficult position.”

The author’s statement highlights his shock and disappointment that his perceptions of the French ruling class was so far off the mark. The author states that he believed the French aristocracy and people would spring to defend Marie Antoinette when the opportunity arose, and it is clear from his language that the failure of the French people to do so caused him to feel shocked and saddened. The phrase “Little did I dream” highlights the author’s feelings of shock.

The author’s tone in this passage is primarily pessimistic and admonishing. “Pessimistic” means *having a negative outlook about past, current, or future events*, and “admonishing” means *condemning or telling off*. The author’s pessimistic tone is evident throughout; one example can be found in the clause “the glory of Europe is extinguished forever.” Likewise, the author admonishes throughout the passage and really gets into his stride in the concluding paragraph, where he criticizes the characteristics of the current French nation by means of discussing their earlier, and opposite, virtues.



READING TEST: ANSWER KEY

33. Passage A compares Mary Stuart, Messalina, and Theodora to Marie Antoinette in order to emphasize that _____.

- A. Marie Antoinette is more famous
- B. Marie Antoinette has been accused more harshly
- C. The listed rulers, who were unpopular, all governed better than Marie Antoinette
- D. Marie Antoinette's current situation isn't worse than what other historic rulers have experienced

34. The quotation that ends Passage A serves what purpose in the text?

- E. It contradicts the author's argument.
- F. It provides evidence that supports one of the author's claims about Marie Antoinette.
- G. It provides a transition to a new idea discussed in the next paragraph in the larger work from which the passage is adapted.
- H. It suggests that Emperor Joseph was as unpopular as Marie Antoinette.

Passage A compares Marie Antoinette with other historical figures in this excerpt:

The popular hatred of Marie Antoinette sprang from a sound instinct. We shall never know how much or how little truth there was in those frightful charges against her, that may still be read in a thousand pamphlets. These imputed depravities far surpass anything that John Knox ever said against Mary Stuart, or that Juvenal has recorded against Messalina; and, perhaps, for the only parallel we must look to the hideous stories of the Byzantine secretary against Theodora, the too-famous empress of Justinian and the persecutor of Belisarius.

The accusations against Marie Antoinette are definitely negatively connoted—"imputed depravities" is very strong wording. We can ignore answer choice D, as the accusations against Marie Antoinette are said to be worse than those that were made against the other listed rulers. The varying famousness of the figures isn't the thing being directly compared, so A isn't correct either. While C is partially true—the listed rulers do seem to have all been unpopular—nothing is said about how they governed in relation to Marie Antoinette. The correct answer is B, that Marie Antoinette has been accused more harshly than the other listed rulers. This is the comparison being directly made in the passage.

The quotation that ends Passage A is attributed to "Emperor Joseph, the Queen's own brother": "The court of France is turned into a gaming-hell . . . if they do not amend, the revolution will be cruel." This ties into the passage's discussion of Marie Antoinette's "incredible dissipations," one of which was "insensate gambling." D isn't correct; nothing about the quotation suggests that Emperor Joseph was as unpopular as Marie Antoinette. The fact that Emperor Joseph is quoted as being concerned about Marie Antoinette's gambling suggests they disagree over it, which suggests that he might be more popular with the French people where Marie Antoinette might not be, but there is no evidence in the passage to go on. While found at the end of the passage, this quotation does not provide a clear transition to a new idea in the paragraph that follows in the larger work from which the passage is taken. Given that the rest of the passage discusses how unpopular Marie Antoinette was and to what actions and behavior this unpopularity can be attributed, A can't be correct, as this quotation is supporting the author's argument, not providing a dissenting opinion that contradicts it. This means that B is correct; the quotation supports one of the author's claims about Marie Antoinette: that she gambled a great deal.



READING TEST: ANSWER KEY

35. Based on the passage, what can the reader infer about Robespierre (underlined in Line 4 in Passage A)?

- A. The author is discussing Marie Antoinette in a piece primarily about Robespierre.
- B. Robespierre provides an example of a highly-praised figure, whereas Marie Antoinette was highly disparaged.
- C. Robespierre was related to Marie Antoinette.
- D. Robespierre greatly disliked Marie Antoinette.

36. Consider Passage A. Where did Marie Antoinette live before she came to France?

- E. Denmark
- F. Belgium
- G. Austria
- H. The Netherlands

Robespierre is mentioned in line 4 in the following way:

But history must be just; and the character of [Marie Antoinette] had far more concern in the disaster of the first five years of the Revolution than had the character of Robespierre.

This is the first line of the passage. Both Robespierre and Marie Antoinette are suggested to have something to do with the French Revolution, but it's not clear that Robespierre was a highly-praised figure. It's suggested somewhat in the indicated line as well as later in the passage that Robespierre is also somewhat blamed for precipitating the French Revolution:

We have to remember that all the revolutionary portraits are distorted by furious passion, and that Marie Antoinette may no more deserve to be compared to Mary Stuart than Robespierre deserves to be compared to Ezzelino or to Alva.

Marie Antoinette's comparison to Mary Stuart isn't a positive one, so we can infer that Robespierre's comparisons to Ezzelino and Alva aren't positive ones either.

Nothing in the passage suggests that Robespierre is related to Marie Antoinette or that he greatly disliked Marie Antoinette. The fact that the author brings up Marie Antoinette in relation to Robespierre and that the piece from which the passage is adapted is titled "Robespierre," A is the best answer.

This question asks you to pick out a particular detail from Passage A: where Marie Antoinette lived before coming to France. This detail is provided subtly in the following line, which concludes with the author's complaints against Marie Antoinette:

It is at least certain that, from the unlucky hour when the Austrian archduchess crossed the French frontier, a childish bride of fourteen, down to the hour when the Queen of France made the attempt to recross it in resentful flight one and twenty years afterwards, Marie Antoinette was ignorant, unteachable, blind to events and deaf to good counsels, a bitter grief to her heroic mother, the evil genius of her husband, the despair of her truest advisers, and an exceedingly bad friend to the people of France.

Look at that first part of the sentence: the author initially refers to Marie Antoinette as "the Austrian archduchess." We can tell that this phrase indicates Marie Antoinette because of the way it parallels "the Queen of France" later in the sentence. The author's use of the phrase "Austrian archduchess" tells us that Marie Antoinette lived in Austria before coming to France.



READING TEST: ANSWER KEY

The following questions ask about both Passages A and B.

37. The author of Passage A _____ Marie Antoinette, while the author of Passage B _____.
- A. is critical of . . . idealizes her
 B. is biased in favor of . . . bemoans her loss
 C. detests . . . reluctantly admits that she had a few good characteristics
 D. praises . . . disparages her
38. Which of the authors considers Marie Antoinette's life in the context of history?
- E. The author of Passage A
 F. The author of Passage B
 → G. Both authors
 H. Neither author

How does the author of Passage A feel about Marie Antoinette? While he does say that we can't completely trust historical pamphlets complaining about her behavior because they are not objective, he also states later in the passage that "Marie Antoinette was ignorant, unteachable, blind to events and deaf to good counsels, a bitter grief to her heroic mother, the evil genius of her husband, the despair of her truest advisers, and an exceedingly bad friend to the people of France." This is not a good portrayal of her. In contrast to this, the author of Passage B idealizes her in extremely positive terms, calling her a "delightful vision" and saying, "I saw her just above the horizon, decorating and cheering the elevated sphere she had just begun to move in, glittering like the morning star full of life and splendor and joy." Based on the author's distinct contrast in opinions, A is the best answer. The author of Passage A is critical of Marie Antoinette, while the author of Passage B idealizes her. One way to make this question easier to parse is to consider each answer choice and figure out whether each part of it is positively or negatively connoted. Notes might look like this:

- A. - / +
 B. + / +
 C. -! / -
 D. + / -

Our analysis tells us that the first column (the answer choice associated with the author of Passage A) should be negative (-), and that the second column (the answer choice associated with the author of Passage B) should be positive (+). Answer choice A is the only one that fits these criteria.

The author of Passage A considers Marie Antoinette's life in the context of history, whereas the author of Passage B presents only first-hand anecdotes. Passage A begins with historical context ("But history must be just . . ."), compares Marie Antoinette to several historical figures, mentions historical evidence ("those frightful charges against her, that may still be read in a thousand pamphlets") and even analyzes the reliability of that evidence to try and present an objective view of Marie Antoinette: ("We have to remember that all the revolutionary portraits are distorted by furious passion, and that Marie Antoinette may no more deserve to be compared to Mary Stuart than Robespierre deserves to be compared to Ezzelino or to Alva.") The author of Passage B offers no historical comparisons or textual evidence to support his claims, which are all based on his own personal but notably distant interaction with Marie Antoinette.



READING TEST: ANSWER KEY

39. If presented with Passage B, the author of Passage A would most likely _____.

- A. point out that Passage B's argument is unsubstantiated opinion
 B. agree with Passage B
 C. refuse to acknowledge that Marie Antoinette was well-liked due to being unable to offer evidence to the contrary
 D. accuse the author of Passage B of building his argument on evidence from unreliable sources

40. A historian is researching how the French population felt about Marie Antoinette in the period leading up to the French Revolution. Which aspect of which passage would this historian find most relevant?

- E. Passage A's discussion of historical pamphlets
 F. Passage A's quotation from Emperor Joseph
 G. The third paragraph of Passage B
 H. Passage B's anecdote about the last time the author saw Marie Antoinette

Imagine the authors of these two passages met each other and the topic of Marie Antoinette came up in conversation. Do you see them getting along and agreeing in their opinions of her? Not at all. We can immediately ignore answer choice B. Now, let's focus on the reaction of the author of Passage A to Passage B. Would he refuse to acknowledge that Marie Antoinette was well-liked because he would be unable to contradict it with evidence? No, that's not correct; the author of Passage A provides some evidence that Marie Antoinette was widely disliked by the French people in the form of mentioned pamphlets and direct quotations. So, would the author of Passage A accuse the author of Passage B of using evidence from unreliable sources (D) or say that Passage B is based on unsubstantiated opinion (A)? In other words, would he say that Passage A's sources are unreliable, or that there aren't any sources used? The latter is correct; the author of Passage B says he saw Marie Antoinette sixteen or seventeen years ago, and this firsthand eyewitness account seems reliable, but the argument that Passage B actually makes isn't substantiated by any other sources.

Time for some inferences! Which of these answer choices is most relevant to figuring out how the French population felt about Marie Antoinette before the French Revolution? Passage B doesn't mention the French population at all; it consists just of the author's opinion and reactions to historical events. Given this, we can ignore answer choices C and D. So, we know that a historian would be more interested in Passage A, but which specific part of Passage A: the quotation from Emperor Joseph or its discussion of historical pamphlets? Context says the pamphlets would be more relevant:

The popular hatred of Marie Antoinette sprang from a sound instinct. We shall never know how much or how little truth there was in those frightful charges against her, that may still be read in a thousand pamphlets.

Note how the author specifically leads into his discussion of these pamphlets by mentioning "the popular hatred of Marie Antoinette." His use of the word "popular" tells us that this "hatred" (and by extension, the pamphlets he mentions immediately afterward) have to do with how the French population felt about Marie Antoinette before the French Revolution. A is the correct answer.





SCIENCE TEST: ANSWER KEY

1. Diabetes is a disease where a person may need to take insulin to help the body cells take up blood sugar. According to the passage, when should a person with diabetes take insulin?

→ A. 1 hour before eating
 B. 5 hours before eating
 C. 3 hours before eating
 D. A person with diabetes should not take insulin.

2. Who or what was the “control” group in Experiment 2?

→ E. Participant A
 F. Participant B
 G. 15 grams of glucose
 H. Insulin levels

3. Experiment 1 is repeated. The team of endocrinologists also monitors glucagon levels of the participants over time. What would the graph of average glucagon levels over time look like?

A. Linear increase
 B. Exponential increase
 → C. Inverse of insulin graph
 D. Horizontal line

Figure 1 depicts the average blood glucose and insulin concentrations for participants over a 5 hour period. At time 0, participants have ingested the tube of glucose. Figure 1 shows that glucose levels increase one hour after eating. Insulin levels increase two hours after eating.

The question clearly states that an individual with diabetes needs insulin, so one answer choice is already eliminated. Since there is a delay between eating, glucose level increase, and insulin increase, an individual would need to have insulin in their system before eating.

One hour before eating is a reasonable time to take the insulin. Five hours before eating is much too early and may cause an unhealthy increase in blood sugar levels.

The control group for an experiment is an individual(s) that is(are) not exposed to the testing criteria. In Experiment 2, the control group was the healthy individual. Therefore, the correct answer is Participant A.

The passage states that insulin and glucagon have opposing functions. Insulin decreases blood sugar levels and glucagon increases blood sugar levels. Therefore, it is most likely that the graph of glucagon would resemble the inverse of the insulin graph.





4. At what time after eating would you expect the highest blood sugar level?

→ E. 1 hour
 F. 2 hours
 G. 3 hours
 H. 4 hours

5. If you were an endocrinologist, what would you recommend to Participant B?

A. Take glucagon injections to increase blood sugar levels
 B. Take insulin injections to increase blood sugar levels
 → C. Take insulin injections to decrease blood sugar levels
 D. Take glucagon injections to increase blood sugar levels

6. Experiment 2 is repeated. Instead of insulin levels, Participant A's glucagon levels are recorded. What would you expect Participant A's blood glucose level to be after 1 hour?

→ E. $130 \frac{mg}{dl}$
 F. $90 \frac{mg}{dl}$
 G. $60 \frac{mg}{dl}$
 H. $200 \frac{mg}{dl}$

This question involves graph interpretation. According to Figure 1, the greatest increase in blood sugar occurs after 1 hour. With the information that the endocrinologists began recording data immediately after participants consumed a tube of sugar, you can expect the greatest increase in blood sugar 1 hour after eating.

The descriptions for Experiment 1 and Experiment 2 specify the requirements for participation. Experiment 1 included all healthy individuals while Experiment 2 included one healthy and one unhealthy participant. Comparing the graphs of glucose concentration from Experiment 1 and Experiment 2, the graph of Participant A's concentration most closely resembles the graph from Experiment 1. Therefore, it is most likely that Participant B has an endocrine disease.

The graph of Participant B's glucose concentration is above average. Therefore, an endocrinologist would recommend Participant B take medication to lower sugar levels. The hormone which lowers blood sugar level is insulin.

By recording glucagon levels instead of insulin levels, there should not be a significant change to the blood sugar levels. Therefore, it would be expected that the blood sugar levels remain the same at $130 \frac{mg}{dl}$ after 1 hour.





7. Which of the following best states the basis for the belief of Scientist 1?
- A. The Moon was formed from the destruction of another planet.
 - B. The Moon was formed from a broken-off piece of the Earth's mantle.
 - C. The Moon was pulled into orbit with the Earth.
 - D. The Moon has the exact same rock composition as the Earth.

8. When it comes to the Moon, both scientists agree that:
- E. the Moon was formed about 4.5 billion years ago
 - F. the Moon takes twenty-seven days to complete an orbit
 - G. the Moon orbits around the Earth
 - H. All of the other answers are correct.

9. What is the main conflicting viewpoint between Scientist 1 or Scientist 2?
- A. Scientist 1 believes that the Moon formed from pieces of the Earth, while Scientist 2 believes the Moon was formed from pieces of the Earth as well as pieces of another planet.
 - B. Scientist 1 believes the Moon was formed from pieces of the Earth as well as pieces of another planet, while Scientist 2 believes that the Moon formed from pieces of the Earth.
 - C. Scientist 1 believes the Moon was formed from debris pulled into Earth's orbit, while Scientist 2 believes the Moon was formed from pieces of the Earth as well as pieces of another planet.
 - D. Scientist 1 believes that the Moon formed from pieces of the Earth, while Scientist 2 believes the Moon was formed from the collision of two planets that orbited Earth.

Scientist 1 believes that "a part of the formation separated from Earth and became the moon." Further, this piece was taken from the Earth's mantle as discussed in the sentence "the rock densities of the moon are similar to the rock densities of the Earth's mantle."

Most of the answer choices list information that is factual stated in the first paragraph. "The Moon orbits around the Earth" is a piece of information that can be inferred to be true after reading both scientist's beliefs.

After reading each Scientist's viewpoint, it is clear Scientist 1 believes the Moon was formed from just the Earth, while Scientist 2 states that the "collective piece" formed from the Earth and another planet created the Moon.





10. What do both the viewpoint of Scientist 1 and the viewpoint of Scientist 2 have in common?
- E. Both agree that pieces of the Earth were used in the formation of the Moon.
- F. A small planet collided with the Earth prior to the existence of the Moon.
- G. It is unlikely, but possible, that the Moon and Earth formed separately.
- H. They both agree that the Moon should have a comet-like elliptical orbit.
11. If research concluded that the Moon's composition was the same as the Earth's composition, which viewpoint would this support?
- A. Scientist 1's
- B. Scientist 2's
- C. Both Scientist 1's and Scientist 2's
- D. Neither Scientist 1's nor Scientist 2's
12. According to the data in Study 1, as density increases, what happens to the velocity of sound?
- E. It increases
- F. It decreases
- G. It either increases or decreases
- H. It does not change
13. According to Study 2, over what temperature interval does velocity begin to decrease as water temperature rises?
- A. 0°C to 10°C
- B. 50°C to 60°C
- C. 60°C to 80°C
- D. 80°C to 100°C

Both Scientists agree pieces of the Earth were used to create the Moon. The viewpoints differ on if the Earth was solely used as opposed to the Earth and an additional planet.

Scientist 1 believes the Moon was created solely from the Earth. This would be supported if the composition of the Moon was the same as the Earth. Scientist 2 believes the Earth and another planet merged to create the Moon; therefore Scientist 2 would want to see data showing the Moon had some of Earth's composition, but not identical.

The velocity of sound in different media is listed in the third column of Table 1. Density of the media increases as you move down the column. Velocity increases along with density except between iron and lead. This means that no direct relationship between density and velocity can be drawn; the velocity of sound can either increase or decrease as density increases.

Table 2 lists velocity of sound in its third column.

As you move down the column, each value corresponds to increasing temperatures. The velocity values increase over each interval until they drop from $1555 \frac{m}{s}$ to $1543.05 \frac{m}{s}$. This decrease in velocity occurs between and 100°C according to the corresponding values in the first column of the table.





14. Assume that density of a substance is the only contributing factor to velocity of sound through that substance. If the students' hypothesis in Study 1 is correct, what might they have predicted for the velocity of sound through lead? (Assume all other values in Table 1 remained the same.)

E. $1300 \frac{m}{s}$

F. $4200 \frac{m}{s}$

G. $5100 \frac{m}{s}$

→ H. $6500 \frac{m}{s}$

15. Which study provides stronger evidence against the students' prediction and why?
- A. Study 1, because using different media shows that velocity increases as density increases
- B. Study 1, because using different media shows that velocity increases as density decreases
- C. Study 2, because using the same medium shows that velocity increases as density decreases
- D. Study 2, because using the same medium shows that velocity decreases as density decreases

16. According to Study 2, water at which of the following temperatures yields the greatest velocity of sound?

E. 0°

F. 60°

→ G. $80^{\circ}C$

H. $100^{\circ}C$

The students hypothesized that velocity of sound through a substance is directly dependent upon the density of that substance. In other words, as density increases, velocity of sound also increases. Because lead is the densest substance in Table 1, only velocities greater than $5130 \frac{m}{s}$ (the velocity through iron) would satisfy the hypothesized relationship between velocity and density.

The students in the passage hypothesized that increased density would result in increased velocity of sound through that medium. Study 2 provides the most evidence to the contrary by showing decreasing densities linked to increasing velocities.

Velocities of sound in water can be found in the third column of Table 2. The highest velocity listed in the table is $1555 \frac{m}{s}$. Temperatures are listed in the first column; this sound velocity occurs in water that is $80^{\circ}C$.





17. In Study 1, if the students were to double the length of the samples of media, what would happen to the velocity of sound through those media?

→ A. It would remain constant.
B. It would increase.
C. It would decrease.
D. More information is necessary.

18. A bacterium that stains red in a Gram stain and requires lactose to grow is most likely to cause what disease?

E. Sinus infection
→ F. Urinary tract infection
G. Pneumonia
H. Diarrhea

19. A patient with a sinus infection goes to the doctor and a culture of mucus is taken to determine the species of bacteria causing the disease. The technician appropriately selects the Chocolate Agar base but forgets to add Factor X. What effect will this have on the bacterial culture?

→ A. No growth
B. Normal growth
C. Excess growth
D. Cannot be predicted

20. A patient with watery diarrhea comes to the doctor after eating spoiled food at a family event. The bacterium most likely responsible for causing the diarrhea would require what growth medium or media?

E. Chocolate agar
→ F. Lactose
G. Lactose and chocolate agar
H. Sheep's blood

According to the passage, velocity of sound is found by dividing distance by time. Any increase in distance traveled would cause the time required for travel to increase proportionally. In this manner, velocity would remain constant if the students were to double the length of media samples in Study 1.

The question asks us to use the provided table to choose the disease most likely caused by an organism with a specific growth media and Gram stain. We can see that a pink-staining bacterium that requires lactose is most likely *E. coli*, which causes a urinary tract infection.

This question asks us to predict the result of an experiment involving selective growth media. Using the chart provided, we can see that chocolate agar requires two factors to aid the growth of bacteria. Thus, we can infer that without these two additions, the bacteria will not be able to grow. This question tests inference based on scientific principles listed in the passage and the chart provided.

According to the table, diarrhea is caused by *B. cerrius*. We can also see that *B. cerrius* requires lactose to grow; thus, lactose would need to be supplemented in the growth medium for this bacterium to grow. We do not need to add additional medium, like chocolate agar, because *B. cerrius* does not cause other diseases according to the information presented in the passage.





21. A technician stains a slide using the gram stain procedure and sees nothing upon looking under the microscope. Which growth medium could be required to determine if a particular bacterium is causing disease?

A. Chocolate agar
B. Lactose

→ C. Sheep's blood
D. None of the other answers

22. Which of the following bacteria is most likely to produce urease?

E. *B. cerrius*
F. *E. coli*

→ G. *H. pylori*
H. *S. pneumoniae*

23. Assume a new growth medium was created that contained a mixture of sheep's blood, lactose, and Factors X and V (chocolate agar) but was also supplemented with penicillin. What type of bacteria could likely be cultured on this new medium?

A. *B. cerrius*

→ B. *E. coli*
C. *H. pylori*
D. *S. pneumoniae*

24. How did the procedures of the two experiments differ?

E. In Experiment 1, temperature was varied, while in Experiment 2, it was held constant.

F. In Experiment 1, temperature was held constant, while in Experiment 2, temperature was varied.

G. In Experiment 1, several solvents were tested, while Experiment 2 tested several solutes.

→ H. In Experiment 1, several solutes were tested, while Experiment 2 tested several solvents.

This question describes a technician that has completed a Gram stain, yet sees nothing under the microscope. Using the provided table, we can see that one bacterium in particular, *H. pylori*, does not Gram stain. We could use sheep's blood to determine if *H. pylori* is present.

Using the last paragraph as a guide, we can see that urease is able to combat the acidic environment of the stomach. Thus, we could predict that *H. pylori*, which causes stomach infections, would most likely produce urease to combat the acidic environment of the stomach.

The question asks us to identify the bacteria that would most likely grow on the selective medium that includes penicillin. The information we need to answer this question is contained in the last paragraph, where we see that *E. coli* is the only bacteria that is resistant to penicillin via the production of beta-lactamase.

This question requires a careful reading of the text, or a good understanding of the definition of solutes and solvents. We can eliminate the choices dealing with temperature, because a cursory glance at Table 1 and Table 2 reveals that temperature was varied in both experiments.

A solvent is the substance present in greater amount. In this passage, all our solvents are liquids.

A solute is the substance present in smaller amount. In this case, our solutes are solids.

In Experiment 1, several solids were tested, while in Experiment 2, only one solid was tested: NaCl. The correct answer reflects this fact: "In Experiment 1, several solutes were tested, while Experiment 2 tested several solvents."



25. In Experiment 2, which of the following combinations of temperature and solvent dissolved the greatest number of moles of HCl?

- A. Water at 10°C
- B. Ethanol at 30°C
- C. HCl at 50°C
- D. Methanol at 50°C

26. Which of the following correctly ranks the solutes from Experiment 1 in decreasing order of solubility in water at 50°C?

- E. KNO_3 , CuSO_4 , AgCl , NaCl
- F. NaCl , KNO_3 , CuSO_4 , AgCl
- G. AgCl , NaCl , KNO_3 , CuSO_4
- H. CuSO_4 , NaCl , KNO_3 , AgCl

27. Which of the following best explains the relationship between temperature and solubility of AgCl ?

- A. As temperature decreases, solubility of AgCl increases.
- B. As temperature increases, solubility of AgCl increases.
- C. As temperature decreases, solubility of AgCl decreases.
- D. As temperature increases, solubility of AgCl increases, then decreases.

To find the correct answer to this question, we need to look for it in Table 2. We are asked to find the greatest number of moles of NaCl that dissolved. Begin by searching in the last column until you find the largest value. Then, look at the temperature and solvent corresponding to that value and see if any answer choices match what you found.

In this case, the largest number of moles dissolved is 0.12029. This corresponds with HCl at 50°C, which is in fact one of our answer choices.

The introduction explains that solubility is a measure of how many moles of a substance can dissolve in a given volume of another substance. This is a very important clue that tells us we need to focus on the “moles added” column rather than the “grams added” column. We need to rank our substances in decreasing order, so we need start with the one with the highest solubility at 50°C.

The number of moles dissolved at 50°C are as follows:

NaCl : 0.04004

KNO_3 : 0.03412

CuSO_4 : 0.1603

AgCl : 1.6047×10^{-5}

So the correct order is NaCl , KNO_3 , CuSO_4 , AgCl .

This question doesn't tell us where to look, but it tells us we need to be concerned with solubility of AgCl . Experiment 1 is the only place where AgCl is mentioned, so begin there. In Table 1, the entries for moles of AgCl are highest at 10°C, and lower for increasing temperature. This means that temperature and solubility are indirectly related; as one increases, the other decreases. Thus, our correct answer is, “As temperature decreases, solubility of AgCl increases,” because our solubility is highest at lower temperatures.





28. Suppose the scientists conducted a third experiment in which they dissolved NaCl in 100mL of methanol at 100°C. Which of the following might have been the number of grams dissolved?

E. 3.42g

F. 4.24g

G. 4.56g

→ H. 6.46g

29. On which of the following points would the students most likely disagree?

- A. Two children of different masses swinging on identical swings would show the exact same swinging period.
- B. Two children of identical masses swinging on swings of different length would show different swinging periods.
- C. A child swinging at a height of one meter would show the same period of swinging as a child swinging at a height of two meters.
- D. Length of a pendulum is not important to consider when measuring period.

This question requires some careful reading and extrapolation. We want to find the number of grams dissolved in 100mL of methanol at 100°C. Experiment 2 is the only place in which methanol is mentioned, so start there.

In Experiment 2, we need two details to guide us:

1) 50mL of each liquid was used.

2) The highest temperature tested for methanol was 50°C, and at that temperature, 4.56g dissolved.

So, the scientists had 4.56g at 50°C in 50mL. We want to find how many grams will dissolve at 100°C. We know that as temperature is increased, more grams of NaCl dissolve in the same amount of methanol.

So, we can eliminate any answers lower than or equal to 4.56g. This leaves us with one answer choice, the correct one: 6.46g.

The correct answer is “Two children of different masses swinging on identical swings would show the exact same swinging period.” According to the passage, only Student 2 would agree that the children would show the same period of swinging, while Student 1 would argue that they would differ. Gravitational force was not mentioned in the passage (although it is a true statement) and both students agree that the length is important to consider.





- 30.** During an earthquake, several chandeliers in a mansion begin to swing. Some of the chandeliers are quite small while others, such as one found in the dining room, are very large. However, all of the chandeliers hang the same exact distance from the ceiling. What would the two students predict would happen?

- **E.** Student 1: Every chandelier would display a different period of swinging
Student 2: Every chandelier would have the same exact period of swinging
- F.** Student 1: Every chandelier would have the same exact period of swinging
Student 2: Every chandelier would display a different period of swinging
- G.** Both Student 1 and Student 2: Every chandelier would have the same exact period of swinging.
- H.** Both Student 1 and Student 2: Every chandelier would display a different period of swinging

- 31.** Before analyzing the data collected, the two students go out into a local playground and use the swing set to test their hypotheses in an approximate manner. Student 1 and Student 2 are almost exactly the same mass, so Student 2 swings wearing his backpack full of books. Both students begin swinging from the same height and swing exactly three times each in exactly twelve seconds. Whose hypothesis has been supported in this brief trial?

- A.** Student 1's
- **B.** Student 2's
- C.** Both Student 1's and Student 2's
- D.** Neither student's

What is important here is to recognize the analogy. The chandeliers are essentially pendulums—masses hanging from a certain point which are allowed to swing. The correct answer is the one in which Student 1 predicts variations in period of swinging and Student 2 predicts no variations. This comes down to their fundamental disagreement, which is that Student 1 believes the mass of a pendulum affects the pendulum's period.

The answer is Student 2's hypothesis. Since the students swung the same amount of times over the same period of time, we can extrapolate that the "period" involved in this pendulum was the same. To derive this, we need an understanding of the definition of a pendulum's period as defined by the passage. Since the students varied in mass because of Student 2's backpack, we can see that this supports the hypothesis that mass does not affect a pendulum's period.





32. According to the data provided, what would we predict would happen if an experiment compared the periods of a pendulum made of a lead weight on a meter-long cord and a pendulum made of a tennis ball on a three-meter-long cord?

- E. The period of the tennis ball pendulum would be longer than the period of the lead ball pendulum.
- F. The period of the tennis ball pendulum would be shorter than the period of the lead ball pendulum.
- G. The period of the tennis ball pendulum would be the same as the period of the lead ball pendulum.
- H. The period of the tennis ball pendulum would be different than the period of the lead ball pendulum. We do not know whether it would be shorter or longer.

33. According to the data presented, what is the apparent relationship between mass m and period T ?

- A. The two variables have a positive linear correlation
- B. The two variables have a positive non-linear correlation
- C. The two variables have a negative linear correlation
- D. The two variables are not related.

34. According to the data, what is the apparent relationship between length l and period T ?

- E. The two variables have a positive linear correlation
- F. The two variables have a positive non-linear correlation
- G. The two variables have a negative linear correlation
- H. The two variables are not related.

The table shows in its last two rows that changing the mass of the bob on a pendulum did not affect the pendulum's period. Therefore, only the length of the pendulum appears to affect the period. We can then ignore the lead ball versus the tennis ball, as the mass of each will not affect the periods of their respective pendulums. We can also see that if the length of a pendulum increases, the period of the pendulum also increases. Therefore, we would most likely predict that the period of the longer pendulum would be greater than the period of the shorter pendulum.

The correct answer is that they are not related. For this question, the most important row in the presented table is the last row, because it demonstrates that changing the mass of the bob had no effect on the period of the pendulum. Increasing the mass of the bob by 4 kilograms did not affect the pendulum's period. Note that this is done while keeping the length of the pendulum constant. Therefore, we conclude that there is no correlation between the mass of a pendulum's bob, m , and the pendulum's period, T . This is indeed true experimentally.

The correct answer is that they are positively and non-linearly related. We can ignore the change in mass because the last two rows of the chart demonstrate that changing the mass of a pendulum's bob does not affect the period of the pendulum. As we can see in the chart, once we ignore the changing mass, we see that increasing the length by a factor of four only tends to increase the period by a factor of two. This shows a non-linear but still positive correlation. Although it wouldn't be hard to discern what exactly this non-linear relationship is, all we need for this question is to know that it is non-linear.



35. If four solutions were made from household chemicals and the red cabbage indicator and resulted in the four following colors, which solution contained the most acidic chemical?

- A. Light Pink
B. Dark Pink
C. Yellow
D. Green

36. Which of the following answer choices lists the four acids lemon juice, vinegar, toilet bowl cleaner, and soda pop in order from weakest to strongest?

- E. Toilet Bowl Cleaner, Soda Pop, Lemon Juice, Vinegar
F. Vinegar, Soda Pop, Lemon Juice, Toilet Bowl Cleaner
→ G. Vinegar, Lemon Juice, Soda Pop, Toilet Bowl Cleaner
H. Toilet Bowl Cleaner, Vinegar, Soda Pop, Lemon Juice

37. The student will attempt to color in Figure 1 with the appropriate color the indicator will turn at various pHs. Which answer choice lists the colors in the correct order, from left, or low pHs, to right, or high pHs?

- A. Purple, Blue, Green, Yellow, Red, Pink
B. Pink, Red, Yellow, Green, Blue, Purple
C. Yellow, Green, Blue, Purple, Pink, Red
→ D. Red, Pink, Purple, Blue, Green, Yellow

38. What can be inferred from the results of Experiment 2?

- E. Baking soda only produces bubbles in the presence of an acid.
F. Baking soda only produces bubbles in the presence of a base.
G. Baking soda only produces bubbles in the presence of a chemical with a neutral pH.
H. Baking soda does not bubble in the presence of any solutions.

Figure 1 shows that the lowest pH values correlate with highest acidity. Consult Table 1 to see the corresponding pH values for the four indicator colors mentioned in the question. Light pink, which indicates a pH of roughly 2.5, is the color of the most acidic solution, as 2.5 is the lowest number correlated with a color mentioned in the question.

Figure 1 shows that the strongest acids have the lowest pH values. So, we need to list these acids from the highest pH to the lowest pH. Note that Table 1 lists the chemicals in order of increasing pH. That means the correct order of increasing acidity will list the acids in order from the bottom up on Table 1, with vinegar first, as it is the weakest acid, and toilet bowl cleaner last, as it is the strongest acid.

Note that Table 1 lists the chemicals from lowest to highest pH. Thus, we must simply list the colors that correspond with these increasing pH values in the order down the list on Table 1, starting with red, and ending with yellow.

Baking soda reacts, or produces bubbles, with lemon juice, vinegar, toilet bowl cleaner, and soda pop only. By cross-referencing Table 1, we see that these substances have a pH of less than 7, which is the definition of an acid as Figure 1 shows. Thus, we know that baking soda only reacts with acids.



39. A new indicator, called Methyl Red, is also used to test the household chemicals from Experiments 1 and 2. It is found that the indicator turns red in the presence of toilet bowl cleaner, soda pop, or lemon juice; it turns orange in the presence of vinegar; it turns yellow in the presence of the rest of the chemicals. Which of the following pH ranges most likely contain the pH value at which Methyl Red has its color transition, or pH at which the indicator will stop being red and change to yellow?

A. 0 – 2
B. 2 – 4
→ C. 4 – 6
D. 6 – 8

40. A universal indicator is a pH indicator that is a mix of several different indicators that have distinct color changes in various ranges of the pH scale in order to precisely tell the pH of any solution. While the red cabbage indicator is a good indicator for most pHs, it has one range that does not have drastic enough color changes to precisely tell the pH within this region. What pH range should a supplemental indicator have drastic colors changes in to improve the red cabbage indicator?

→ E. 2 – 4
F. 6 – 8
G. 8 – 10
H. 12 – 14

It appears that the color transition happens at the pH range around the pH of vinegar, given that the indicator in vinegar is orange, and orange is produced by a combination of red and yellow. As the pH of vinegar, according to Table 1, is 4.5, the range of 4 to 6 is the correct answer as it contains the value.

Table 1 shows that the indicator does not change much from the pH of vinegar, or 4.5, to the pH of soda pop, 2.5, since it is some shade of pink at all pH values in this range. Thus, the red cabbage indicator could be improved by adding an indicator that changes color drastically in this region. The answer choice that lists the range from 2 to 4 is the closest answer choice.

Score Conversion Tables

English Section

Raw English Score	Scaled English Score	English Score Percentile
75	36	99
74	35	99
73	34	99
72	33	98
71	32	96
70	31	94
69	30	93
68	29	90
67	28	89
65-66	27	86
63-64	26	83
61-62	25	80
59-60	24	74
57-58	23	69
55-56	22	63
52-54	21	57
49-51	20	50
45-47	19	43
42-44	18	37
39-41	17	33
37-38	16	28
35-36	15	23
33-34	14	18
31-32	13	14
29-30	12	11
26-28	11	8
23-25	10	6
20-22	9	3
17-19	8	2
14-16	7	1
11-13	6	1
9-10	5	1
7-8	4	1
5-6	3	1
3-4	2	1
0-2	1	1

Math Section

Raw Math Score	Scaled Math Score	Math Score Percentile
60	36	99
59	35	99
58	34	99
57	33	98
55-56	32	97
53-54	31	96
51-52	30	95
49-50	29	92
47-48	28	90
45-46	27	87
43-44	26	84
41-42	25	79
39-40	24	73
37-38	23	68
35-36	22	62
32-34	21	57
28-31	20	51
25-27	19	47
22-24	18	41
19-21	17	35
16-18	16	24
13-15	15	15
11-12	14	6
9-10	13	3
7-8	12	2
5-6	11	1
4	10	1
-	9	1
3	8	1
-	7	1
2	6	1
-	5	1
1	4	1
-	3	1
-	2	1
0	1	1



Reading Section

Raw Reading Score	Scaled Reading Score	Reading Score Percentile
40	36	99
39	35	99
38	34	99
37	33	97
36	32	95
35	31	93
33-34	30	90
32	29	88
31	28	85
30	27	80
28-29	26	77
27	25	74
26	24	70
25	23	66
24	22	60
22-23	21	53
20-21	20	49
19	19	41
18	18	35
17	17	30
16	16	25
15	15	19
13-14	14	14
11-12	13	10
9-10	12	7
7-8	11	3
6	10	1
5	9	1
-	8	1
4	7	1
3	6	1
-	5	1
2	4	1
-	3	1
1	2	1
0	1	1

Science Section

Raw Science Score	Scaled Science Score	Science Score Percentile
40	36	99
39	35	99
-	34	99
38	33	99
37	32	98
-	31	97
36	30	96
35	29	95
34	28	94
33	27	92
32	26	87
31	25	83
30	24	77
28-29	23	72
26-27	22	64
24-25	21	57
22-23	20	50
20-21	19	42
18-19	18	33
17	17	26
15-16	16	18
14	15	14
12-13	14	10
10-11	13	6
9	12	4
8	11	2
7	10	1
6	9	1
5	8	1
4	7	1
3	6	1
2	5	1
-	4	1
1	3	1
-	2	1
0	1	1



Calculating Your Composite Score

To find your final composite score, calculate each of the four section scores and find the average. Add all section scores together, and divide by 4. The result will be the composite score.

Composite Score Calculation	
English Scaled Score	
Math Scaled Score	
Reading Scaled Score	
Science Scaled Score	
$(\text{English} + \text{Math} + \text{Reading} + \text{Science}) \div 4 =$	$(\text{ } + \text{ } + \text{ } + \text{ }) \div 4 =$
Composite Score	

Composite Score	Composite Score Percentile
36	99
35	99
34	99
33	99
32	98
31	97
30	96
29	94
28	91
27	88
26	84
25	80
24	74
23	69
22	62
21	55
20	48
19	41

Composite Score	Composite Score Percentile
18	34
17	27
16	21
15	15
14	10
13	6
12	3
11	1
10	1
9	1
8	1
7	1
6	1
5	1
4	1
3	1
2	1
1	1

